Notes on Intuitionistic Fuzzy Sets

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The role of operations over intuitionistic fuzzy index matrices in color image analysis

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Abstract: In this paper, an attempt has been made to define contrast intensification operator on intuitionistic fuzzy index matrices, which is useful for enhancing color images. Further, an algorithm is designed and developed for image preprocessing. The validity is verified by using a few real RGB images.

Keywords: Contrast intensification operator, Image processing algorithm, Intuitionistic fuzzy sets, Intuitionistic fuzzy index matrices.

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1 Introduction

In 1965, Zadeh introduced the idea of fuzzy sets, which are the extension of crisp sets. Following this, in 1983, Atanassov introduced intuitionistic fuzzy sets (IFSs), [1]. IFSs are suitable tool for



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modelling the hesitancy arising from imperfect information. Intuitionistic fuzzy sets are defined by two characteristic functions, namely the membership and the non-membership. Membership function plays key role in systems which need to apply the idea of fuzziness. It finds extensive application in image processing areas to describe vague concepts, which can be represented in the form of membership functions. The membership values lie in the interval [0,1] with 0 signifying no membership and 1 signifying full membership. Atanassov also found that another uncertainty or hesitation underlying the definition of the membership function. Intuitionistic fuzzy sets introduce uncertainty in the membership degree which is known as the hesitancy degree. It is well known that traditional fuzzy set theory assigns a membership degree to each element and the non-membership degree is classically computed as one minus the membership degree. However, human thinking often involves uncertainty or imprecision and sometimes standard fuzzy sets are not enough because of the presence of hesitance or uncertainty or the partial knowledge in the membership definition. Following this idea, Atanassov [1,2] introduced the concept of intuitionistic fuzzy set, which aims to reflect the fact that the degree of non-membership is not always equal to one minus the membership degree due to the presence of some hesitation.

The concept of Index Matrix (IM) was introduced in 1984 by Atanassov, but during the next 25 years some of its properties have only been studied and in general the concept has only been used as an auxiliary tool in generalized nets [3] and in the theory of intuitionistic fuzzy sets [6]. A further modification of the intuitionistic fuzzy index matrices (IFIMs) are the extended intuitionistic fuzzy index matrices (EIFIMs) [4], whose indices of the rows and columns are evaluated by intuitionistic fuzzy pairs. The 3-dimensional intuitionistic fuzzy index matrices (3-D IFIMs) were introduced in [5].

Subsequently, some basic operations and modal operators over the 3-D IFIMS were investigated in [23]. A few applications of the apparatus of IM are discussed in [21,24].

An RGB image of 8-bit having the maximum colors is red, green and blue. Under this condition each RGB color pixel that is a triplet of values (R, G, B) is said to have a depth of 24 bits. Krassimir T. Atanassov introduced operations over 3D-IMs [6]. This serves as a motivation to introduce 3-folded intuitionistic fuzzy index matrix (3f-IFIM) and is used for representing RGB images. On 3f-IFIM, operations are defined.

The paper is organized as follows: In Section 2, review of literature is given. Section 3 is devoted to the basic concepts of sets, fuzzy sets, intuitionistic fuzzy sets. Section 4 deals with preliminaries of IFIMs and 3f-IFIMs. In Section 5, the contrast intensification operator on IFIMs is defined. Intuitionistic fuzzy inference system is explained in Section 6. Algorithm, its implementation and discussion are given in Sections 7 and 8.

2 Literature review

In [6], Krassimir T. Atanassov defined IMs and discussed different types of IMs. Further, the definitions of the operations, relations and operators over IMs with real number elements (R-IM) are introduced. The author also defined IFIM. Standard operations over EIFIMs, which are analogous to the usual matrix operations of addition and multiplication, are defined. From the

concept of three dimensional index matrices and intuitionistic fuzzy index matrices the author introduced 3-folded intuitionistic fuzzy index matrices. The author also developed the concept of operations over intuitionistic fuzzy index matrices by the reference of operation over 3-dimensional index matrices (3D-IMs). In [7], Atanassov et al. illustrate the concepts of two- and three-dimensional intuitionistic fuzzy index matrices with suitable examples.

In [10], Bureva et al. introduced eight new operations over intuitionistic fuzzy index matrices and discussed some of their basic properties. In [8], Atanassov and Pencheva defined Cartesian products on intuitionistic fuzzy index matrices and discussed their properties.

In [21], Parvathi et al. introduced operations like addition, vertex-wise multiplication, multiplication, and structural subtraction on intuitionistic fuzzy graphs using index matrices.

In [25], Traneva et al. present an index matrix interpretation of on-line analytic processing (OLAP) cube. The aim is to present the basic OLAP operations with similar operations from the index matrix theory. The operations are discussed in terms of relational algebra and multidimensional mode.

In [11], Chaira discussed a new approach to color clustering and use of IFS theory in pathological cell images using various color models RGB, HSV. The algorithm is also tested on conventional fuzzy C-means algorithm (FCM). It is observed that intuitionistic fuzzy clusters give better results than FCM. The reason is that when the membership function is not accurately defined due to personal error, then the intuitionistic fuzzy set gives better results where uncertainty in the form of a hesitation degree is used.

Couto et al. [14] proposed a general methodology for RGB color image segmentation using IFSs (termed by the authors as A-IFSs). The proposed methodology is based on a multilevel color thresholding framework that uses Atanassov's intuitionistic index values for representing the uncertainty present in assigning pixels to different regions. This framework is to be applied to each RGB component separately to finally aggregate the results of the three components. The number of thresholds is automatically established by the methodology based on the image entropy, [14].

In [13], the video processing algorithm VIPROC is developed with temporal intuitionistic fuzzy sets to enhance videos. VIPROC algorithm is designed using contrast intensification operation for video enhancement. The results are encouraging in comparison with the original test videos and the results are discussed taking into account the several frames of the test video. VIPROC algorithm is very useful in real-time and medical video processing to enhance video quality.

In [9], Bouchet et al. proposed a new method for color image segmentation of leukocytes based on intuitionistic fuzzy sets.

After reading and reviewing the existing research work and tools on image representation and enhancement, the authors got motivated to use 3f-IFIMs to represent RGB images and proceed further with its applications in enhancing quality of RGB images.

3 Image representation using crisp sets, fuzzy sets and intuitionistic fuzzy sets

In this section, an overview is given for sets, FSs and IFSs. A set A of a universal set X can be defined by its characteristic function χ_A as a mapping from the elements of the universal set X to the values of the set $\{0,1\}$: that is, $\chi_A: X \to \{0,1\}$.

The mapping can be represented as a set of ordered pairs $\{\langle x, \chi_A(x) \rangle\}$ with exactly one ordered pair present for each element of X. The first element of the ordered pair is an element of the set X and the second is its value in $\{0,1\}$. The value 0 is used to represent non-membership and the value 1 is used to represent the membership of the element of A. The truth or falsity of the statement "x is A" is determined by the ordered pair. The statement is true if the second element of the ordered pair is 1, and the statement is false if it is 0.

Let X be a non-empty set. A *fuzzy set* (FS) A in X is characterized by its membership function $\mu_A: x \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of elements X in the fuzzy set A, for each $x \in X$. A is completely determined by $A = \{\langle x, \mu_A(x) \rangle \mid x \in X\}$.

Let X be a non-empty set. An *intuitionistic fuzzy set* (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for each $x \in X$. The intuitionistic fuzzy set can also be written in the form $A = \langle \mu_A(x), \nu_A(x) \rangle$ or simply, $A = \langle \mu_A, \nu_A \rangle$.

The value $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the *degree of non-determinacy* (or the *degree of uncertainty*) of the element $x \in X$ to the intuitionistic fuzzy set A.

A monochrome image, or simply image, is represented mathematically by a spatial brightness function f(m,n) where (m,n) denotes the spatial coordinates of a point in the (flat) image. The value of $f(m,n), 0 < f(m,n) < \infty$, is proportional to the brightness value or gray level of the image at the point (m,n). For computer processing, the continuous function f(m,n) has been discretized both in spatial coordinates and in brightness. Such an approximated image X (digitized) can be considered as an $m \times n$ array.

$$X = f(m,n) = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix}$$

whose row and column indices identify a point (m, n) in the image and the corresponding matrix element value x_{mn} denotes the gray level at that point. Each element of the matrix, which is a discrete quantity, is referred to as an image element or picture element (pixel). For processing, this image along with the coordinates of its pixels is stored in the computer in the form of an array of mn numbers.

As crisp set A is a two valued logic, binary (Black and White) images are represented by the

sets. In matrix notation, a binary image A with m rows and n columns is represented as,

$$A = [\langle x_{ij}, \chi_A(x_{ij}) \rangle]_{m \times n}.$$

A fuzzy set A is described by its membership function $\mu_A: X \to [0,1]$ which may be interpreted as many-valued logic. Hence, gray images are represented by fuzzy sets. Therefore, a gray image A with m rows and n columns is mathematically represented in matrix form as

$$A = [\langle x_{ij}, \mu_A(x_{ij}) \rangle]_{m \times n}.$$

Gray scale is one with 0 for white, 1 for black and any value between 0 and 1 for gray.

An intuitionistic fuzzy set A is described not only by its membership function $\mu_A: X \to [0,1]$ but also by its non-membership function $\nu_A: X \to [0,1]$ such that $0 \le \mu_A(x) + \nu_A(x) \le 1$. Specifically, hesitancy degree defined by the function, $\pi_A: X \to [0,1]$ also plays role in describing IFS satisfying $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$ for every $x \in X$.

Hence, it is noteworthy that an RGB image can be represented by an IFS, by taking three functions to define three colors. Therefore, an RGB image A is represented in matrix notation as

$$A = [\langle x_{ij}, \mu_A(x_{ij}), \nu_A(x_{ij}), \pi_A(x_{ij}) \rangle]_{m \times n}.$$

4 IFIM and 3f-IFIM

Definition 4.1 [17] Let I be a fixed set. *Intuitionistic fuzzy index matrix (IFIM)* with index sets K and L $(K, L \subset I)$ is an object of the form:

where for every $1 \le i \le m, 1 \le j \le n : 0 \le \mu_{k_i, l_j}, \nu_{k_i, l_j}, \mu_{k_i, l_j} + \nu_{k_i, l_j} \le 1, K = \{k_1, k_2, \dots, k_m\}, L = \{l_1, l_2, \dots, l_n\}$ for $1 \le i \le m$, and $1 \le j \le n : \mu_{k_i, l_j}, \nu_{k_i, l_j}, \mu_{k_i, l_j} + \nu_{k_i, l_j} \in [0, 1].$

Definition 4.2 [17] Let I be a fixed set. A 3-folded intuitionistic fuzzy index matrix (3f-IFIM) of rectangular RGB images with index sets R, G and B where R, G, $B \subseteq \{0, 1, 2, 3, \dots, 255\}$, take the form:

$$\left[R, G, B, \left\{ \left\langle \mu_{r_{ij}}, \nu_{g_{ij}}, \pi_{b_{ij}} \right\rangle \right\} \right]$$

$$= \left\{ \begin{array}{c|ccc} b_{ij} & g_{i1} & g_{i2} & \cdots & g_{in} \\ \hline r_{1j} & \left\langle \mu_{r_{11}}, \nu_{g_{11}}, \pi_{b_{11}} \right\rangle & \left\langle \mu_{r_{12}}, \nu_{g_{12}}, \pi_{b_{12}} \right\rangle & \cdots & \left\langle \mu_{r_{1n}}, \nu_{g_{1n}}, \pi_{b_{1n}} \right\rangle \\ \hline r_{2j} & \left\langle \mu_{r_{21}}, \nu_{g_{21}}, \pi_{b_{21}} \right\rangle & \left\langle \mu_{r_{22}}, \nu_{g_{22}}, \pi_{b_{22}} \right\rangle & \cdots & \left\langle \mu_{r_{2n}}, \nu_{g_{2n}}, \pi_{b_{2n}} \right\rangle \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline r_{mj} & \left\langle \mu_{r_{m1}}, \nu_{g_{m1}}, \pi_{b_{m1}} \right\rangle & \left\langle \mu_{r_{m2}}, \nu_{g_{m2}}, \pi_{b_{m2}} \right\rangle & \cdots & \left\langle \mu_{r_{mn}}, \nu_{g_{mn}}, \pi_{b_{mn}} \right\rangle \end{array} \right\},$$

where for every $1 \le i \le m, 1 \le j \le n : 0 \le \mu_{r_{ij}}, \nu_{g_{ij}}, \pi_{b_{ij}} \le 1, 0 \le \mu_{r_{ij}} + \nu_{g_{ij}} + \pi_{b_{ij}} \le 3$ and $R = \{r_{ij}\}, G = \{g_{ij}\}, B = \{b_{ij}\}$ for every i and j, are crisp subsets of $\{0, 1, 2, \dots, 255\}$.

5 Contrast intensification operator

In this section, the basic concept of contrast intensification operator on intuitionistic fuzzy images is defined and discussed.

In [18, 13] the contrast intensification operator on an IFS A of the universe X has been defined. The contrast intensification operator is denoted by INTEN(A) and defined as $INTEN(A) = \{\langle x, \mu_{INTEN}(x), \nu_{INTEN}(x) \rangle \mid x \in X\}$, where

$$\mu_{INTEN}(x) = 1 - (1 - \mu_A(x)^2)^2,$$

$$\nu_{INTEN}(x) = [1 - (1 - \mu_A(x))^2]^2.$$

6 Intuitionistic fuzzy inference system

The design of the traditional logic controller [16] usually requires a mathematical model of the process involved. The construction of such a model is difficult for many real-world problems due to partial or unreliable information. The imprecise description of the problem can be handled as an alternative approach by expert human operations. This modelling leads to the usage of fuzzy concepts that are closer to human perception than the traditional logical system. The basic structure of an intuitionistic fuzzy inference system is shown below.

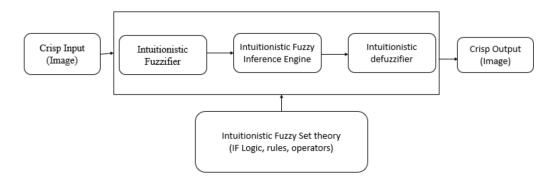


Figure 1. Structure of an intuitionistic fuzzy inference system, [13]

The intuitionistic fuzzy inference system includes the following components.

- (1) *Intuitionistic fuzzifier*: Intuitionistic fuzzification transforms input crisp values into intuitionistic fuzzy values. Nine types of intuitionistic fuzzification functions are defined based on different shapes of the membership and non-membership functions. Appropriate intuitionistic fuzzy membership function can be selected for intuitionistic fuzzification, [20].
- (2) *Intuitionistic fuzzy inference engine*: Intuitionistic fuzzy values are then used by intuitionistic fuzzy inference engine where necessary modifications (using either intuitionistic fuzzy *IF-THEN* rules or intuitionistic fuzzy logic, or intuitionistic fuzzy operations) are done to obtain the desired result. Here, the membership and non-membership functions are modified to improve the quality of the input objects. This is where the applications of intuitionistic fuzzy sets are implemented.

(3) *Intuitionistic defuzzifier*: Intuitionistic defuzzification function transforms intuitionistic fuzzy (output from inference engine) values into crisp values. Various types of intuitionistic defuzzification functions such as triangular, trapezoidal, L-trapezoidal, R-trapezoidal, Gaussian, S-shaped, Z-shaped functions are defined [19]. A suitable intuitionistic fuzzy defuzzification function should be selected for defuzzification.

Remarks:

- (1) If any of the defined fuzzification functions is not found suitable for the specific problem, the user can define an appropriate fuzzification function, based on the requirement.
- (2) Unlike defuzzification to a single value in the fuzzy controller, intuitionistic defuzzification gives a matrix of defuzzified values in [0, 1], corresponding to the gray levels of the given image.

7 Algorithm

The stepwise procedure for enhancing images is given below:

Step 1: Read the input RGB image.

Step 2: Separate Red, Green and Blue channels from the input image.

$$R = [R_{ij}]_{m \times n}, G = [G_{ij}]_{m \times n}, B = [B_{ij}]_{m \times n}.$$

Step 3: Set the arbitrary constants c_1, c_2 and calculate X_{max} .

Step 4: Using intuitionistic fuzzification function, define membership function:

$$\mu_{mn} = \left[1 + \left(\frac{r_{\max} - r_{mn}}{c_2}\right)\right]^{-c_1}$$

Step 5: Calculate the non-membership values in terms of membership values.

$$\nu_{mn} = \begin{cases} \frac{1}{2} \max[|1 - \mu_{mn}|, |0 - \mu_{mn}|], 0 \le \mu_{mn} \le 0.5, \\ \frac{1}{2} \min[|1 - \mu_{mn}|, |0 - \mu_{mn}|], 0.5 \le \mu_{mn} \le 1, \end{cases}$$

such that $0 \le \mu_{mn} + \nu_{mn} \le 1$.

Step 6: Modify membership and non-membership values using contrast intensification on IFS.

$$\mu'_{mn} = 1 - (1 - \mu_{mn}^2)^2, 0 \le \mu_{mn} \le 1,$$

$$\nu'_{mn} = (1 - (1 - \nu_{mn})^2)^2, 0 \le \nu_{mn} \le 1,$$

Step 7: Calculate the new gray level using the modified membership and non-membership values.

$$g'_{mn} = g_{\text{max}} - c_2 * ((\sqrt{\mu'_{mn} * (c_3 - \nu'_{mn})})^{-\frac{1}{c_4 * c_1}}) + c_2),$$

where c_3 , c_4 are arbitrary constants.

Step 8: Convert gray image to an RGB image.

Step 9: After merge the output RGB images by using MATLAB functions the output RGB flower image is obtain.

Note:

- The algorithm is explained by taking Red image values. The same process is followed for Green and Blue images.
- The parameters c_1, c_2, c_3 and c_4 are arbitrary and can be tuned using techniques of perameter tuning.

7.1 Implementation

To verify the algorithm, a test RGB image is taken for the processing. It is read pixel by pixel. For the implementation process, flower image is considered. It is shown in Figure 2. For computational convenience, 4×3 matrices are consided for R, G, B channels.

Step 1: Read the input image, for spliting Red, Green and Blue channels.



Figure 2. Input image

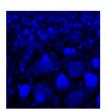
Step 2: Read the Red, Green and Blue channels from the input images, as shown in Figure 3.



(a) Red channel



(b) Green channel



(c) Blue channel

Figure 3. Red, Green and Blue channels

The matrices obtained are of order 109×109 . For the sake of computational convenience, the original matrices are resized to matrices of order 4×3 as given below.

$$R_{mn} = \begin{bmatrix} 236 & 232 & 233 \\ 236 & 236 & 236 \\ 236 & 236 & 236 \\ 235 & 236 & 236 \end{bmatrix}$$

$$G_{mn} = \begin{bmatrix} 174 & 147 & 148 \\ 119 & 132 & 130 \\ 123 & 142 & 150 \\ 107 & 114 & 142 \end{bmatrix}$$

$$\begin{bmatrix}
63 & 34 & 54 \\
45 & 2 & 37 \\
49 & 33 & 64 \\
68 & 32 & 55
\end{bmatrix}$$

Step 3: Set the arbitrary constants c_1, c_2 .

$$c_1 = 25,$$

 $c_2 = 15.$

$$c_1 = 2.1,$$

 $c_2 = 15.47.$

$$c_1=2,$$

$$c_2 = 15.$$

Step 4: Using intuitionistic fuzzification function, find the membership values of R, G and B channels. For example,

$$\mu_{R_{11}} = \left[1 + \left(\frac{237 - 236}{15}\right)\right]^{-25} = 0.4564$$

Same process for all the remaining values of Red, Green and Blue channels.

$\mu_{R_{mn}} =$				$\mu_{G_{mn}} =$				ŀ	$\mu_{B_{mn}} =$			
	0.4564	0.5266	0.5575		0.0722	0.0873	0.0942		0.0553	0.0264	0.0434	
	0.3986	0.3447	0.2807		0.0345	0.0502	0.0469		0.0335	0.0144	0.0283	
	0.2122	0.3778	0.3341		0.0386	0.0730	0.0994		0.0377	0.0255	0.0580	
	0.0645	0.2987	0.5165		0.0256	0.0303	0.0715		0.0654	0.0252	0.0445	l

Step 5: Obtain the non-membership values in terms of membership values for R, G and B channels.

<u>Step 6:</u> Modify the membership and non-membership values by using the contrast intensification on operator IFS for Red, Green and Blue channels R, G and B, as given below.

• New membership values:

$$\mu_{R_{mn}}' = \mu_{G_{mn}}' = \mu_{G_{mn}}' = \mu_{B_{mn}}' =$$

$$\begin{bmatrix} 0.4564 & 0.5266 & 0.5575 \\ 0.3986 & 0.3447 & 0.2807 \\ 0.2122 & 0.3778 & 0.3341 \\ 0.0645 & 0.2987 & 0.5165 \end{bmatrix} \begin{bmatrix} 0.0430 & 0.0458 & 0.0579 \\ 0.0056 & 0.0114 & 0.0193 \\ 0.0107 & 0.0287 & 0.0488 \\ 0.0062 & 0.0041 & 0.0287 \end{bmatrix} \begin{bmatrix} 0.0426 & 0.0137 & 0.0307 \\ 0.0241 & 0.0006 & 0.0166 \\ 0.0287 & 0.0104 & 0.0324 \\ 0.0514 & 0.0152 & 0.0320 \end{bmatrix}$$

• New non-membership values:

Step 7: The new gray level using the new membership and non-membership values are calculated for R, G and B channels and the respective gray images are shown in Figure 4: Set the arbitrary constants c_3 and c_4 as follows:

$$c_3 = 145,$$
 $c_3 = 30,$ $c_3 = 10,$ $c_4 = 255.$ $c_4 = 20.$ $c_4 = 2.$

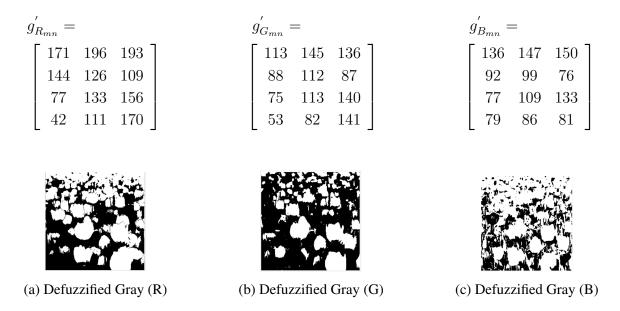


Figure 4. Defuzzified Gray channels for Red, Green and Blue

Step 8: Convert gray images to R, G and B images as given in Figure 5:

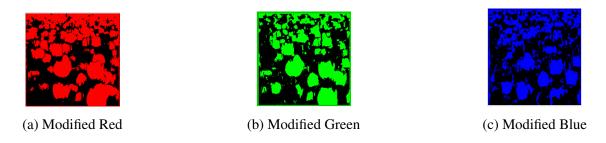


Figure 5. Converted Red, Green and Blue channels

Step 9: After merge the output R, G and B images by using MATLAB functions and obtain the output RGB image as shown in Figure 6.



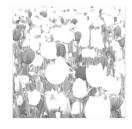
Figure 6. Final output RGB image

8 Results and discussion

In this section, a comparison is made with other existing algorithms. For the same input image given in Figure 2, the contrast intensified images obtained using fuzzy, intuitionistic fuzzy and temporal intuitionistic fuzzy algorithms are shown in Figure 7, [18, 13].







(a) Fuzzy image, [18]

(b) Intuitionistic fuzzy image, [18]

(c) Temporal IF image, [13]

Figure 7. Output images using the existing algorithm

It is concluded from Figure 6 and Figure 7 that for an RGB color image, a gray output image is obtained in the existing algorithms, whereas the proposed algorithm gives an RGB output image which is more necessary in color image preprocessing. Hence, the proposed novel algorithm provides better results than the existing algorithms.

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