Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
2025, Volume 31, Number 2, 154–171
DOI: 10.7546/nifs.2025.31.2.154-171

An extended framework for autocratic multi-parameter group decision making using interval-valued intuitionistic fuzzy numbers

Mousumi Akter [1], Sahadat Hossain [2], Fazlul Hoque [3], Rafiqul Islam [4], and Nasimul Karim [5]

[1] Department of Mathematics, Pabna University of Science and Technology
Pabna-6600, Bangladesh
e-mail: mousumiakter@pust.ac.bd

[2] Department of Mathematics, University of Rajshahi
Rajshahi-6205, Bangladesh
e-mail: sahadat@ru.ac.bd

[3] Department of Mathematics, Pabna University of Science and Technology
Pabna-6600, Bangladesh
e-mail: fazlulmath@pust.ac.bd

[4] Department of Mathematics, Pabna University of Science and Technology
Pabna-6600, Bangladesh
e-mail: rafiq.math@pust.ac.bd

[5] Department of Mathematics, University of Rajshahi
Rajshahi-6205, Bangladesh
e-mail: nasimulkarim.ru@gmail.com

Received: 10 January 2025 **Revised:** 22 March 2025
Accepted: 8 May 2025 **Online First:** 27 May 2025

Abstract: The objective of this paper is to develop a more generalized autocratic multi-parameter group decision-making (AMPGDM) model that provides an optimal solution for choosing an ideal object among several choices in AMPGDM circumstances. Then we apply

interval-valued intuitionistic fuzzy numbers (IVIFNs) to form the weight of the parameters and entries of the decision matrices. Finally, we introduce two modified methods to deal with AMPGDM problems that will provide the same outcome but in a short amount of time.
Keywords: Autocratic multi-parameter group decision-making, Interval-valued intuitionistic fuzzy set, Interval-valued intuitionistic fuzzy number, Weight vector, Resultant matrix, Weight evaluation matrix.
2020 Mathematics Subject Classification: 03E72, 68T27, 15A30.

1 Introduction

Atanassov [1, 2, 4] was the first to develop the concept of intuitionistic fuzzy set (IFS) theory, which is an extension of fuzzy set and takes into account both the membership and non-membership grade of each element of a given set. After a while, Atanassov [3, 6] studied and brought out the knowledge of modal operators over intuitionistic fuzzy sets that was developed and enriched by mathematician Bhattacharya [10]. In [8], Atanassov and Gargov, and in [5], Atanassov initiated the idea of an interval-valued intuitionistic fuzzy set (IVIFS) as an amplification of IFS that includes both interval-valued membership and non-membership degrees, which perchance showed as a satisfactory approach to dealing with the ambiguity and imprecise situation of a given decision making problem. In [9], Atanassov *et al.* proposed a collaborative and adaptive method based on intuitionistic fuzzy aggregation to effectively integrate multiple expert decisions and multiple measurement tools. This method guarantees a flexible and effective decision making procedure, especially in scenarios requiring consensus. Several multi-attribute decision making (MADM) and multi-attribute group decision making (MAGDM) models based on intuitionistic fuzzy sets (IFSs) and interval-valued intuitionistic fuzzy sets (IVIFSs) have been proposed in recent years [11–15, 18, 19]. Introducing intuitionistic L-fuzzy T_1 spaces [17] will enhance MADGM by strengthening the mathematical foundation of IVIFSs, leading to greater accuracy and flexibility in decision-making. To overcome modern life decision-making problems, most of the experts are generally accustomed to applying interval-valued intuitionistic fuzzy numbers (IVIFNs) rather than IVIFSs to draw out their evaluation. A considerable number of methods of multi-parameter group decision making (MPGDM) developed on IVIFNs have been taken into account as a significant research topic [22, 25, 26]. In this study, the weights of the parameters are defined with IVIFNs.

In recent years, autocratic multi-parameter group decision-making (AMPGDM) has been a useful research topic for the integration and rationalization of simultaneous modern life problems that are rife with ambiguity. To deal with convoluted decision-making circumstances, AMPGDM methods have become imperative in modern science, more precisely in the fields of uncertainty and vagueness, such as supply chain management, social economy, engineering, and environmental sustainability. Conventional multi-parameter decision making (MPDM) approaches are often unable to demonstrate the involute risk preferences of decision-makers or to rationalize the interdependencies between parameters, which are pivotal to accurately epitomizing real-world decision complexities. To overcome these limitations, researchers have increasingly studied rudiments with IVIFNs, which is an extension of fuzzy set theory including membership and non-membership degrees that impart a comprehensive framework for conducting uncertainty in decision-making. Several foundational studies have provided critical enhancements to MADM by refining and building on IVIFN frameworks. For instance, Xu [21–23] proposed several aggregation methodologies for intuitionistic fuzzy sets and set down a significant foundational framework for subsequent MADM circumstances. Ye [24] initiated the concept of improved similarity measures to enhance the legibility of multi-criteria decision-making (MCDM) by allowing for the more accurate evaluation of parameter dependencies, which is a fundamental development in fields requiring fine distinctions between choices, such as investment evaluation. Applying the idea of relative closeness between the distinct choices and the ideal choice, Yue [25, 26] proposed the TOPSIS method to rank the weights of the experts. Subsequently, Yue [25, 26] established the assumption that the ideal weight is the average of all distinct weights of the expert. In the past few years, to address these AMPGDM problems, a number of useful methods have been developed. In [20], Wibowo introduced a multi-criteria group decision-making (MCGDM) methodology that utilizes the idea of the IVIFS framework to sort out any hotel location selection problem. The Wibowo's technique, on the contrary, appears too intricate for use with any selection problem and instead depends on the concept of perfect solutions to assess the overall effectiveness of all hotel locations with regard to each selection parameter. Zhang and Xu [27] developed an AMPGDM model related to ranking methods that focus on IVIFNs, in which they employ IVIFNs to express each component of the decision matrix and weight of the parameters provided by the experts. In order to overcome the drawbacks of Wibowo's [20] strategy, Cheng [16] developed a novel autocratic multi-attribute group decision making (AMAGDM) method that uses interval-valued intuitionistic fuzzy values (IVIFVs) to deal with such kinds of selection challenges.

All these novel concepts are used in our study to develop a modified AMPGDM model combining the Zhang *et al.* [27] and Cheng [16] techniques to solve any type of AMPGDM problem with an emphasis on IVIFSs and IVIFNs, in which the weight of the parameters provided by the experts is formed with IVIFNs. The research aim is to establish an AMPGDM model to resolve interval-type intuitionistic AMPGDM difficulties with regard to ranking the preference data focused on the ideal resultant matrix. In decision-making and data analysis, matrices are very effective for addressing various everyday problems. Traditional matrices often face challenges with uncertain issues. To overcome these challenges, Atanassov [7] introduced the index matrix theory, which has been proven useful in addressing complex circumstances, especially in intuitionistic fuzzy environments. This study is developed on the fundamental principles of IVIFNs and interval-valued intuitionistic fuzzy matrices (IVIFMs) and their combinations, providing a comprehensive framework for dealing with AMPGDM problems. To do so, in Section 2, we give an overview of the theoretical foundations of IVIFS and IVIFN. Section 3 is devoted to the methodological developments of these concepts and an established methodology based on IVIFN and IVIFM to get an optimal result for the supplier selection problem. In Section 4, we demonstrate a hypothetical example to illustrate the functioning of the proposed methods, with a particular focus on their role in improving decision-making processes by enhancing assessment accuracy through organized analysis and assimilation of uncertainty. Discussions and consequences are presented in Section 5.

2 Preliminaries

Definition 2.1. [1] Consider K be the domain of discourse. An IFS over K is defined as the set of ordered triplets $\vartheta = \{(\alpha, \mu_\vartheta(\alpha), \nu_\vartheta(\alpha)) \mid \alpha \in K\}$, where $\mu_\vartheta : K \to [0,1]$ and $\nu_\vartheta : K \to [0,1]$, respectively describe the membership and non-membership grade for each element $\alpha \in K$, and the relation $(\mu_\vartheta(\alpha) + \nu_\vartheta(\alpha)) \le 1$ holds true for each $\alpha \in K$.

Definition 2.2. [8] Consider K be the domain of discourse. An IVIFS over K is defined as the set of ordered triplets $\vartheta = \{(\alpha, \mu_\vartheta(\alpha), \nu_\vartheta(\alpha)) \mid \alpha \in K\}$, where $\mu_\vartheta(\alpha) = [\mu_\vartheta^l(\alpha), \mu_\vartheta^u(\alpha)] \subseteq [0,1]$ and $\nu_\vartheta(\alpha) = [\nu_\vartheta^l(\alpha), \nu_\vartheta^u(\alpha)] \subseteq [0,1]$ are intervals that respectively describe the membership and non-membership grade for each element $\alpha \in K$, and the relation $(\mu_\vartheta(\alpha) + \nu_\vartheta(\alpha)) \le 1$ holds true for each $\alpha \in K$.

Definition 2.3. [18] For simplicity, Xu [18] describes $\bar{a} = (\mu_{\bar{a}}, \nu_{\bar{a}})$ as an IVIFN and represents it with $\bar{a} = ([\alpha, \beta], [\lambda, \delta])$, where $[\alpha, \beta], [\lambda, \delta] \subseteq [0,1]$ and the relation $\beta + \delta \le 1$ exists for each element $a \in \Re$.

Definition 2.4. [5, 8] Consider $\bar{a}_1 = ([\alpha_1, \beta_1], [\lambda_1, \delta_1])$ and $\bar{a}_2 = ([\alpha_2, \beta_2], [\lambda_2, \delta_2])$ to be two IVIFNs, then the addition and multiplication operations of these two IVIFNs can be conducted as

(i) $\bar{a}_1 \oplus \bar{a}_2 = ([\alpha_1 + \alpha_2 - \alpha_1\alpha_2, \beta_1 + \beta_2 - \beta_1\beta_2], [\lambda_1\lambda_2, \delta_1\delta_2])$, (1)

(ii) $\bar{a}_1 \otimes \bar{a}_2 = ([\alpha_1\alpha_2, \beta_1\beta_2], [\lambda_1 + \lambda_2 - \lambda_1\lambda_2, \delta_1 + \delta_2 - \delta_1\delta_2])$, (2)

where the notations $\oplus$ and $\otimes$ respectively represent the additive and multiplicative operations of IVIFNs.

In order to quantify the consequences of IVIFNs, Xu [18] initiated the idea of score and accuracy functions of IVIFN. These functions established a structured framework to evaluate the membership and non-membership degrees.

Definition 2.5. The modified score and accuracy functions of the IVIFN $\bar{a} = ([\alpha, \beta], [\lambda, \delta])$ can be derived as follows

$$S(\bar{a}) = \frac{1}{2}[\alpha + \beta - (\lambda + \delta)] + 1, \tag{3}$$

$$A(\bar{a}) = \frac{1}{2}[\alpha + \beta + \lambda + \delta]. \tag{4}$$

Definition 2.6. In accordance with score and accuracy functions, the IVIFN ranking methodology for two IVIFNs $\bar{a}_1 = ([\alpha_1, \beta_1], [\lambda_1, \delta_1])$ and $\bar{a}_2 = ([\alpha_2, \beta_2], [\lambda_2, \delta_2])$ can be described like and so

(i) if $S(\bar{a}_1) < S(\bar{a}_2)$, then $\bar{a}_1 < \bar{a}_2$, (5)

(ii) if $A(\bar{a}_1) = A(\bar{a}_2)$, then $\begin{cases} if\ S(\bar{a}_1) = S(\bar{a}_2), \text{ then } \bar{a}_1 = \bar{a}_2 \\ if\ S(\bar{a}_1) < S(\bar{a}_2), \text{ then } \bar{a}_1 < \bar{a}_2 \end{cases}$. (6)

Definition 2.7. We may describe the interval-value intuitionistic fuzzy averaging operator (IVIFAO) for a collection $\{\bar{a}_j\}_{j=1}^{s} = ([\alpha_j, \beta_j], [\lambda_j, \delta_j])$ of IVIFNs using the following formula

$$IVIFA(\bar{a}_1, \bar{a}_2, \cdots, \bar{a}_s) = \frac{1}{s}\sum_{j=1}^{s}\bar{a}_j = \left(\left[1-\prod_{j=1}^{s}(1-\alpha_j)^{\frac{1}{s}}, 1-\prod_{j=1}^{s}(1-\beta_j)^{\frac{1}{s}}\right], \left[\prod_{j=1}^{s}(\lambda_j)^{\frac{1}{s}}, \prod_{j=1}^{s}(\delta_j)^{\frac{1}{s}}\right]\right). \quad (7)$$

Definition 2.8. [7] For two index sets $I = \{i_1, i_2, \ldots, i_m\}$ and $J = \{j_1, j_2, \ldots, j_n\}$, an index matrix can be formed as $M = (m_{ij})_{(I,J)}$, where each entry m_{ij} can be any structured object.

Definition 2.9. [7] For two index sets $I = \{i_1, i_2, \ldots, i_m\}$ and $J = \{j_1, j_2, \ldots, j_n\}$, an intuitionistic fuzzy matrix can be formed as $M = (m_{ij})_{(I,J)}$, where each entry m_{ij} is an intuitionistic fuzzy number.

Definition 2.10. For two index sets $I = \{i_1, i_2, \ldots, i_m\}$ and $J = \{j_1, j_2, \ldots, j_n\}$, an interval-valued intuitionistic fuzzy matrix can be formed as $M = (m_{ij})_{(I,J)}$, where each entry m_{ij} is an interval-valued intuitionistic fuzzy number.

3 Ideal approaches for AMPGDM difficulties based on interval-valued intuitionistic fuzzy numbers

In this section, focusing on the fuzzy TOPSIS method and AMPGDM, we develop two optimal decision-making models to deal with AMPGDM difficulties, in which the weights of the parameters and experts are predefined and are formed with IVIFNs. The developed model involves the following steps (for the convenience of this study, we presume $N_1 = \{1, 2, 3, \ldots, r\}$, $N_2 = \{1, 2, 3, \ldots, s\}$, and $N_3 = \{1, 2, 3, \ldots, t\}$, in all respect):

Step 1: In the context of interval-valued intuitionistic fuzzy circumstances, let $C = \{C_i\}_{i \in N_1}$ describe a discrete set of r feasible choices provided by the t experts $E = \{e_k\}_{k \in N_3}$ consequent to the s parameters $P = \{\varepsilon_j\}_{j \in N_2}$. Let $\omega_k = (\tilde{\omega}_j^k)_{1\times s}$ represent the weight vector of the parameters ε_j demonstrated by the t experts $E = \{e_k\}_{k \in N_3}$, which can be illustrated as

$$\begin{matrix} & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \cdots & \varepsilon_s \\ \omega_k = & [\tilde{\omega}_1^k & \tilde{\omega}_2^k & \tilde{\omega}_3^k & \cdots & \tilde{\omega}_s^k], \end{matrix}$$

in which each $\tilde{\omega}_j^k$ formed with IVIFN, can be deduced as $\tilde{\omega}_j^k = ([\alpha_j^k, \beta_j^k], [\gamma_j^k, \delta_j^k])$, where $0 \le \alpha_j^k \le \beta_j^k \le 1$, $0 \le \gamma_j^k \le \delta_j^k \le 1$ and $0 \le \beta_j + \delta_j \le 1$.

Step 2: Consider $R_k = (r_{ij}^k)_{r\times s}$ to be the interval-valued intuitionistic fuzzy evaluation matrices of the choices C_i regarding the parameters ε_j provided by the experts e_k. Consequently, the AMPGDM problem with IVIFNs can be expressed in the following matrix form

$$R_k = \left(r_{ij}^k\right)_{r\times s} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_r \end{array} \begin{array}{c} \begin{array}{cccc} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_s \end{array} \\ \begin{bmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1s}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2s}^k \\ \vdots & \vdots & \vdots & \vdots \\ r_{r1}^k & r_{r2}^k & \cdots & r_{rs}^k \end{bmatrix} \end{array},$$

where each r_{ij}^k is an IVIFN that explains the evaluation value of the choices C_i, which are provided by the experts e_k regarding each parameter ε_j, and is deduced as $r_{ij}^k = \left(\left[x_{ij}^k, y_{ij}^k\right], \left[z_{ij}^k, w_{ij}^k\right]\right)$, where $0 \le x_{ij}^k \le y_{ij}^k \le 1$, $0 \le z_{ij}^k \le w_{ij}^k \le 1$ and $0 \le y_{ij}^k + w_{ij}^k \le 1$.

Step 3: Using Eq. (2) of Definition 2.4, we compute the weight evaluation matrix $W_k = \left(d_{ij}^k\right)$, of all experts e_k and represent them in matrix form in the following way:

$$W_k = \left(d_{ij}^k\right)_{r\times s} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_s \end{array} \\ \begin{bmatrix} r_{11}^k \otimes \tilde{\omega}_1^k & r_{12}^k \otimes \tilde{\omega}_2^k & \cdots & r_{1s}^k \otimes \tilde{\omega}_s^k \\ r_{21}^k \otimes \tilde{\omega}_1^k & r_{22}^k \otimes \tilde{\omega}_2^k & \cdots & r_{2s}^k \otimes \tilde{\omega}_s^k \\ \vdots & \vdots & \vdots & \vdots \\ r_{r1}^k \otimes \tilde{\omega}_1^k & r_{r2}^k \otimes \tilde{\omega}_2^k & \cdots & r_{rs}^k \otimes \tilde{\omega}_s^k \end{bmatrix} \end{array},$$

where each $d_{ij}^k = r_{ij}^k \otimes \tilde{\omega}_j^k$ is an IVIFN.

Step 4: In a feasible AMPGDM approach, the decision of the group should be presented to the greatest extent with the evaluation of the particular expert. For simplicity of study, let us speculate that the aggregated evaluation of the group is its collective will, which is provided by the ideal expert. Perhaps the consensus of the decisions drawn up by all the individual experts provides the optimal collective decision. Therefore, the aggregated ideal decision matrix can be determined by performing the following method:

$$W^* = \left(d_{ij}^*\right)_{r\times s} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_r \end{array} \begin{array}{c} \begin{array}{cccc} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_s \end{array} \\ \begin{bmatrix} d_{11}^* & d_{12}^* & \cdots & d_{1s}^* \\ d_{21}^* & d_{22}^* & \cdots & d_{2s}^* \\ \vdots & \vdots & \vdots & \vdots \\ d_{r1}^* & d_{r2}^* & \cdots & d_{rs}^* \end{bmatrix} \end{array},$$

in which each d_{ij}^* is formed with IVIFN as $d_{ij}^* = \left(\left[x_{ij}^*, y_{ij}^*\right], \left[z_{ij}^*, w_{ij}^*\right]\right)$, where $0 \le x_{ij}^* \le y_{ij}^* \le 1$, $0 \le z_{ij}^* \le w_{ij}^* \le 1$ and $0 \le y_{ij}^* + w_{ij}^* \le 1$.

Using Eq. (7), we can explain d_{ij}^* mathematically in the following way:

$$d_{ij}^* = \frac{1}{t}\sum_{k=1}^{t} d_{ij}^k,$$

where $x_{ij}^* = 1 - \prod_{k=1}^{t}\left(1 - x_{ij}^k\right)^{\frac{1}{t}}$, $y_{ij}^* = 1 - \prod_{k=1}^{t}\left(1 - y_{ij}^k\right)^{\frac{1}{t}}$, $z_{ij}^* = \prod_{k=1}^{t}\left(x_{ij}^k\right)^{\frac{1}{t}}$, $w_{ij}^* = \prod_{k=1}^{t}\left(w_{ij}^k\right)^{\frac{1}{t}}$.

3.1 Modified Zhang's method

In the modified Zhang's method, we illustrate the ideal decision matrix by the aggregation of the weight evaluation matrices supplied by each expert, taking into account the respected parameters. As the ideal decision matrices are obtained by the aggregation of the weight vector with the decision matrices, thus it can be considered as the final resultant matrix.

Step 5: In the following, we use Eq. (3) to evaluate the modified score values $S(d_{ij}^{*})$ of each d_{ij}^{*} from the optimal decision matrix W^{*}, considered as final decision matrix and the score matrix $\Psi = \left(S\left(d_{ij}^{*}\right)\right)_{r\times s}$ can be formed in the following way

$$\Psi = \left(S\left(d_{ij}^{*}\right)\right)_{r\times s} = \begin{matrix} & \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_s \\ c_1 & S\left(d_{11}^{*}\right) & S\left(d_{12}^{*}\right) & \cdots & S\left(d_{1s}^{*}\right) \\ c_2 & S\left(d_{21}^{*}\right) & S\left(d_{22}\right) & \cdots & S\left(d_{2p}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_r & S\left(d_{r1}^{*}\right) & S\left(d_{r2}^{*}\right) & \cdots & S\left(d_{rs}^{*}\right) \end{matrix}.$$

Step 6: Afterwards, determine the interval-valued intuitionistic fuzzy positive ideal solution (PIS) C^{+} and interval-value intuitionistic fuzzy negative ideal solution (NIS) C^{-} consequently according to the formula

$$\begin{aligned} C^{+} &= \left\{\left\langle \varepsilon_j, \left([x_j^{*+}, y_j^{*+}],[z_j^{*+}, w_j^{*+}]\right)\right\rangle \mid \text{for which } S(d_{ij}^{*}) \text{ is maximum for each } i\right\} \\ \Rightarrow C^{+} &= \left\{\left\langle \varepsilon_1, \left([x_1^{*+}, y_1^{*+}],[z_1^{*+}, w_1^{*+}]\right)\right\rangle, \left\langle \varepsilon_2, \left([x_2^{*+}, y_2^{*+}],[z_2^{*+}, w_2^{*+}]\right)\right\rangle, ..., \left\langle \varepsilon_s, \left([x_s^{*+}, y_s^{*+}],[z_s^{*+}, w_s^{*+}]\right)\right\rangle \right\}, \end{aligned} \tag{8}$$

$$\begin{aligned} C^{-} &= \left\{\left\langle \varepsilon_j, \left([x_j^{*-}, y_j^{*-}],[z_j^{*-}, w_j^{*-}]\right)\right\rangle \mid \text{for which } S(d_{ij}^{*}) \text{ is maximum for each } i\right\} \\ \Rightarrow C^{-} &= \left\{\left\langle \varepsilon_1, \left([x_1^{*-}, y_1^{*-}],[z_1^{*-}, w_1^{*-}]\right)\right\rangle, \left\langle \varepsilon_2, \left([x_2^{*-}, y_2^{*-}],[z_2^{*-}, w_2^{*-}]\right)\right\rangle, ..., \left\langle \varepsilon_s, \left([x_s^{*-}, y_s^{*-}],[z_s^{*-}, w_s^{*-}]\right)\right\rangle \right\}, \end{aligned} \tag{9}$$

Step 7: Next, focusing on PIS C^{+} and NIS C^{-}, we can exhibit the distance measures Δ_i^{+} and Δ_i^{-}, employing formulas accordingly shown as

$$\Delta_i^{+} = \sum_{j=1}^{s} \partial(d_{ij}^{*}, d_j^{*+})\lambda_j = \sum_{j=1}^{s} \lambda_j \sqrt{\frac{1}{4}\left(\left(x_{ij}^{*} - x_j^{*+}\right)^2 + \left(y_{ij}^{*} - y_j^{*+}\right)^2 + \left(z_{ij}^{*} - z_j^{*+}\right)^2 + \left(w_{ij}^{*} - w_j^{*+}\right)^2\right)}, \mathrm{i} \in \mathrm{N}_1, \tag{10}$$

$$\Delta_i^{-} = \sum_{j=1}^{s} \partial(d_{ij}^{*}, d_j^{*-})\lambda_j = \sum_{j=1}^{s} \lambda_j \sqrt{\frac{1}{4}\left(\left(x_{ij}^{*} - x_j^{*-}\right)^2 + \left(y_{ij}^{*} - y_j^{*-}\right)^2 + \left(z_{ij}^{*} - z_j^{*-}\right)^2 + \left(w_{ij}^{*} - w_j^{*-}\right)^2\right)}, \mathrm{i} \in \mathrm{N}_1, \tag{11}$$

where λ_j represents the ideal weight of the parameters with respect to the experts that we can compute from the predefined weight vectors $\omega_k = \left(\tilde{\omega}_j^{k}\right)_{1\times s}$ of the parameters using the following formula

$$\lambda_j = \frac{\sum_{k=1}^{t} S\left(\tilde{\omega}_j^k\right)}{\sum_{j=1}^{r}\sum_{k=1}^{t} S\left(\tilde{\omega}_j^k\right)}. \tag{12}$$

Step 8: Finally, the relative closeness of the indices computed as

$$cl_i = \frac{\Delta_i^-}{\Delta_i^- + \Delta_i^+}. \tag{13}$$

If $cl_m \le cl_n$, it will be ideal to select supplier C_n rather than supplier C_m.

3.1 Modified Cheng's Method

Step 5: Based on the ideal decision matrix find out the optimal weight evaluation vector (WEV) $\aleph_i$ of choice C_i accordingly as:

$$\aleph_i = d_{i1}^* \oplus d_{i2}^* \oplus d_{i3}^* \oplus \cdots \oplus d_{ij.}^* \tag{14}$$

Step 6: Thereinafter, employing Equation (4), the modified score values $S(\aleph_i)$ appertaining to each $\aleph_i$ are evaluated from aggregated optimal weight evaluation vector. If $S(\aleph_{C_a}) \le S(\aleph_{C_b})$, it will be ideal to select supplier C_b rather than supplier C_a.

4 Illustrative example

In the following, an illustrative example is given to demonstrate the effectual mode of the developed AMPGDM models to act with certain decision making problem and the parallel evaluation of conferred results is also drafted to show its novelty.

A renowned garment factory exports winter cloths in different countries of Europe. To extend their business and contend with competitive business world, factory board of directors decides to produce new products. According to their concern, a resolution committee of three experts $E = \{e_1, e_2, e_3\}$ has been created to choose material suppliers to acquire basic components from six proficient suppliers $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ based on the following five parameters ε_1 (product quality), ε_2 (price structure), ε_3 (financial stability), ε_4 (delivery capacity), ε_5 (communication and collaboration). The predefined weight vectors ω_1, ω_2, ω_3 of the experts $E = \{e_1, e_2, e_3\}$ corresponding to the five parameters are evaluated accordingly as follows:

$$\omega_1 = \{([0.7,0.8],[0.1,0.2]),([0.5,0.6],[0.3,0.4]),([0.5,0.8],[0.1,0.2]),([0.5,0.7],[0.2,0.3])\\ ([0.6,0.7],[0.1,0.3])\},$$
$$\omega_2 = \{([0.4,0.5],[0.3,0.4]),([0.7,0.8],[0.1,0.2]),([0.3,0.6],[0.2,0.4]),([0.7,0.8],[0.1,0.2]),\\ ([0.5,0.7],[0.2,0.3])\},$$
$$\omega_3 = \{([0.5,0.8],[0.1,0.2]),([0.3,0.6],[0.3,0.4]),([0.6,0.7],[0.2,0.3]),([0.6,0.8],[0.1,0.2]),\\ ([0.4,0.5],[0.3,0.4])\}.$$

The evaluation matrix of the six suppliers $\{C_1, C_2, C_3, C_4, C_5, C_6\}$, in accordance to the five parameters $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$, given by the experts $\{e_1, e_2, e_3\}$ are formed with IVIFNs, as demonstrated in Tables 1–3.

Table 1. Evaluation matrix R_1.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.3,0.5], [0.4,0.5])	([0.1,0.2],[0.7,0.8])	([0.1,0.2],[0.5,0.8])	([0.4,0.6],[0.2,0.3])	([0.2,0.3],[0.5,0.6])
C_2	([0.4,0.5],[0.3,0.5])	([0.7,0.8],[0.1,0.2])	([0.5,0.8],[0.1,0.2])	([0.5,0.8],[0.1,0.2])	([0.1,0.4],[0.3,0.5])
C_3	([0.3,0.4],[0.4,0.6])	([0.1,0.3],[0.2,0.4])	([0.4,0.5],[0.3,0.5])	([0.7,0.8],[0.1,0.2])	([0.3,0.5],[0.4,0.5])
C_4	([0.5,0.6],[0.2,0.3])	([0.3,0.5],[0.4,0.5])	([0.7,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.2,0.4],[0.4,0.6])
C_5	([0.2,0.4],[0.5,0.6])	([0.6,0.7],[0.2,0.3])	([0.5,0.6],[0.2,0.3])	([0.4,0.5],[0.3,0.5])	([0.3,0.4],[0.5,0.8])
C_6	([0.6,0.7],[0.2,0.3])	([0.3,0.4],[0.4,0.5])	([0.6,0.7],[0.2,0.3])	([0.5,0.6],[0.2,0.3])	([0.2,0.3],[0.4,0.7])

Table 2. Evaluation matrix R_2.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.3,0.6],[0.2,0.3])	([0.4,0.5],[0.3,0.5])	([0.4,0.6],[0.2,0.3])	([0.4,0.5],[0.3,0.4])	([0.2,0.3],[0.3,0.5])
C_2	([0.4,0.7],[0.1,0.2])	([0.3,0.5],[0.2,0.3])	([0.2,0.5],[0.3,0.4])	([0.5,0.7],[0.1,0.3])	([0.1,0.4],[0.3,0.5])
C_3	([0.5,0.7],[0.1,0.2])	([0.4,0.5],[0.2,0.4])	([0.6,0.8],[0.1,0.2])	([0.6,0.7],[0.2,0.3])	([0.3,0.5],[0.2,0.4])
C_4	([0.6,0.8],[0.1,0.2])	([0.5,0.8],[0.1,0.2])	([0.3,0.4],[0.2,0.4])	([0.5,0.8],[0.1,0.2])	([0.4,0.5],[0.2,0.3])
C_5	([0.3,0.5],[0.3,0.4])	([0.5,0.7],[0.1,0.2])	([0.7,0.8],[0.1,0.2])	([0.2,0.4],[0.3,0.4])	([0.2,0.3],[0.4,0.6])
C_6	([0.5,0.6],[0.1,0.3])	([0.3,0.6],[0.2,0.4])	([0.6,0.7],[0.1,0.2])	([0.4,0.6],[0.1,0.3])	([0.3,0.6],[0.2,0.3])

Table 3. Evaluation matrix R_3.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.3,0.5],[0.4,0.5])	([0.6,0.7],[0.1,0.3])	([0.5,0.6],[0.2,0.3])	([0.4,0.6],[0.2,0.3])	([0.3,0.4],[0.4,0.6])
C_2	([0.6,0.7],[0.1,0.2])	([0.4,0.6],[0.3,0.4])	([0.7,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.3,0.5],[0.4,0.5])
C_3	([0.5,0.6],[0.2,0.3])	([0.2,0.4],[0.2,0.3])	([0.6,0.8],[0.1,0.2])	([0.5,0.6],[0.3,0.4])	([0.1,0.2],[0.6,0.8])
C_4	([0.6,0.8],[0.1,0.2])	([0.3,0.5],[0.4,0.5])	([0.4,0.6],[0.2,0.3])	([0.7,0.8],[0.1,0.2])	([0.2,0.3],[0.6,0.7])
C_5	([0.2,0.3],[0.3,0.5])	([0.7,0.8],[0.1,0.2])	([0.5,0.7],[0.1,0.2])	([0.4,0.5],[0.3,0.4])	([0.1,0.2],[0.3,0.5])
C_6	([0.4,0.6],[0.2,0.3])	([0.5,0.6],[0.2,0.3])	([0.4,0.5],[0.2,0.4])	([0.5,0.7],[0.2,0.3])	([0.2,0.5],[0.2,0.4])

In what follows, we apply the developed methodology to deal with the illustrated decision making problem. Foremost, employing the multiplication operation of IVIFNs does explain in Eq. (2), we demonstrate the weight evaluation matrices W_1, W_2 and W_3 that are listed in Tables 4–6, respectively as follows.

Table 4. Weight evaluation matrix W_1 for expert e_1.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.21,0.4],[0.46,0.6])	([0.05,0.12],[0.79,0.88])	([0.05,0.16],[0.55,0.84])	([0.2,0.42],[0.36,0.51])	([0.12,0.21],[0.55,0.72])
C_2	([0.28,0.4],[0.37,0.6])	([0.35,0.48],[0.37,0.52])	([0.25,0.64],[0.19,0.36])	([0.25,0.56],[0.28,0.44])	([0.06,0.28],[0.37,0.65])
C_3	([0.21,0.32],[0.46,0.68])	([0.05,0.18],[0.44,0.64])	([0.2,0.4], [0.37,0.6])	([0.35,0.56],[0.28,0.44])	([0.18,0.35],[0.46,0.65])
C_4	([0.35,0.48],[0.28,0.44)]	([0.15,0.3],[0.58,0.7])	([0.35,0.64],[0.19,0.36])	([0.3,0.49],[0.28,0.44])	([0.12,0.28],[0.46,0.72])
C_5	([0.14,0.32],[0.55,0.68])	([0.3,0.42],[0.44,0.58])	([0.25,0.48],[0.28,0.44])	([0.2,0.35],[0.44,0.65])	([0.18,0.28],[0.55,0.86])
C_6	([0.42,0.56],[0.28,0.44])	([0.15,0.24],[0.58,0.7])	([0.3,0.56],[0.28,0.44])	([0.25,0.42],[0.36,0.51])	([0.12,0.21],[0.46,0.79])

Table 5. Weight evaluation matrix W_2 for expert e_2.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.12,0.3],[0.44,0.58])	([0.28,0.4],[0.37,0.6])	([0.12,0.36],[0.36,0.58])	([0.28,0.4],[0.37,0.52])	([0.1,0.21],[0.44,0.65])
C_2	([0.16,0.35],[0.37,0.52])	([0.21,0.4],[0.28,0.44])	([0.06,0.3],[0.44,0.64])	([0.35,0.56],[0.19,0.44])	([0.05,0.28],[0.44,0.65])
C_3	([0.2,0.35],[0.37,0.52])	([0.28,0.4],[0.28,0.52])	([0.18,0.48],[0.28,0.52])	([0.42,0.56],[0.28,0.44])	([0.15,0.35],[0.36,0.58])
C_4	([0.24,0.4],[0.37,0.52])	([0.35,0.64],[0.19,0.36])	([0.09,0.24],[0.36,0.64])	([0.35,0.64],[0.19,0.36])	([0.2,0.35],[0.36,0.51])
C_5	([0.12,0.25],[0.51,0.64])	([0.35,0.56],[0.19,0.36])	([0.21,0.48],[0.28,0.52])	([0.14,0.32],[0.37,0.52])	([0.1,0.21],[0.52,0.72])
C_6	([0.2,0.3], [0.37,0.58])	([0.21,0.48],[0.28,0.52])	([0.18,0.42],[0.28,0.52])	([0.28,0.48],[0.19,0.44])	([0.15,0.42],[0.36,0.51])

Table 6. Weight evaluation matrix W_3 for expert e_3.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.15,0.4],[0.46,0.6])	([0.18,0.42],[0.37,0.58])	([0.3,0.42],[0.36,0.51])	([0.24,0.48],[0.28,0.44])	([0.12,0.2],[0.58,0.76])
C_2	([0.3,0.56],[0.19,0.36)	([0.12,0.36],[0.51,0.64])	([0.42,0.56],[0.28,0.44])	([0.36,0.56],[0.19,0.36])	([0.12,0.25],[0.58,0.7])
C_3	([0.25,0.48],[0.28,0.44])	([0.06,0.24],[0.44,0.58])	([0.36,0.56],[0.28,0.44])	([0.3,0.48],[0.37,0.52])	([0.04,0.1],[0.72,0.88])
C_4	([0.3,0.64],[0.19,0.36])	([0.09,0.3],[0.58,0.7])	([0.24,0.42],[0.28,0.44])	([0.42,0.64],[0.19,0.36])	([0.08,0.15],[0.72,0.82])
C_5	([0.1,024.],[0.37,0.6])	([0.21,0.48],[0.37,0.52])	([0.3,0.49],[0.28,0.44])	([0.24,0.4],[0.37,0.52])	([0.04,0.1],[0.51,0.7])
C_6	([0.2,0.48],[0.28,0.44])	([0.15,0.36],[0.44,0.58])	([0.24,0.35],[0.36,0.58])	([0.3,0.56],[0.28,0.44])	([0.08,0.25],[0.44,0.64])

Next, utilizing Eq. (7) we compute the ideal decision matrix W^* and is formed in Table 7.

Table 7. Ideal decision matrix W^*.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.161,0.368], [0.453,0.593])	([0.175,0.326], [0.476,0.674])	([0.163,0.322], [0.415,0.629])	([0.241,0.434], [0.334,0.489])	([0.113,0.207], [0.520,0.708])
C_2	([0.249,0.444], [0.296,0.482])	([0.233,0.415], [0.375,0.527])	([0.258,0.519], [0.286,0.466])	([0.322,0.56], [0.216,0.411])	([0.077,0.270], [0.455,0.666])
C_3	([0.220,0.387], [0.362,0.538])	([0.137,0.279], [0.378,0.578])	([0.251,0.484], [0.307,0.516])	([0.358,0.535], [0.307,0.465])	([0.125,0.275], [0.492,0.692])
C_4	([0.298,0.517], [0.270,0.435])	([0.205,0.439], [0.400,0.561])	([0.234,0.459], [0.291,0.490])	([0.358,0.596], [0.216,0.385])	([0.135,0.264], [0.492,0.670])
C_5	([0.120,0.271], [0.470,0.639)	([0.289,0.490], [0.314,0.477])	([0.254,0.483], [0.28,0.465])	([0.194,0.357], [0.392,0.560])	([0.108,0.200], [0.526,0.757])
C_6	([0.281,0.457], [0.307,0.482])	([0.170,0.367], [0.4150.595])	([0.241,0.450], [0.304,0.510])	([0.277,0.490], [0.267,0.462])	([0.117,0.299], [0.418,0.636])

4.1 Modified Zhang's method

Using Eq. (3) we evaluate the modified score values $S(d_{ij}^*)$ of each d_{ij}^* from the ideal decision matrix W^*, considered as final resultant matrix and the computed score matrix we have

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	0.741358	0.675392	0.721079	0.926154	0.545924
C_2	0.957322	0.872905	1.012508	1.127005	0.612852
C_3	0.853678	0.730042	0.956109	1.060461	0.608209
C_4	1.055256	0.841672	0.955915	1.176567	0.618449
C_5	0.640965	0.993971	0.996202	0.799872	0.512690
C_6	0.974277	0.763833	0.938797	1.018572	0.681262

In what follows, employing Eq. (8) and Eq. (9), we determine the interval-valued intuitionistic fuzzy PIS C^+ and interval-value intuitionistic fuzzy NIS C^- consequently as

$$C^+ = \{\langle([0.298,0.517],[0.270,0.435])\rangle, \langle([0.289,0.490],[0.314,0.477])\rangle, \langle([0.258,0.519],[0.286,0.466])\rangle, \langle([0.358,0.596],[0.216,0.385])\rangle, \langle([0.117,0.299],[0.418,0.636])\rangle\},$$

$$C^- = \{\langle([0.120,0.271],[0.470,0.640])\rangle, \langle([0.175,0.326],[0.476,0.674])\rangle, \langle([0.163,0.321],[0.415,0.629])\rangle, \langle([0.194,0.357],[0.392,0.560])\rangle, \langle([0.108,0.200],[0.526,0.757])\rangle\}.$$

Thereinafter, employing Eq. (12) and Eq. (10–11), for the computed ideal weight vector and distance measure of each choice C_i we obtain

$$\lambda_j = [0.204, 0.190, 0.197, 0.219, 0.190],$$

$$\Delta_1^+ = 0.1352, \Delta_2^+ = 0.0349, \Delta_3^+ = 0.0785, \Delta_4^+ = 0.0301, \Delta_5^+ = 0.1060, \Delta_6^+ = 0.057,$$

$$\Delta_1^- = 0.0308, \Delta_2^- = 0.1316, \Delta_3^- = 0.0999, \Delta_4^- = 0.1363, \Delta_5^- = 0.0587, \Delta_6^- = 0.1088.$$

Finally, utilizing Eq. (13), the relative closeness of the indices is respectively computed as:

$$cl_1 = 0.1856, cl_2 = 0.7904, cl_3 = 0.5601, cl_4 = 0.8189, cl_5 = 0.3563, cl_6 = 0.6563.$$

To find the best supplier we just need to rank the cl_i in descending order

$$cl_4 > cl_2 > cl_6 > cl_3 > cl_5 > cl_1.$$

and we observe that the relative closeness degree of choice C_4 is greater than others. Thus it will be ideal to select C_4.

4.2 Modified Cheng's method

In conformity with Eq. (14) and concentrating on Table 7, we demonstrate the optimal WEV $\aleph_i$ of each choice C_i:

$$\aleph_i = \left\langle \begin{matrix} ([0.609962, 0.870373],[0.015542, 0.087038]), ([0.732533, 0.949748],[0.00312, 0.032401]), \\ ([0.716776, 0.923116],[0.006345, 0.051632]), ([0.762598, 0.956412],[0.00334, 0.030845]), \\ ([0.664, 0.901],[0.00852, 0.060084]), ([0.710834, 0.932414],[0.004323, 0.042977]) \end{matrix} \right\rangle$$

Consequently upon to Eq. (3), the modified score values $S(\aleph_i)$ appertaining to each $\aleph_i$ are illustrated below

$$\varphi(\aleph_i) = \langle 1.688878, 1.82338, 1.790958, 1.842413, 1.748198, 1.797974 \rangle.$$

Based on Definition 2.6, this gives the ranking of the suppliers as

$$C_1 \leq C_5 \leq C_3 \leq C_6 \leq C_2 \leq C_4.$$

Thus the ideal decision is to select C_4.

4.3 Result exhibited for Zhang's method for a predefined weight vector of the parameters

We presume that the weight vector of the parameters is $\theta = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\} = \{0.30, 0.20, 0.15, 0.20, 0.15\}$. Utilizing Eq. (15) of [27], the computed decision matrix R^* is formed in Table 8.

Table 8. Ideal decision matrix R^*.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.3,0.536], [0.317,0.422])	([0.4,0.507], [0.276,0.493])	([0.354,0.496], [0.271,0.416])	([0.4,0.596], [0.229,0.330])	([0.235,0.335], [0.391,0.564])
C_2	([0.476,0.644], [0.144,0.271])	([0.499,0.658], [0.182,0.288])	([0.507,0.728], [0.144,0.252])	([0.536,0.738], [0.1,0.229])	([0.172,0.435], [0.330,0.5])
C_3	([0.441,0.584], [0.2,0.330])	([0.244,0.406], [0.2,0.363])	([0.542,0.728], [0.144,0.271])	([0.608,0.711], [0.182,0.288])	([0.239,0.415], [0.363,0.543])
C_4	([0.569,0.748], [0.126,0.229])	([0.374,0.632], [0.252,0.368])	([0.499,0.636], [0.159,0.288])	([0.608,0.771], [0.1,0.2])	([0.273,0.406], [0.363,0.501])
C_5	([0.235,0.406], [0.356,0.493)	([0.608,0.738], [0.126,0.229])	([0.578,0.711], [0.126,0.229])	([0.340,0.469], [0.3,0.431])	([0.204,0.305], [0.391,0.621])
C_6	([0.507,0.636], [0.159,0.3])	([0.374,0.542], [0.252,0.391])	([0.542,0.644], [0.159,0.288])	([0.467,0.636], [0.159,0.3])	([0.235,0.481], [0.252,0.438])

Hereinafter, in conformity with Eq. (12) of [27], the ranking table for the appraisement degree of the choice set C corresponding to the parameters ε_j evaluated by experts e_1, e_2 and e_3 (focus on to R_1, R_2, R_3 and ideal decision matrix R^*) is demonstrated in Table 9, as given below:

Table 9. Ranking table for R_1, R_2, R_3 and R^*.

	Ranking table for R_1	Ranking table for R_2	Ranking table for R_3	Ranking table for R^*
ε_1	$C_6 \geq C_4 \geq C_2 \geq C_1 \geq C_3 \geq C_5$	$C_4 \geq C_3 \geq C_2 \geq C_6 \geq C_1 \geq C_5$	$C_4 \geq C_2 \geq C_3 \geq C_6 \geq C_1 \geq C_5$	$C_4 \geq C_2 \geq C_6 \geq C_3 \geq C_1 \geq C_5$
ε_2	$C_2 \geq C_5 \geq C_4 \geq C_3 \geq C_6 \geq C_1$	$C_4 \geq C_5 \geq C_2 \geq C_6 = C_3 \geq C_1$	$C_5 \geq C_1 \geq C_6 \geq C_2 \geq C_3 \geq C_4$	$C_5 \geq C_2 \geq C_4 \geq C_6 \geq C_1 \geq C_3$
ε_3	$C_4 \geq C_2 \geq C_6 \geq C_5 \geq C_3 \geq C_1$	$c_5 \geq c_3 \geq c_6 \geq c_1 \geq c_4 \geq c_2$	$C_2 \geq C_3 \geq C_5 \geq C_1 \geq C_4 \geq C_6$	$C_5 \geq C_3 \geq C_2 \geq C_6 \geq C_4 \geq C_1$
ε_4	$C_3 \geq C_2 = C_4 \geq C_6 \geq C_1 \geq C_5$	$C_4 \geq C_2 = C_3 \geq C_6 \geq C_1 \geq C_5$	$C_4 \geq C_2 \geq C_6 \geq C_1 \geq C_3 \geq C_5$	$C_4 \geq C_2 \geq C_3 \geq C_6 \geq C_1 \geq C_5$
ε_5	$C_3 \geq C_2 \geq C_4 \geq C_6 = C_1 = C_5$	$C_6 = C_4 \geq C_3 \geq C_2 = C_1 \geq C_5$	$C_6 \geq C_2 \geq C_1 \geq C_5 \geq C_4 \geq C_3$	$C_6 \geq C_4 \geq C_2 \geq C_3 \geq C_1 \geq C_5$

Next, employing the dominance classes of the choices $C_i(i=1,2,3,4,5,6)$ according to the parameters ε_j (j = 1, 2, 3, 4, 5) depend on Table 9 provided by the experts $e_k(j=1,2,3)$ and Eq. (16) of [27] we demonstrate the consensus indices $\Im_k(\varepsilon_j)$ between the experts {e_1, e_2, e_3} and the ideal expert e^* regarding the choices made in accordance to the parameters, which are illustrated in Table 10.

Table 10. Consensus indices $\Im_k(\varepsilon_j)$.

	ε_1	ε_2	ε_3	ε_4	ε_5
$\Im_1(\varepsilon_j)$	0.805	0.833	0.655	0.833	0.333
$\Im_2(\varepsilon_j)$	0.800	0.808	0.48	1.000	0.550
$\Im_3(\varepsilon_j)$	0.858	0.464	0.683	0.819	0.530

Afterwards, using Eq. (17) of [27] and concentrating Table 10, the weight sum (WS_k) of the consensus degrees between the experts {e_1, e_2, e_3} and the ideal expert e^* for the choices C_i according to the parameters ε_j are computed as below

$$WS_1 = 0.7229, WS_2 = 0.7561, WS_3 = 0.69595.$$

Thereafter, by utilizing Eq. (19) of [27] the optimal weight vector (OWV) of the experts {e_1, e_2, e_3}, we obtain $\vartheta = \{\vartheta_1^*, \vartheta_2^*, \vartheta_3^*\} = \{0.332375, 0.34764, 0.319984\}$.

After computing the optimal weight vector of the experts, we should compute the collective decision matrix by employing Eq. (20) of [27] and this is illustrated in Table 11.

Table 11. Collective decision matrix Λ.

	ε_1	ε_2	ε_3	ε_4	ε_5
C_1	([0.3,0.539], [0.310,0.415])	([0.401,0.507], [0.276,0.491])	([0.356,0.502], [0.267,0.409])	([0.4,0.566], [0.232,0.333])	([0.233,0.333], [0.386,0.561])
C_2	([0.473,0.647], [0.141,0.267])	([0.490,0.651], [0.183,0.289])	([0.496,0.720], [0.149,0.258])	([0.534,0.736], [0.1,0.232])	([0.169,0.434], [0.329,0.5])
C_3	([0.444,0.590], [0.193,0.322])	([0.252,0.411], [0.2,0.365])	([0.545,0.733], [0.141,0.267])	([0.608,0.711], [0.183,0.289])	([0.241,0.419], [0.353,0.535])
C_4	([0.571,0.751], [0.124,0.227])	([0.381,0.642], [0.241,0.358])	([0.490,0.627], [0.1610.293])	([0.604,0.773], [0.1,0.2])	([0.280,0.410], [0.353,0.489])
C_5	([0.238,0.410], [0.352,0.488)	([0.604,0.736], [0.124,0.227])	([0.585,0.717], [0.124,0.227])	([0.333,0.465], [0.3,0.429])	([0.204,0.304], [0.391,0.620])
C_6	([0.506,0.635], [0.155,0.3])	([0.371,0.545], [0.249,0.391])	([0.545,0.647], [0.155,0.283])	([0.465,0.635], [0.155,0.3])	([0.238,0.487], [0.249,0.430])

Afterwards, using Eq. (21) and Eq. (22) of [27], we exhibited the interval-valued intuitionistic fuzzy PIS C^+ and interval-value intuitionistic fuzzy NIS C^- consequently represented as:

C^{+} = {⟨([0.571,0.751],[0.124,0.227])⟩, ⟨([0.604,0.736],[0.124,0.227])⟩, ⟨([0.585,0.717],[0.124,0.227])⟩, ⟨([0.604,0.772],[0.1,0.2])⟩, ⟨([0.238,0.487],[0.249,0.430])⟩}

C^{-} = {⟨([0.238,0.410],[0.352,0.488])⟩, ⟨([0.252,0.411],[0.2,0.365])⟩, ⟨([0.356,0.502],[0.267,0.409])⟩, ⟨([0.333,0.465],[0.3,0.429])⟩, ⟨([0.204,0.304],[0.391,0.620])⟩}.

Next, focus onto PIS C^+ and NIS C^- employing Eq. (23) and Eq. (24) of [27], we compute the distance measure of each choice C_i, accordingly shown as

$$\Delta_1^+ = 0.19046, \Delta_2^+ = 0.065092, \Delta_3^+ = 0.116286, \Delta_4^+ = 0.052336, \Delta_5^+ = 0.161996, \Delta_6^+ = 0.090056,$$

$$\Delta_1^- = 0.070264, \Delta_2^- = 0.184870, \Delta_3^- = 0.13263, \Delta_4^- = 0.198381, \Delta_5^- = 0.079632, \Delta_6^- = 0.158715.$$

Finally, the relative closeness of the indices is computed according to the formula $cl_i = \frac{\Delta_i^-}{\Delta_i^- + \Delta_i^+}$, which produces $cl_1 = 0.269, cl_2 = 0.739, cl_3 = 0.532, cl_4 = 0.791, cl_5 = 0.329, cl_6 = 0.638.$ After arranging the cl_i's in descending order: $cl_4 > cl_2 > cl_6 > cl_3 > cl_5 > cl_1,$ we observe that the ideal choice is C_4.

4.4 Result exhibited for Cheng's method for predefined weight vector of the parameters and experts

Using the addition operation of IVIFNs illustrated in Eq. (1), in addition to the weighted evaluation matrices and the operation $a_{ik} = d_{i1}^k \oplus d_{i2}^k \oplus \cdots \oplus d_{in}^k,$ the aggregated evaluation matrix of the choices regarding to {e_1, e_2, e_3} is illustrated in Table 12.

Table 12. Aggregated evaluation matrix.

	e_1	e_2	e_3
C_1	([0.498066,0.796779], [0.039574,0.162861])	([0.638696,0.872589], [0.009541,0.068222])	([0.673692,0.916035], [0.009951,0.059349])
C_2	([0.752545,0.964417], [0.002695,0.032124])	([0.614813,0.913514], [0.003811,0.04188])	([0.79878,0.959112], [0.00299,0.025547])
C_3	([0.679987,0.904316], [0.009646,0.07468])	([0.767146,0.941999], [0.002924,0.030363])	([0.696794,0.91862], [0.00919,0.051383])
C_4	([0.778779,0.951882], [0.003974,0.035127])	([0.766239,0.961587], [0.001731,0.021997])	([0.741674,0.955275], [0.005427,0.037939])
C_5	([0.85552,0.960008], [0.016398,0.097007])	([0.650245,0.907816], [0.00522,0.044856])	([0.636878,0.891162], [0.007233,0.04997])
C_6	([0.772234,0.932582], [0.00753,0.054601])	([0.682837,0.936326], [0.001984,0.035193])	([0.667181,0.928614], [0.005464,0.041681])

Now, the score values $S(a_{ik})$ and aggregated group evaluation values g_i, of each choice according to each expert are demonstrated in Table 13. (Initially Cheng takes the weight vector of the experts is $W_t^1 = \frac{1}{3}$.)

Table 13. Score values $S(a_{ik})$ and aggregated group evaluation values g_i.

	e_1	e_2	e_3	$g_i = \sum_{k=1}^{p}\left(W_t^1 \times S(a_{ik})\right)$
C_1	1.546205	1.716761	1.760214	1.674393
C_2	1.841072	1.741318	1.864678	1.815689
C_3	1.749989	1.837929	1.777421	1.788446
C_4	1.845780	1.852049	1.826792	1.841540
C_5	1.851062	1.753993	1.735419	1.780158
C_6	1.821343	1.790993	1.774325	1.795554

Based on Xu's ranking method [18] of IVIFNs, the preference vectors and group preference vector are illustrated thus and so

$$\begin{array}{c} \quad c_1 \;\; c_2 \;\; c_3 \;\; c_4 \;\; c_5 \;\; c_6 \\ O^1 = [6 \quad 3 \quad 5 \quad 2 \quad 1 \quad 4], \\ O^2 = [6 \quad 5 \quad 2 \quad 1 \quad 4 \quad 3], \\ O^3 = [5 \quad 1 \quad 3 \quad 2 \quad 6 \quad 4], \\ T = [6 \quad 2 \quad 4 \quad 1 \quad 5 \quad 3]. \end{array}$$

This gives the value of weight similarity degrees $S(T,O^1)=1, S(T,O^2)=11, S(T,O^3)=0.$ Also $z_1 = 0.08333, z_2 = 0.91667, z_3 = 0$ and group consensus degree $GC^1 = 0.33333.$ Since Cheng's method depends on the consensus degree, the higher the consensus degree that occurs, the more ideal and reliable the obtained result can be considered.

To get a maximum consensus degree, Cheng proposes to repeat the steps as long as the consensus degree is greater or equal to a predefined threshold value. Let us take a threshold value of 0.8000. New weight W_t^2 of the experts at the second round, according to the formula

$$W_t^{l+1} = \frac{\ell_t^{l+1}}{\sum_{t=1}^{k} \ell_t^{l+1}},$$

where $\left(\ell_t^{l+1} = W_t^l \times (1 + z_t)\right) \in [0,2],$ is calculated as

$$W_1^2 = 0.270833, W_2^2 = 0.479167, W_3^2 = 0.25.$$

Over and over, conducting Step 5 to Step 8 of the Cheng's method, the obtained group consensus degree at the 7-th round is 0.853037, which we observe to be greater than the predefined threshold value. At this stage, the process stops and finally we obtain the group preference vector

$$\begin{array}{c} \quad c_1 \;\; c_2 \;\; c_3 \;\; c_4 \;\; c_5 \;\; c_6 \\ G^7 = [6 \quad 5 \quad 2 \quad 1 \quad 4 \quad 3] \end{array}$$

That provided the preference rank of the supplier's choices $C_i\ (i=1, 2, 3, 4, 5, 6)$ as:

$$C_4 < C_3 < C_6 < C_5 < C_2 < C_1.$$

Thus, the ideal choice is C_4.

5 Conclusions

In this study, we have developed two modified methods for any AMPGDM problem combining Zhang *et al.* [27] and Cheng [16] models. Our method differs significantly in that we utilize IVIFN's to specify the weight vectors of the parameters. These weight vectors are established by experts and utilized to derive an ideal vector. This ideal vector is then used to construct the ideal resultant matrix. This modification optimizes the strategy by providing a more accessible and ideal solution that was developed specifically with expert decision in mind. Our method is effective due to the fact that, in spite of its simplification, it yields results that are nearly identical to those of the previous methods. We have a plan to employ distance and similarity measures in the proposed method in future work. This will assist us in improving the proposed strategy, especially studying further modifications that ensure the continued improvement of our methodology and provide valuable insights in decision-making problems.

Acknowledgements

We extend our sincere gratitude to the experts whose valuable insights and guidance shaped the weight vectors used in this study. Their contributions were instrumental in deriving the ideal vectors and resultant matrices that underpin our modified methodology. We are deeply thankful for the foundational work of Zhang *et al.* [27] and Cheng [16], which provided the basis for our proposed enhancements. Finally, we acknowledge our institution for its assistance in providing the resources and an ideal environment required to conduct this study.

References

[1] Atanassov, K. (1983). Intuitionistic fuzzy sets. *VII ITKR's Session, Sofia, June 1983.* Deposed in Central Sci.-Techn. Library of Bulg. Acad. of Sci., 1697/84) (in Bulgarian). Reprinted: *Int. J. Bioautomation*, 2016, 20(S1), S1–S6.

[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Set and Systems*, 20(1), 87–96.

[3] Atanassov, K. (1994). New operations defined over Intuitionistic fuzzy sets. *Fuzzy Sets and Systems,* 61(2), 137–142.

[4] Atanassov, K. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*. Springer Physica-Verlag, Heidelberg.

[5] Atanassov, K. (1999). Interval valued intuitionistic fuzzy sets. In: *Intuitionistic Fuzzy Sets: Theory and Applications* (pp. 139–177). Springer Physica-Verlag, Heidelberg.

[6] Atanassov, K. (1997). Some operators in Intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 3(4), 28–33.

[7] Atanassov, K. T. (2014). *Index Matrices: Towards an Augmented Matrix Calculus*. Studies in Computational Intelligence, Vol. 573. Cham: Springer International Publishing.

[8] Atanassov, K. & Gargov, G. (1989). Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 31(3), 343–349.

[9] Atanassov, K., Pasi, G., & Yager, R. (2005). Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making. *International Journal of Systems Science*, 36(14), 859–868.

[10] Bhattacharya, J. (2023). Some new results on intuitionistic fuzzy operators. *Notes on Intuitionistic Fuzzy Sets*, 29(3), 247–260.

[11] Chang, Y. H., Yeh, C. H., & Chang, Y. W. (2013). A new method selection approach for fuzzy group multicriteria decision making. *Applied Soft Computing*, 13(4), 2179–2187.

[12] Chen, T. Y. (2015). The inclusion-based TOPSIS method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision making. *Applied Soft Computing*, 26, 57–73.

[13] Chen, S. M., & Chang, C. H. (2016). Fuzzy multiattribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators. *Information Sciences*, 352, 133–149.

[14] Chen, S. M., Cheng, S. H., & Lan, T. C. (2016). Multicriteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values. *Information Sciences*, 367, 279–295.

[15] Chen, S. M., Cheng, S. H., & Tsai, W. H. (2016). Multiple attribute group decision making based on interval-valued intuitionistic fuzzy aggregation operators and transformation techniques of interval-valued intuitionistic fuzzy values. *Information Sciences*, 367, 418–442.

[16] Cheng, S. H. (2018). Autocratic multiattribute group decision making for hotel location selection based on interval-valued intuitionistic fuzzy sets. *Information Sciences*, 427, 77–87.

[17] Islam, R., Hossain, M. S., & Hoque, M. F. (2020). A study on intuitionistic L-fuzzy T_1 spaces. *Notes on Intuitionistic Fuzzy Sets*, 26(3), 33–42.

[18] Jiang, Y., Xu, Z., & Shu, Y. (2017). Interval-valued intuitionistic multiplicative aggregation in group decision making. *Granular Computing*, 2, 387–407.

[19] Wang, L. (2020). A multi-criteria group decision making method and its applications based on improved intuitionistic fuzzy entropy and information integration operator. *International Journal of Information and Management Science*, 31, 375–392.

[20] Wibowo, S. (2013). Interval-valued intuitionistic fuzzy multicriteria group decision making approach for hotel selection. *International Journal of Machine Learning and Computing*, 3(1), 65–69.

[21] Xu, Z. (2007). Intuitionistic fuzzy aggregation operators. *IEEE Transactions on Fuzzy Systems*, 15(6), 1179–1187.

[22] Xu, Z. (2007). Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. *Control and Decision*, 22(2), 215–219.

[23] Xu, Z. (2010). A method based on distance measure for interval-valued intuitionistic fuzzy group decision making. *Information sciences*, 180(1), 181–190.

[24] Ye, J. (2012). Multicriteria decision-making method using the Dice similarity measure based on the reduct intuitionistic fuzzy sets of interval-valued intuitionistic fuzzy sets. *Applied Mathematical Modelling*, 36(9), 4466–4472.

[25] Yue, Z. (2011). Deriving decision maker's weights based on distance measure for interval-valued intuitionistic fuzzy group decision making. *Expert Systems with Applications*, 38(9), 11665–11670.

[26] Yue, Z. (2012). Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling*, 36(7), 2900–2910.

[27] Zhang, X., & Xu, Z. (2015). Soft computing based on maximizing consensus and fuzzy TOPSIS approach to interval-valued intuitionistic fuzzy group decision making. *Applied Soft Computing*, 26, 42–56.