Notes on Intuitionistic Fuzzy Sets
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Uniformly expanding intuitionistic fuzzy operator

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Abstract: A new intuitionistic fuzzy topological operator is introduced. Some of its basic properties are studied.
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In memory of Prof. Ilarion Yanchev
(1937 – 2016)

1 Introduction

Let E be a fixed universe and let $A \subset E$ be a fixed set. The object

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

is called an Intuitionistic Fuzzy Set (IFS, see, e.g., [1, 2]), where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the *degree of membership* and the *degree of non-membership* of the element $x \in E$ to the set A, respectively, and for every $x \in E$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Below, we use only the IFS A^* and by this reason, we omit the asterisk, writing A instead of A^*.

Let

$$E^* = \{\langle x, 1, 0\rangle | x \in E\},$$

$$O^* = \{\langle x, 0, 1\rangle | x \in E\}.$$

The topological operators C and I are defined for every IFS A, by

$$C(A) = \{\langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y)\rangle | x \in E\},$$

$$I(A) = \{\langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y)\rangle | x \in E\}.$$

The modal operators $\Box$ and $\Diamond$ are defined for every IFS A, by

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\},$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.$$

The extended modal operator $G_{\alpha,\beta}$ is defined for the real numbers $\alpha, \beta \in [0, 1]$ and for every IFS A, by

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x)\rangle | x \in E\}.$$

2 Uniformly expanding intuitionistic fuzzy operator

Let us define the operator U over the IFS A by

$$\begin{aligned} &\sup_y \mu_A(y) > \inf_y \mu_A(y), \\ &\sup_y \nu_A(y) > \inf_y \nu_A(y), \end{aligned} \tag{1}$$

by:

$$U(A) = \left\{ \left\langle x, \frac{\mu_A(x) - \inf_y \mu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)}, \frac{\nu_A(x) - \inf_y \nu_A(y)}{\sup_y \nu_A(y) - \inf_y \nu_A(y)} \right\rangle | x \in E \right\}.$$

Immediately, we can see that if we denote

$$U(A, x) \equiv U(A, \langle \mu_A(x), \nu_A(x)\rangle) = \left\langle \frac{\mu_A(x) - \inf_y \mu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)}, \frac{\nu_A(x) - \inf_y \nu_A(y)}{\sup_y \nu_A(y) - \inf_y \nu_A(y)} \right\rangle,$$

where for $x \in E$, $\langle x, \mu_A(x), \nu_A(x)\rangle \in A$, then we obtain the degrees of membership and non-membership of element $x \in E$ after applying operator U over IFS A.

The geometrical interpretation of the new operator is given on Fig. 1., where the intuitionistic fuzzy interpretational triangle is shown.

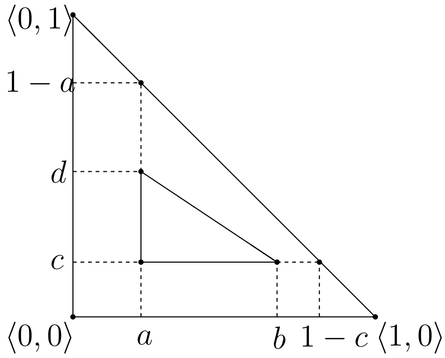

Fig. 1.

Therefore,

$$U(A, \langle a, c\rangle) = \langle 0, 0\rangle,$$
$$U(A, \langle b, c\rangle) = \langle 1, 0\rangle,$$
$$U(A, \langle a, d\rangle) = \langle 0, 1\rangle.$$

Now, we must prove that the definition is correct if inequalities (1) are valid, i.e.,

$$X \equiv \frac{\mu_A(x) - \inf_y \mu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)} + \frac{\nu_A(x) - \inf_y \nu_A(y)}{\sup_y \nu_A(y) - \inf_y \nu_A(y)} \leq 1.$$

If we construct the line, connecting the points with coordinates $\langle a, c\rangle$ and $\langle b, d\rangle$, its analytical equation will be

$$\alpha p + \beta q = 1. \qquad (2)$$

Therefore, we obtain the system

$$\left| \begin{array}{l} b\alpha + c\beta = 1 \\ a\alpha + d\beta = 1 \end{array} \right.,$$

that has solution

$$\alpha = \frac{d-c}{bd-ac} \quad \beta = \frac{b-a}{bd-ac}.$$

Therefore, (2) has the form

$$\frac{d-c}{bd-ac}x + \frac{b-a}{bd-ac}y = 1.$$

Hence, if $p \in [a, b]$ and $q \in [c, d]$, then

$$\frac{d-c}{bd-ac}p + \frac{b-a}{bd-ac}q \leq 1$$

and therefore,

$$q \leq \frac{bd-ac}{b-a}\left(1 - \frac{d-c}{bd-ac}p\right). \qquad (3)$$

Now, using (3) and putting

$$a = \inf_y \mu_A(y),$$

$$b = \sup_y \mu_A(y),$$

$$c = \inf_y \nu_A(y),$$

$$d = \sup_y \nu_A(y),$$

the check of definition's validity is the following

$$X \leq \frac{\mu_A(x) - \inf_y \mu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)}$$

$$+ \frac{\frac{\sup_y \mu_A(y).\sup_y \nu_A(y) - \inf_y \mu_A(y).\inf_y \nu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)} \left(1 - \frac{\sup_y \nu_A(y) - \inf_y \nu_A(y)}{\sup_y \mu_A(y).\sup_y \nu_A(y) - \inf_y \mu_A(y).\inf_y \nu_A(y)} \mu_A(x)\right) - \inf_y \nu_A(y)}{\sup_y \nu_A(y) - \inf_y \nu_A(y)}$$

$$= \frac{\mu_A(x) - \inf_y \mu_A(y)}{\sup_y \mu_A(y) - \inf_y \mu_A(y)}$$

$$+ \frac{\sup_y \mu_A(y).\sup_y \nu_A(y) - \inf_y \mu_A(y).\inf_y \nu_A(y)}{(\sup_y \mu_A(y) - \inf_y \mu_A(y))(\sup_y \nu_A(y) - \inf_y \nu_A(y))}$$

$$- \frac{(\sup_y \nu_A(y) - \inf_y \nu_A(y))\mu_A(x) - (\sup_y \mu_A(y) - \inf_y \mu_A(y)) \inf_y \nu_A(y)}{(\sup_y \mu_A(y) - \inf_y \mu_A(y))(\sup_y \nu_A(y) - \inf_y \nu_A(y))}$$

$$= \frac{1}{(\sup_y \mu_A(y) - \inf_y \mu_A(y))(\sup_y \nu_A(y) - \inf_y \nu_A(y))} \Big(\sup_y \mu_A(y).\sup_y \nu_A(y) - \inf_y \mu_A(y).\inf_y \nu_A(y)$$

$$- (\sup_y \nu_A(y) - \inf_y \nu_A(y)) \inf_y \mu_A(x) - (\sup_y \mu_A(y) - \inf_y \mu_A(y)) \inf_y \nu_A(y)\Big)$$

$$= \frac{1}{(\sup_y \mu_A(y) - \inf_y \mu_A(y))(\sup_y \nu_A(y) - \inf_y \nu_A(y))} \Big(\sup_y \mu_A(y).\sup_y \nu_A(y) - \inf_y \mu_A(y).\inf_y \nu_A(y)$$

$$- \sup_y \nu_A(y) \inf_y \mu_A(x) + \inf_y \nu_A(y) \inf_y \mu_A(x) - \sup_y \mu_A(y) \inf_y \nu_A(y) + \inf_y \mu_A(y) \inf_y \nu_A(y)\Big)$$

$$= \frac{1}{(\sup_y \mu_A(y) - \inf_y \mu_A(y))(\sup_y \nu_A(y) - \inf_y \nu_A(y))} \Big(\sup_y \mu_A(y).\sup_y \nu_A(y)$$

$$- \sup_y \nu_A(y) \inf_y \mu_A(x) + \inf_y \nu_A(y) \inf_y \mu_A(x) - \sup_y \mu_A(y) \inf_y \nu_A(y)\Big) = 1.$$

Therefore, the definition of operator U is correct. Since it uniformly expands the intuitionistic fuzzy interpretational triangle, we call it "Uniformly expanding intuitionistic fuzzy operator".

The following assertions are proved analogously.

Theorem 1. For every IFS A satisfying (1):
(a) $U(C(A)) = E^* = C(U(A))$,

(b) $U(I(A)) = O^* = I(U(A))$,

(c) $U(U(A)) = U(A)$,

(d) $\neg U(\neg A) = U(A)$.

Theorem 2. For every IFS A satisfying (1):

(a) $U(\Box A) = \Box U(A)$,

(b) $U(\Diamond A) = \Diamond U(A)$.

Theorem 3. For every IFS A satisfying (1) and for every two real numbers $\alpha, \beta \in [0, 1]$:

$$U(G_{\alpha,\beta}(A)) = U(A).$$

3 Conclusion

The so introduced operator can be a basis for defining a new quantifier in intuitionistic fuzzy predicate logic, that will be a theme of the next author's research.

The operator U can be used for the aims of Intercriterial analysis (see, e.g. [3, 4]) for modifying the regions in the intuitionistic fuzzy interpretational triangle, when the results of this analysis are collected in the central parts of the triangle. So, we can increase the region of the these results.

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References

[1] Atanassov, K. (1999) *Intuitionistic Fuzzy Sets,* Springer, Heidelberg.

[2] Atanassov, K. (2012) *On Intuitionistic Fuzzy Sets Theory,* Springer, Berlin.

[3] Atanassov, K. (2014) *Index Matrices: Towards an Augmented Matrix Calculus*, Springer, Cham.

[4] Atanassov K., Mavrov, D. & Atanassova, V. (2014) Intercriteria Decision Making: A New Approach for Multicriteria Decision Making, Based on Index Matrices and Intuitionistic Fuzzy Sets. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets*, Vol. 11, 1–8.