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Remark on equalities between intuitionistic fuzzy sets

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Abstract: Four equalities between intuitionistic fuzzy sets are given. An open problem is formulated. described.

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In [1, 2], the following equality is proved for two fixed Intuitionistic Fuzzy Sets (IFSs, see [1]) A and B:

$$((A \cap B) + (A \cup B))@((A \cap B).(A \cup B)) = A@B,$$

where operations "$\cap$", "$\cup$", "$+$", "." and "@" are defined by

$$A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle \mid x \in E\},$$

$$A.B = \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle \mid x \in E\},$$

$$A@B = \{\langle x, (\frac{\mu_A(x)+\mu_B(x))}{2}, \frac{(\nu_A(x)+\nu_B(x))}{2}\rangle | x \in E\}.$$

Here, we shall formulate and prove four other elementary, but interesting equalities.

Theorem 1: For every two IFSs A and B:

$$(A \cap B)@(A \cup B)) = (A + B)@(A.B).$$

Proof: We shall use the fact that for every two real numbers a and b it follows

$$\max(a, b) + \min(a, b) = a + b.$$

Then

$$(A \cap B)@(A \cup B))$$

$$= (\{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\}$$

$$@\{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | x \in E\})$$

$$= \{\langle x, \frac{\min(\mu_A(x),\mu_B(x)),\max(\mu_A(x),\mu_B(x))}{2}, \frac{\max(\nu_A(x),\nu_B(x))+\min(\nu_A(x),\nu_B(x))}{2}\rangle | x \in E\})$$

$$
\begin{aligned}
&= \{\langle x, \tfrac{\mu_A(x)+\mu_B(x)}{2}, \tfrac{\nu_A(x)+\nu_B(x)}{2}\rangle | x \in E\}) \\
&= \{\langle x, \tfrac{(\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x))+\mu_A(x).\mu_B(x)}{2}, \\
&\quad \tfrac{\nu_A(x).\nu_B(x)+(\nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x))}{2}\rangle | x \in E\} \\
&= \{\langle x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\} \\
&\quad @\{\langle x, \mu_A(x).\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x)\rangle | x \in E\} \\
&= (A+B)@(A.B).
\end{aligned}
$$

The proofs of the next assertions are analogous.

Theorem 2: For every two IFSs A and B:

$$(A \cap B) \rightarrow (A \cup B) = (A \rightarrow B) \cup (B \rightarrow A),$$

where operation "$\rightarrow$" is defined by

$$A \rightarrow B = \{\langle x, max(\nu_A(x), \mu_B(x)), min(\mu_A(x), \nu_B(x))\rangle | x \in E\}.$$

It is the standard intuitionistic fuzzy implication that is an extension of Kleene-Dienes implicaion (see [3]).

Theorem 3: For every three IFSs A, B and C:

$$(A \rightarrow B) \cup (B \rightarrow C) \cup (C \rightarrow A) = (A \rightarrow C) \cup (C \rightarrow B) \cup (B \rightarrow A),$$

$$(A \rightarrow B) \cap (B \rightarrow C) \cap (C \rightarrow A) = (A \rightarrow C) \cap (C \rightarrow B) \cap (B \rightarrow A).$$

At the moment, it is an **open problem** whether there are equalities similar the the last one, but for the rest intuitionistic fuzzy implications.

References

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[3] Klir, G. and Bo Yuan, Fuzzy Sets and Fuzzy Logic. Prentice Hall, New Jersey, 1995.