11th Int. Workshop on IFSs, Banská Bystrica, Slovakia, 30 Oct. 2015
Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 21, 2015, No. 5, 20–23

Intercriteria analysis of the intuitionistic fuzzy implication properties

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Abstract: The apparatus of the intercriteria analysis is used for studying of some intuitionistic fuzzy implication properties. For this aim it is checked which axioms of the intuitionistic logic are satisfied by the separate implications.
Keywords: Implication, Intuitionistic fuzzy logic, Intuitionistic logic.
AMS Classification: 03E72.

1 Introduction

The Intercriteria Analysis (ICrA, see [2, 4, 5]) is a new tool for searching of relations between given criteria about some objects. Here, we use the notation from [2]. In a series of papers, different intuitionistic fuzzy implications are defined. All they are collected in [3]. Here, we check which axioms of the intuitionistic logic are satisfied by the separate implications. We use the following 17 axioms of the intuitionistic logic (see, e.g. [7]). If A, B and C are arbitrary propositional forms, then:

(IL1) $A \to A$,

(IL2) $A \to (B \to A)$,

(IL3) $A \to (B \to (A \& B))$,

(IL4) $(A \to (B \to C)) \to (B \to (A \to C))$,

(IL5) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$,

(IL6) $A \to \neg\neg A$,

(IL7) $\neg(A \& \neg A)$,

(IL8) $(\neg A \vee B) \to (A \to B)$,

(IL9) $\neg(A \vee B) \to (\neg A \& \neg B)$,

(IL10) $(\neg A \& \neg B) \to \neg(A \vee B)$,

(IL11) $(\neg A \vee \neg B) \to \neg(A \& B)$,

(IL12) $(A \to B) \to (\neg B \to \neg A)$,

(IL13) $(A \to \neg B) \to (B \to \neg A)$,

(IL14) $\neg\neg\neg A \to \neg A$,

(IL15) $\neg A \to \neg\neg\neg A$,

(IL16) $\neg\neg(A \to B) \to (A \to \neg\neg B)$,

(IL17) $(C \to A) \to ((C \to (A \to B)) \to (C \to B))$.

Now, for our purposes, we construct the following index matrix (IM, see [2]):

$$\begin{array}{c|ccc} & IL1 & \dots & IL17 \\ \hline \to_1 & \sigma_{1,1} & \dots & \sigma_{1,17} \\ \vdots & \vdots & \vdots & \vdots \\ \to_{185} & \sigma_{185,1} & \dots & \sigma_{185,17} \end{array},$$

where for every $i \in \{1, 2, \dots, 185\}$ and for every $j \in \{1, 2, \dots, 17\}$, $\sigma_{i,j}$ is equal to " $+$ ", if the i-th implication satisfies j-th axiom, or " $-$ ", in otherwise. Now, using the ICrA, we can construct the IM

$$\begin{array}{c|ccc} & \to_1 & \dots & \to_{185} \\ \hline \to_1 & \langle \mu_{1,1}, \nu_{1,1} \rangle & \dots & \langle \mu_{1,185k}, \nu_{1,185} \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \to_{185} & \langle \mu_{185,1}, \nu_{185,1} \rangle & \dots & \langle \mu_{185,185k}, \nu_{185,185} \rangle \end{array},$$

where for $k \in \{1, 2, \dots, 185\}$, $\mu_{i,k} = \frac{M(i,k)}{185}$, $\nu_{i,k} = \frac{N(i,k)}{185}$, $M(i,k)$ is the number of the cases when $\sigma_{i,j} = \sigma_{k,j} =$ "+", $N(i,k)$ is the number of the cases when $\sigma_{i,j} = \sigma_{k,j} =$ "$-$". Therefore, $\mu_{i,k}, \nu_{i,k} \in [0,1]$ and $\mu_{i,k} + \nu_{i,k} \leq 1$, i.e., $\langle \mu_{i,k}, \nu_{i,k} \rangle$ is an Intuitionistic Fuzzy Pair (IFP, see [6]).

In a lot of papers and books (see, e.g., [1]) the IFP $\langle \mu_{i,k}, \nu_{i,k} \rangle$ is called an Intuitionistic Fuzzy Tautology (IFT) if and only if $\mu_{i,k} \geq \nu_{i,k}$, and it is called a tautology, if $\mu_{i,k} = 1$ and $\nu_{i,k} = 0$.

If we like to study the IFPs that are tautologies, we obtain that these pairs are generated with participation of the implications $\to_3, \to_{11}, \to_{14}, \to_{20}, \to_{23}, \to_{74}, \to_{77}, \to_{153}$ only. These pairs are the following:

3	11	3	74	11	20	11	153	14	77	20	77	23	153
3	14	3	77	11	23	14	20	14	153	20	153	74	77
3	20	3	153	11	74	14	23	20	23	23	74	74	153
3	23	11	14	11	77	14	74	20	74	23	77	77	153

Moreover, the implications that generate IFPs that are IFTs, are: $\to_1, \to_3, \to_4, \to_5, \to_9, \to_{11}$, $\to_{13}$, $\to_{14}$, $\to_{17}$, $\to_{18}$, $\to_{20}$, $\to_{21}$, $\to_{22}$, $\to_{23}$, $\to_{27}$, $\to_{28}$, $\to_{29}$, $\to_{61}$, $\to_{66}$, $\to_{71}$, $\to_{74}$, $\to_{75}$, $\to_{76}$, $\to_{77}$, $\to_{79}$, $\to_{81}$, $\to_{100}$, $\to_{101}$, $\to_{102}$, $\to_{109}$, $\to_{110}$, $\to_{111}$, $\to_{112}$, $\to_{113}$, $\to_{118}$, $\to_{124}$, $\to_{125}$, $\to_{126}$, $\to_{127}$, $\to_{128}$, $\to_{151}$, $\to_{153}$, $\to_{158}$, $\to_{159}$, $\to_{160}$, $\to_{166}$, $\to_{167}$, $\to_{169}$, $\to_{170}$, $\to_{182}$, $\to_{185}$.

2 Conclusion

In the paper, we have shown all pairs of intuitionistic fuzzy implications that have similar behaviour about axioms of the intuitionistic logic. In a next research, we will show their behaviour about other axiomatic systems.

Acknowledgement

The first two authors are grateful for the support provided by the project DFNI-I-02-5 "InterCriteria Analysis A New Approach to Decision Making" funded by the Bulgarian National Science Fund.

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