Notes on Intuitionistic Fuzzy Sets
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Note on multiattribute decision making in intuitionistic fuzzy context

Shiny Jose[1] and Sunny Kuriakose[2]

[1] St. George's College Aruvithura, Kerala, India
email: shinyjosedavis@gmail.com

[2] B. P. C. College Piravom, Kerala, India
email: asunnyk@gmail.com

Abstract: Zhi Pei and Li Zheng [8], introduced a particular score function and an accuracy function to rank intuitionistic fuzzy sets (IFSs). They considered the degree of membership, degree of non membership and degree of hesitation with descending order of importance. We further recall an optimization model to estimate the relative degree of importance of each quantity given by the above mentioned authors. In this paper we identify certain anomalies in the accuracy function given in [8], we propose a new accuracy function and a revised optimization model.
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1 Introduction

Following the introduction of Fuzzy set (FS) by L. A. Zadeh in 1965, Krassimir Atanassov introduced the notion of IFS which has been found a better tool to model decision problems. Multi-criteria decision making methods based on IFS theoretical tools were introduced in the decision theory in 2007 by Z. S. Xu. This was extended to IVIFS [4]. Later many researchers studied the problem of ranking IFSs. Zhi Pei and Li Zheng studied this problem and proposed a score function and an accuracy function by giving relative importance to degree of membership, degree of non membership and to the degree of hesitation [8]. By rectifying the error in the accuracy function given in [8], we propose another accuracy function which can be used to solve MADM problems. Section 2 contains basic definitions and results. In Section 3, we define an accuracy function and discuss its properties. Also, we define an optimization model in this section. An illustration is given in Section 4.

2 Preliminaries

Definition 2.1 [1]. Let X be a given set. An Intuitionistic fuzzy set A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\},$$

where $\mu_A, \nu_A : X \to [0,1]$, $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of non membership of x in A, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. For each $x \in X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the degree of hesitation.

Definition 2.2 [1]. If A and B are IFSs in X, then
(i) Complement of A, i.e. $\overline{A} = \{(x,\ \nu_A(x),\ \mu_A(x)), x \in X\}$
(ii) Intersection,$A \cap B = \{(x, min(\mu_A(x),\ \mu_B(x)),\ max(\nu_A(x),\ \nu_B(x)))|x \in X\}$
(iii) Union, $A \cup B = \{(x,\ max(\mu_A(x),\ \mu_B(x)),\ min(\nu_A(x),\ \nu_B(x)))|x \in X\}$
(iv) Scalar Multiplication, nA = $\{(x,\ 1-(1-\mu_A(x))^n, (\nu_A(x))^n)|x \in X\}$
(v) Power, $A^n = \{(x,\ (\mu_A(x))^n),\ 1-(1-\nu_A(x))^n)|x \in X\}$.
Following the introduction of operations of IFSs by Atanassov, Xu and Yager [5], Xu [6, 7] introduced the notion of aggregation operator of intuitionistic fuzzy numbers (IFNs for short).
Definition 2.3 [3]. A fuzzy set A on R, the set of real numbers, is said to be a fuzzy number if A satisfies the following properties
(i) A must be a normal fuzzy subset of R;
(ii) each α cut of A must be closed interval for every $\alpha \in (0,1]$;
(iii) the support of A, ${}^{0+}A$, must be bounded.
Definition 2.4 [1]. An intuitionistic fuzzy set(IFS) $A = (\mu_A,\ \nu_A)$ of R is said to be an intuitionistic fuzzy number if μ_A and ν_A are fuzzy numbers with $\mu_A,\ \nu_A \in [0,1]$ and $\mu_A + \nu_A \leq 1$.
Final ranking of the alternatives in MADM problems is determined by the ranking of the corresponding IFNs [6].
Definition 2.5 [6, 7]. Assume $a_i = (\mu_{a_i},\ \nu_{a_i})$ are IFNs, and $b_i = (\mu_{b_i},\ \nu_{b_i})$ are ordered IFNs of $a_i = (\mu_{a_i},\ \nu_{a_i})$ from large to small for $i = 1, ..., n$.
(i) If $\omega = (\omega_1, ..., \omega_n)$ is the weight vector of $(a_1, ..., a_n)$, then the aggregation operator of intuitionistic fuzzy weighted average is defined by

$$IFWA_\omega(a_1, ..., a_n) = \omega_1 a_1 + ... + \omega_n a_n.$$

(ii) If $\omega = (\omega_1, ...\omega_n)$ is the exponential weight vector of $(a_1, ..., a_n)$, then the aggregation operator of intuitionistic fuzzy weighted geometric is defined by

$$IFWG_\omega(a_1, ..., a_n) = a_1^{\omega_1} ... a_n^{\omega_n}.$$

(iii) If $\omega = (\omega_1, ...\omega_n)$ is the weight vector of position, then the aggregation operator of intuitionistic fuzzy ordered weighted average is defined by

$$IFOWA_\omega(a_1, ..., a_n) = \omega_1 b_1 + ... + \omega_n b_n.$$

(iv) If $\omega = (\omega_1, ...\omega_n)$ is the exponential weight vector of position, then the aggregation operator of intuitionistic fuzzy ordered weighted geometric is defined by

$$IFOWG_\omega(a_1, ..., a_n) = b_1^{\omega_1} ... b_n^{\omega_n}.$$

In 2007, Xu [6] applied aggregation operators as a better tool to obtain a single IFN for each alternative, and then compared the aggregated IFNs. Each aggregated IFN represents one alternative respectively.

Later, besides Xu, many researchers studied the problem of ranking of IFNs. Noted among them are Lakshmana Gomathi Nayagam, V., Venkatesvari, G. and Geetha Sivaraman [4]. Zhi Pei and Li Zheng [8], ranked the IFNs and based on that, they ranked the alternatives, by giving decending order of importance to the degree of membership, degree of non membership and to degree of hesitation. In [8], they defined a score function and an accuracy function to rank the alternatives in MADM problems. Now we recall the definition of score function and accuracy function introduced by Zhi Pei and Li Zheng.

Definition 2.6 [8]. Consider a decision making model with n alternatives $\{A_1, ..., A_n\}$, with respect to the m attributes $\{T_1, ..., T_m\}$. If μ_{ij} and ν_{ij} denote the degree to which ith alternative satisfies jth attribute and the degree to which ith alternative does not satisfies jth attribute respectively. Also if the IFN (μ_i, ν_i) for the ith alternative A_i is, $\mu_i = \sum_{j=1}^{m} w_j \mu_{ij}$ and $\nu_i = \sum_{j=1}^{m} w_j \nu_{ij}$ where w_j is the normalized weight of the j^{th} attribute, which satisfying $w_j > 0$ and $\sum_{j=1}^{m} w_j = 1$. Then the score function and accuracy function for the i^{th} alternative A_i are respectively defined by

$$S_i = \alpha\mu_i - \beta\nu_i \tag{1}$$

and

$$H_i = \alpha\mu_i - \gamma\pi_i, \tag{2}$$

where α stands for the relative importance of the degree of membership, similarly β for non membership and γ for hesitation. Also $\alpha, \beta, \gamma \in [0, 1]$ with $\alpha \geq \beta \geq \gamma$ and $\alpha + \beta + \gamma = 1$. Also in [8] they used the following linear programming model, which could be solved by simplex method to obtain the values of α, β and γ.
max

$$\sum_{i=1}^{n} S_i = \sum_{i=1}^{n} (\alpha\mu_i - \beta\nu_i) \tag{3}$$

such that,

$$\begin{aligned} \alpha' &\leq \alpha \leq \alpha'', \\ \beta' &\leq \beta \leq \beta'', \\ \gamma' &\leq \gamma \leq \gamma'', \\ \alpha &\geq \beta \geq \gamma, \\ \alpha + \beta + \gamma &= 1, \end{aligned}$$

where the objective function is the sum of the score functions for all the alternatives. By substituting the values of α and β in equation (1), Zhi Pei and Li Zheng, ranked the alternatives A_i. If $S_i = S_j$ for $i \neq j$, they used equations (1) and (2) to rank the alternatives.

Even though the score function given by them can rank the alternatives successfully, their accuracy function is insufficient to rank the alternatives.

For two comparable alternatives A_1 and A_2 with A_1 better than A_2,

$$H_1 - H_2 \;=\; (\mu_1 - \mu_2)(1-\beta) \;+\; (\nu_1 - \nu_2)(1-\alpha-\beta).$$

The first term is positive, while the second is negative. So we are not sure whether $H_1 \;>\; H_2$.

It is clear in the following counter example.

Let $A_1 \;=\; (.55, .35)$, and $A_2 \;=\; (.54, .43)$, clearly A_1 better than A_2. But H_1 = .265 and H_2 = .267, by rectifying this error, in the next section we introduce an accuracy function which ranks the alternatives correctly.

3 New accuracy function

Now we define an accuracy function and an optimization model as follows.

Definition 3.1. Let $A_1, ..., A_n$ be n alternatives having $T_1, ..., T_m$ as their attributes, with μ_{ij}, ν_{ij}, π_{ij}, μ_i, ν_i, π_i were as defined in Definition 2.6 such that $\mu_{ij} + \nu_{ij} + \pi_{ij} \;=\; 1$ for $i = 1, ..., n$ and $j = 1, ..., m$. π_{ij} is the hesitancy part. Also $\mu_i \;+\; \nu_i \;+\; \pi_i = 1$. Then the new accuracy function is defined as

$$S(A_i) \;=\; \alpha\mu_i \;-\; \beta\nu_i \;-\; \gamma\pi_i \tag{4}$$

where α, β, γ are parameters, which are obtained by solving the following linear programming model by simplex method.

$$\max \sum_{i=1}^{n} S_i \;=\; \sum_{i=1}^{n} (\alpha\mu_i \;-\; \beta\nu_i \;-\; \gamma\pi_i) \tag{5}$$

with

(i) $\alpha' \;\leq\; \alpha \;\leq\; \alpha'', \beta' \;\leq\; \beta \;\leq\; \beta'' \;\; \gamma' \;\leq\; \gamma \;\leq\; \gamma\gamma''$,

(ii) $\alpha \;\geq\; \beta \;\geq\; \gamma$, and

(iii) $\alpha \;+\; \beta + \;\gamma \;=\; 1$,

where $\alpha, \alpha', \alpha'', \beta, \beta', \beta'', \gamma, \gamma', \gamma'' \;\in\; [0, 1]$.

Theorem 3.1. For any two comparable IFSs A and B, if $A \subset B$, then $S(A) < S(B)$.

Proof. Let $A \;=\; (\mu_1, \nu_1)$ and $B \;=\; (\mu_2, \nu_2)$ be two comparable IFSs such that $A \;\subset\; B$, then $\mu_1 < \mu_2$ and $\nu_1 > \nu_2$. Then, $S(B) - S(A) = (\alpha\mu_2 - \beta\nu_2 - \gamma\pi_2) - (\alpha\mu_1 - \beta\nu_1 - \gamma\pi_1)$.

By substituting γ with $1 - \alpha - \beta$ and π_i with $1 - \mu_i - \nu_i$, we get

$$S(B) - S(A) = (\mu_2 - \mu_1)(1-\beta) + (\nu_1 - \nu_2)(2\beta + \alpha - 1) > 0.$$

4 Illustrative example

A man was offered five jobs, $A_1, A_2, ..., A_5$. He has to decide which among the five jobs to choose, with regard to the following three attributes: close driving distance (T_1), good salary (T_2), chance for promotion (T_3).

By including the hesitancy part also, the decision matrix with weight vector of the attributes $T_j (j = 1, 2, 3)$ as $w = (0.3, 0.4, 0.3)$ can be given by

	T_1	T_2	T_3
A_1	$(.7, .2, .1)$	$(.9, .1, 0)$	$(.8, .2, 0)$
A_2	$(.7, .3, 0)$	$(.6, .1, .3)$	$(.7, .2, .1)$
A_3	$(.4, .1007, .4993)$	$(.375, .3, .325)$	$(.6, .01, .39)$
A_4	$(.9, .1, 0)$	$(.8, .1, .1)$	$(.5, .4, .1)$
A_5	$(.55, .35, .1)$	$(.462, .28, .258)$	$(.4874, .235, .2776)$

The weighted average value of the degree of membership, non membership and the hesitancy of the alternatives are

$$\begin{aligned} \mu_1 &= (.7 \times .3) + (.9 \times .4) + (.8 \times .3) = .81 \\ \nu_1 &= (.2 \times .3) + (.1 \times .4) + (.2 \times .3) = .16 \\ \pi_1 &= (.1 \times .3) + (0 \times .4) + (0 \times .3) = .03 \end{aligned}$$

Similarly,

$$\begin{aligned} \mu_2 &= .66,\ \mu_3 = .45,\ \mu_4 = .74,\ \mu_5 = .496 \\ \nu_2 &= .19,\ \nu_3 = .1532,\ \nu_4 = .19,\ \nu_5 = .2875 \\ \pi_2 &= .15,\ \pi_3 = .3968,\ \pi_4 = .07,\ \pi_5 = .2165 \end{aligned}$$

Using the linear programming model (5), the objective function is

$$\max \sum_{i=1}^{n} S_i = 3.156\,\alpha - .9807\,\beta - .8633\,\gamma \tag{6}$$

with $.30 \leq \alpha \leq .90$, $.25 \leq \beta \leq .80$, $.02 \leq \gamma \leq .60$, where $\alpha \geq \beta \geq \gamma$, and $\alpha + \beta + \gamma = 1$.

Solving (6) using simplex method, we get $\alpha = .73,\ \beta = .25, \gamma = .02$

By using the score function given in equation (1) , we could get the score for each alternatives as $S'_1 = .5513,\ S'_2 = .4343,\ S'_3 = .2902, S'_4 = .4927,\ S'_5 = .2902$.

As the third and the fifth alternatives have the same score function value and as the accuracy function given in equation (2) is insufficient, we proceed with the revised accuracy function (4) , to get the accuracy function values of the alternatives, as follows,

$$H'_1 = .5507,\ H'_2 = .4313,\ H'_3 = .2823,\ H'_4 = .4913,\ H'_5 = .2858.$$

Now all the alternatives in the MADM problem has been distinguished as

$$H_1' > H_4' > H_2' > H_5' > H_3'.$$

Therefore, the final ranking of the alternatives should be

$$A_1 > A_4 > A_2 > A_5 > A_3.$$

5 Conclusion

In this paper we introduced a method of solving multi-attribute decision making problems in intuitionistic fuzzy environment. A score function and an accuracy function given by Zhi Pei and Li Zheng is introduced. Also, we introduced an optimization model to estimate the relative degree of importance of membership, non membership and hesitation by the above mentioned authors. We proposed another accuracy function, by rectifying the drawbacks in the accuracy function given by Zhi Pei and Li Zheng. Also, we proposed a revised optimization model to estimate the relative degree of importance of membership, non membership and hesitation.

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