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A NOTE ON UNION AND INTERSECTION OF INTUITIONISTIC FUZZY SETS

Kankana Chakrabarty
Ranjit Biswas
Sudarsan Nanda
Department of Mathematics
Indian Institute of Technology
Kharagpur-721302
West Bengal
INDIA.

Abstract

The notions of union and intersection of intuitionistic fuzzy sets as proposed by Atanassov is generalized in this paper. Some real life-problems are discussed in order to justify the motivation behind such attempt.

Keywords: Intuitionistic fuzzy set(IFS), union of intuitionistic fuzzy sets, intersection of intuitionistic fuzzy sets.

In their recent work [?,?] the authors have generalized the notions of union and intersection of fuzzy sets. These can be regarded as the generalization of Zadeh's definition of union/intersection due to the fact that Zadeh's definition is just a particular case of our definition.

Keeping in view that the notion of intuitionistic fuzzy set is a generalization of that of a fuzzy set, there is an analogous need of incorporating the same kind of generalization in case of intuitionistic fuzzy sets too. Consequently, in the present paper the generalization of Atanassov's notions of union and intersection of intuitionistic fuzzy sets [?] has been done. The concept of the union/intersection of two intuitionistic fuzzy sets A and B in the universe X could be treated as a particular case of union/intersection of the following two intuitionistic fuzzy sets:

(I) intuitionistic fuzzy set A in the universe X, and

(II) intuitionistic fuzzy set B in the universe Y,

where X and Y are two different universes, in general.

The utility of this kind of generalization of union and intersection is obvious because in practical situation, many problems call for such type of operations. For example, suppose

X is the set of all System Analysts of Dept-I and Y be the set of all Research Assistants of Dept-II of a company. Now if we consider the collection of all those employees of this company who are either expert system analysts of Dept-I or efficient research assistants of Dept-II, then this collection is not a set. Besides, the present definition of union of two IF-sets [?] does not support this type of union.
Naturally the question arises that if A is an IF-set (expert system analysts) of X and B is an IF-set (efficient research assistants) of Y, then what is the union of A and B.

We propose here the following definitions on union and intersection of two intuitionistic fuzzy sets.

Definition 1

Let A be an IF-set of X with the membership function μ_A and the non-membership function ν_A. Also let B be an IF-set of Y with membership function μ_B and non-membership function ν_B. The union of these two IF-sets A and B, denoted by $A\tilde{\cup}B$, is an IF-set of $X \cup Y$ with the membership function $\mu_{A\tilde{\cup}B}$ and the non-membership function $\nu_{A\tilde{\cup}B}$ defined by

$$\begin{aligned} \mu_{A\tilde{\cup}B}(z) &= \mu_A(z), \quad if\ z \in X - Y \\ &= \mu_B(z), \quad if\ z \in Y - X \\ &= max\{\mu_A(z), \mu_B(z)\}, \quad if\ z \in X \cap Y. \\ \text{and} \quad \nu_{A\tilde{\cup}B}(z) &= min\{\nu_A(z), \nu_B(z)\} \quad \forall\, z \in X \cup Y. \end{aligned}$$

Note that the union of two IF-sets as defined by Atanassov [?] is a particular case of the above defined union when $X = Y$.

Definition 2

Let A be an IF-set of X with the membership function μ_A and the non-membership function ν_A. Also let B be an IF-set of Y with the membership function μ_B and the non-membership function ν_B. The intersection of A and B, denoted by $A\tilde{\cap}B$, is an IF-set of $X \cup Y$ with the membership function $\mu_{A\tilde{\cap}B}$ and the non-membership function $\nu_{A\tilde{\cap}B}$ defined by

$$\begin{aligned} \mu_{A\tilde{\cap}B}(z) &= min\{\mu_A(z), \mu_B(z)\}, \quad \forall\, z \in X \cup Y. \\ \text{and} \quad \nu_{A\tilde{\cap}B}(z) &= \nu_A(z), \quad if\ z \in X - Y \\ &= \nu_B(z), \quad if\ z \in Y - X \\ &= max\{\nu_A(z), \nu_B(z)\}, \quad if\ z \in X \cap Y. \end{aligned}$$

It may be observed that the intersection of IF-sets as defined by Atanassov[?] is a particular case of the above defined intersection when $X = Y$.

Example 1

Let $X = \{p, q, r, s\}$ be a set and $A = \{p/(.3, .1), q/(.5, .4), r/(.6, .4), s/(.2, .7)\}$ be an IF-set of X. Also let $Y = \{m, n, p, r\}$ be a set and $B = \{m/(.5, .3), n/(.2, .2), p/(.8, .2), r/(.5, .4)\}$ be an IF-set of Y. Then,

$$\begin{aligned} A\tilde{\cup}B &= \{p/(.8, .1), q/(.5, 0), r/(.6, .4), s/(.2, 0), m/(.5, 0), n/(.2, 0)\}, \\ \text{and} \quad A\tilde{\cap}B &= \{p/(.3, .2), q/(0, .4), r/(.5, .4), s/(0, .7), m/(0, .3), n/(0, .2)\}. \end{aligned}$$

The following propositions are straightforward.

Proposition 1

For any IF-set A in X,

(I) $A\tilde{\cup}A = A$ $\quad$ $(II) A\tilde{\cap}A = A$.

Proposition 2

For any two IF-sets A and B in X and Y respectively, the following holds:

(I) $A\tilde{\cup}B = B\tilde{\cup}A$ $\quad$ $(II) A\tilde{\cap}B = B\tilde{\cap}A$.

Proposition 3

For any two IF-sets A and B in X and Y respectively, the following holds:

(I) $(A\tilde{\cup}B)^c = A^c\tilde{\cap}B^c$

(II) $(A\tilde{\cap}B)^c = A^c\tilde{\cup}B^c$.

where $(A\tilde{\cup}B)^c$ and $(A\tilde{\cap}B)^c$ are the complements of $(A\tilde{\cup}B)$ and $(A\tilde{\cap}B)$ in $X \cup Y$ respectively, A^c is the complement of A in X and B^c is the complement of B in Y.

Proof (I): Case-1: Consider $x \in X - Y$, where $\mu_A(x) = p$ and $\nu_A(x) = q$. Therefore, $\mu_B(x) = 0$ and $\nu_B(x) = 1$.

$$\begin{aligned} \text{Hence,} \quad \mu_{A^c\tilde{\cap}B^c}(x) &= min\{\mu_{A^c}(x), \mu_{B^c}(x)\} \\ &= min\{1 - p, 1\} \\ &= 1 - p. \\ &= 1 - \mu_{A\tilde{\cup}B}(x) \\ &= \mu_{(A\tilde{\cup}B)^c}(x). \end{aligned}$$

$$\begin{aligned} \text{Again,} \quad \nu_{A^c\tilde{\cap}B^c}(x) &= \nu_{A^c}(x) \\ &= 1 - q \\ &= 1 - \nu_{A\tilde{\cup}B}(x) \\ &= \nu_{(A\tilde{\cup}B)^c}(x). \end{aligned}$$

Case-2: If $x \in Y - X$, then the proof is similar to that of Case-1.

Case-3: Suppose $x \in X \cap Y$.

$$\begin{aligned} \text{Therefore,}\quad \mu_{A^c \tilde{\cap} B^c}(x) &= min\{\mu_{A^c}(x), \mu_{B^c}(x)\} \\ &= 1 - max\{\mu_A(x), \mu_B(x)\} \\ &= \mu_{(A \tilde{\cup} B)^c}(x). \end{aligned}$$

$$\begin{aligned} \text{Again}\quad \nu_{A^c \tilde{\cap} B^c}(x) &= max\{\nu_{A^c}(x), \nu_{B^c}(x)\} \\ &= 1 - min\{\nu_A(x), \nu_B(x)\} \\ &= \nu_{(A \tilde{\cup} B)^c}(x). \end{aligned}$$

Hence proved.

Proof (II): Proof is similar to that of (I).

CONCLUSION: The union/intersection of two IF-sets from two different universes is very common in many cases of everyday life. Hence, in this work, we generalize Atanassov's notion of union and intersection of IF-sets. With these notions, it will be possible to find the union or intersection of IFS's obtained from different universes of discourse.

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Editor's remark: Obviously, the authors do not know about the paper
Atanassov K., Intuitionistic fuzzy sets over different universes, Second Scientific Session of the `Mathematical Foundation of Artificial Intelligence` Seminar, Sofia, March 30, 1990, Preprint IM-MFAIS-1-90, Sofia, 1990, 6-9.
where similar operations are defined. The third case of the formulas is not discussed there.

Graphical representation of the union of two IF – sets:-

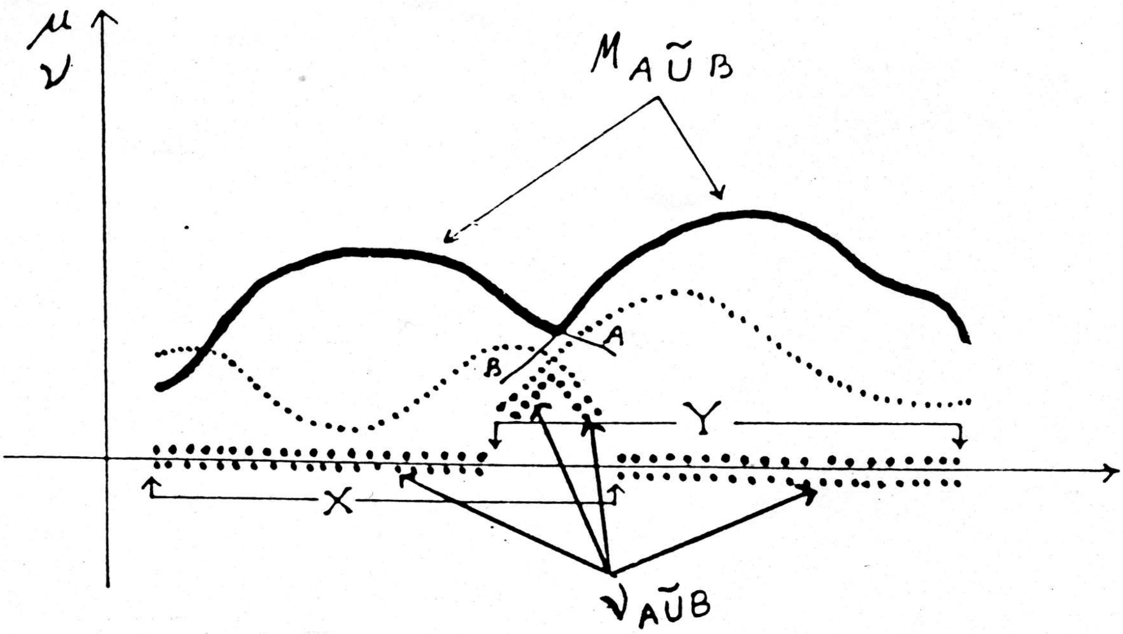

Fig.1

Graphical representation of the intersection of two IF – sets:-

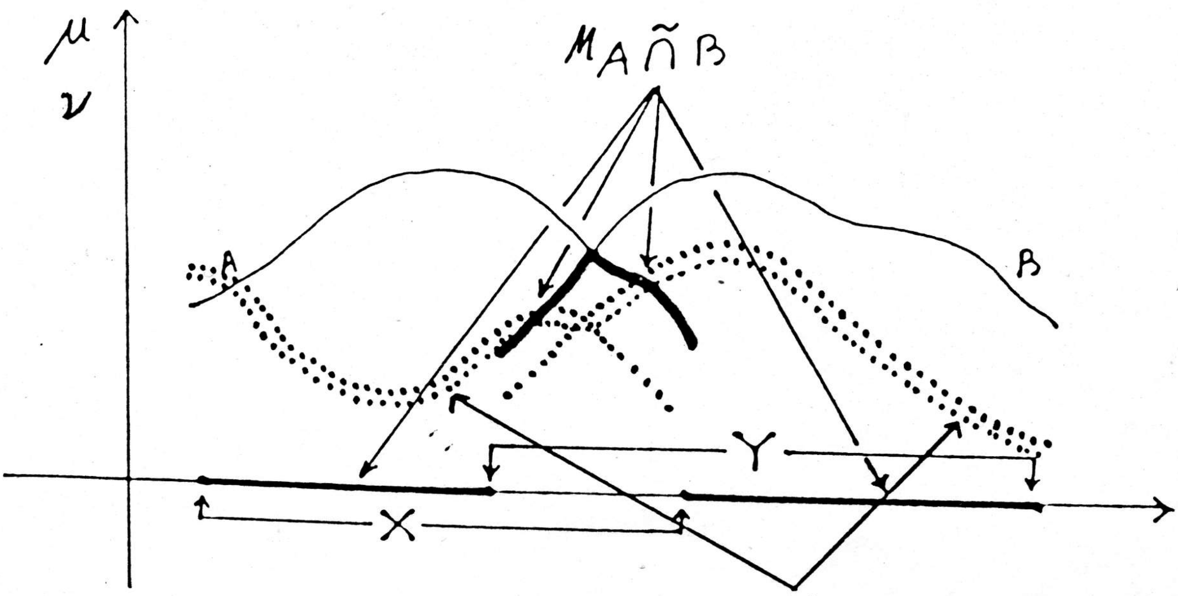

Fig.2