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IF Nearness

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Abstract

The concept of a fuzzy nearness was introduced to solve some practical situations, where the T-equivalence is not suitable. We provide an IF version of a fuzzy nearness and show how it can be used to describe a situation with imprecise information and present examples of IF nearnesses.

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There are several methods, how to estimate the difference of given sets under the conditions of incomplete information (see e.g. [5], [6], [7] or [8]). Perhaps the ones used most frequently are methods of fuzzy metric spaces or various kinds of similarity measures. Let us consider one of these concepts, the T-equivalence, a little deeper. For the reader's convenience we will repeat its definition.

Definition 1 *Let X be a nonempty set and T a triangular norm. The mapping $E : X^2 \to [0;1]$ is called a fuzzy equivalence (T-equivalence), if*

- $E(x,x) = 1$ *for all* $x \in X$,
- $E(x,y) = E(y,x)$ *for all* $x, y \in X$,
- $T(E(x,y), E(y,z)) \leq E(x,z)$ *for all* $x, y, z \in X$.

Although thourougly studied and frequently used, De Cock and Kerre in [1] claim, that *T-equivalences are not suitable to model some real-life situations of approximate equality.* Shortly, the reason can be explained in the following consideration: If x is "almost like"y and y is "almost like"z, then x by far need not be "almost like"z – just think of resemblance between three generations in a family.

To avoid the above mentioned problems, at least in a metric space, the authors in [2] define the relation of a resemblence in the following way.

Definition 2 *Let (X, ρ) be a metric space, let T be a triangular norm. Then $R : X^2 \to [0;1]$ is a resemblance relation, if*

- $R(x, x) = 1$ *for all* $x \in X$,
- $R(x, y) = R(y, x)$ *for all* $x, y \in X$,
- $R(x, y) \geq R(x, z)$ *whenever* $\rho(x, y) \leq \rho(x, z)$.

Independently, Kalina in [3] defines the relation of a fuzzy nearness as follows:

Definition 3 *The binary fuzzy relation* N *on a real line with values in the interval [0; 1] is called a fuzzy nearness if for each* $x, y, t \in R$ *there is*

- $xNx = 1$,
- $xNy = yNx$,
- *if* t *is between* x *and* y *then* $xNt \geq xNy$.

Although this nearness is defined for the real line, it can be clearly (and in later works was) used for an arbitrary metric space or in [4] for a linear space. This will also be a base for our consideration.

The motivation for one possible extention of above metioned nearness relation is the following: Let us consider two finite sequences of the symbols 0,1:
$x = 1110001010$
$y = 1111001000$
As 2 of 10 characters are different, the Hamming distance h of these sequences is $h(x, y) = 2$, or, their normalised Hamming distance h^* is $h^*(x, y) = 0.2$.

Let us suppose now, that the characters on the third and the ninth position are not readable. Then the sequences are
$x = 11?00010?0$
$y = 11?10010?0$
Clearly the number of mutually different characters is at least 1 (the fourth position) and the number of mutually equal characters is 7. Using normalised forms, we can say that the grade of dissimilarity is 0.1 and the grade of similarity is 0.7.

More precisely, we can define the following:

Definition 4 *Let* (X, ρ, N, M) *be a quadruple such that* (X, ρ) *is a metric space, let* (N, M) *satisfy the following:*

- $N(x, x) = 1$ *for all* $x \in X$,
- $N(x, y) = N(y, x)$ *for all* $x, y \in X$,
- $N(x, y) \geq N(x, z)$ *whenever* $\rho(x, y) \leq \rho(x, z)$,
- $M(x, x) = 0$ *for all* $x \in X$,
- $M(x, y) = M(y, x)$ *for all* $x, y \in X$,
- $M(x, y) \leq M(x, z)$ *whenever* $\rho(x, y) \leq \rho(x, z)$,
- $N(x, y) + M(x, y) \leq 1$ *for all* $x, y \in X$.

Then (M, N) is called an IF-nearness on X.

The term nearness is used just to be in line with the terminology used in [3], but clearly only the function N reflects the nearness of x and y, while the function M corresponds to their "remoteness".

In a similar way we can define the IF-nearness in a linear space.

Definition 5 *Let (X, N, M) be a triple such that X is a linear space, let (N, M) satisfy the following:*

- $N(x, x) = 1$ *for all* $x \in X$,
- $N(x, y) = N(y, x)$ *for all* $x, y \in X$,
- $N(x, y) \geq N(x, z)$ *whenever* $y = \lambda x + (1 - \lambda)z, 0 < \lambda < 1$,
- $M(x, x) = 0$ *for all* $x \in X$,
- $M(x, y) = M(y, x)$ *for all* $x, y \in X$,
- $M(x, y) \leq M(x, z)$ *whenever* $y = \lambda x + (1 - \lambda)z, 0 < \lambda < 1$,
- $N(x, y) + M(x, y) \leq 1$ *for all* $x, y \in X$.

Then (M, N) is called an IF-nearness on X.

Finally, we introduce some examples of IF-nearnesses.

Example 1. Let N be a usual nearness in a metric space, let $M = 1 - N$. Then (M, N) is an IF-nearness.

Example 2. Let (X, ρ) be a metric space, let $x, y \in X$, let k, s be real numbers. Put $N(x, y) = \max\{0, 1 - k\rho(x, y)\}$ and $M(x, y) = \min\{1, s\rho(x, y)\}$. Then (M, N) is an IF-nearness if and only if $k \geq s > 0$.

Example 3. Let (X, ρ) be a metric space, let $a, b \in X$, let k, s be real numbers. Put $N(x, y) = a^{\rho(x,y)}$ and $M(x, y) = 1 - b^{\rho(x,y)}$. Then (M, N) is an IF-nearness if and only if $a \geq b > 0$.

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