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AN EXAMPLE FOR A DIFFERENCE BETWEEN ORDINARY (CRISP), FUZZY AND INTUITIONISTIC FUZZY SETS

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ABSTRACT: An interesting difference between the ordinary (crisp), fuzzy and intuitionistic fuzzy sets is discussed.

KEYWORDS: Fuzzy set, Intuitionistic fuzzy set.

It is well know that for each two ordinary (crisp) sets the equality

$$(A \cup B) \cap (\overline{A} \cup \overline{B}) = (\overline{A} \cap B) \cup (A \cap \overline{B}) \tag{1}$$

is valid. In [1] it is noted that (1) is not always valid for the case of fuzzy sets. Here we shall study the cases, in which (1) is valid and these ones in which (1) is not valid for the case of Intuitionistic Fuzzy Sets (IFSs, for all used notations related to IFS see [2]).

Let for a fixed universe E, A and B be two IFSs such that

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\},$$

where the functions $\mu_A, \mu_b : E \to [0,1]$ and $\nu_A, \nu_B : E \to [0,1]$ define the degrees of membership and the degrees of non-membership of the element $x \in E$, respectively, and for every $x \in E$ and let:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1,$$

$$0 \leq \mu_B(x) + \nu_B(x) \leq 1.$$

Then

$$(A \cup B) \cap (\overline{A} \cup \overline{B}) = \{\langle x, \min(\max(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))),$$

$$\max(\min(\nu_A(x), \nu_B(x)), \min(\mu_A(x), \mu_B(x)))\rangle | x \in E\} \tag{2}$$

and

$$(\overline{A} \cap B) \cup (A \cap \overline{B}) = \{\langle x, \max(\min(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x))),$$

$$\min(\max(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)))\rangle | x \in E\} \tag{3}$$

Now, we shall study the cases in which the IFSs from (2) and (3) coincide; the cases in which one of both sets is included in the other, and the cases in which these sets are not in one of the two above relations, having in mind that

$$\begin{array}{lll} A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \end{array}$$

Therefore, we must check the relations between expressions

$$Z_1 = \min(\max(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))),$$

$$Z_2 = \max(\min(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x))),$$

$$Z_3 = \max(\min(\nu_A(x), \nu_B(x)), \min(\mu_A(x), \mu_B(x))),$$

$$Z_4 = \min(\max(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))).$$

We shall construct the following table

orders between $\mu_A(x), \mu_B(x), \nu_A(x)$ and $\nu_B(x)$	relations between Z_1 and Z_2	relations between Z_3 and Z_4
$\mu_A(x) \leq \mu_B(x) \leq \nu_A(x) \leq \nu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_A(x) \leq \mu_B(x) \leq \nu_B(x) \leq \nu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_A(x) \leq \nu_A(x) \leq \mu_B(x) \leq \nu_B(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\mu_A(x) \leq \nu_A(x) \leq \nu_B(x) \leq \mu_B(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\mu_A(x) \leq \nu_B(x) \leq \mu_B(x) \leq \nu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_A(x) \leq \nu_B(x) \leq \nu_A(x) \leq \mu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_B(x) \leq \mu_A(x) \leq \nu_A(x) \leq \nu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_B(x) \leq \mu_A(x) \leq \nu_B(x) \leq \nu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_B(x) \leq \nu_A(x) \leq \mu_A(x) \leq \nu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_B(x) \leq \nu_A(x) \leq \nu_B(x) \leq \mu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\mu_B(x) \leq \nu_B(x) \leq \mu_A(x) \leq \nu_A(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\mu_B(x) \leq \nu_B(x) \leq \nu_A(x) \leq \mu_A(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\nu_A(x) \leq \mu_B(x) \leq \mu_A(x) \leq \nu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_A(x) \leq \mu_B(x) \leq \nu_B(x) \leq \mu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_A(x) \leq \mu_A(x) \leq \mu_B(x) \leq \nu_B(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\nu_A(x) \leq \mu_A(x) \leq \nu_B(x) \leq \mu_B(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\nu_A(x) \leq \nu_B(x) \leq \mu_B(x) \leq \mu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_A(x) \leq \nu_B(x) \leq \mu_A(x) \leq \mu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_B(x) \leq \mu_B(x) \leq \nu_A(x) \leq \mu_A(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\nu_B(x) \leq \mu_B(x) \leq \mu_A(x) \leq \nu_A(x)$	$Z_1 \geq Z_2$	$Z_3 \leq Z_4$
$\nu_B(x) \leq \nu_A(x) \leq \mu_B(x) \leq \mu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_B(x) \leq \nu_A(x) \leq \mu_A(x) \leq \mu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_B(x) \leq \mu_A(x) \leq \mu_B(x) \leq \nu_A(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$
$\nu_B(x) \leq \mu_A(x) \leq \nu_A(x) \leq \mu_B(x)$	$Z_1 = Z_2$	$Z_3 = Z_4$

From the table we obtain directly the validity of the following

THEOREM: For every two IFSs A and B:

$$(\overline{A} \cap B) \cup (A \cap \overline{B}) \subset (A \cup B) \cap (\overline{A} \cup \overline{B}) \tag{4}$$

The above Table shows the orders for which the equality is valid, too. It can be directly seen that inclusion (4) is valid for fuzzy sets, too.

Inclusion (4) is an example of a difference between ordinary (crisp) stes from one hand and fuzzy and IFSs from another. In [2] there are examples that equalities for fuzzy sets are transformed in (strong) inequalities in the case of IFSs.

References:

[1] Cross, V., T. Sudkamp, Similarity and Compatibility in Fuzzy Set Theory, Springer Physica-Verlag, Berlin, 2002.

[2] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.