Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
2026, Volume 32, Number 3, 231–244
DOI: 10.7546/nifs.2026.32.3.231-244

Intuitionistic fuzzy topological operators via controlled sets

Sinem Tarsuslu (Yılmaz) [1] **and Gökhan Çuvalcıoğlu** [2]

[1] Department of Natural and Math. Sciences, Faculty of Engineering, Tarsus University, Tarsus, Türkiye
e-mail: sinemtarsuslu@tarsus.edu.tr

[2] Department of Mathematics, Faculty of Sciences, Mersin University, Mersin, Türkiye
e-mail: gcuvalcioglu@mersin.edu.tr

Received: 4 September 2026 **Revised:** 30 September 2026
Accepted: 23 October 2026 **Online First:** 6 November 2026

Abstract: Topological concepts are widely used in the mathematical modeling of spatial objects with uncertain boundaries. Determining suitable operators for the interior, closure, and boundary is one of the main problems that researchers are interested in. Therefore, fuzzy topological and intuitionistic fuzzy topological operators have been examined in such problems. Different methods are used to fuzzify the elements of a region under study. However, determining the non-membership degrees of a region based on its membership degrees remains a separate problem.

In this study, this problem is addressed by applying controlled set theory. The resulting intuitionistic fuzzy representations are then processed using intuitionistic fuzzy pre-interior and pre-closure operators to examine the interior, closure, and boundary characteristics of the spatial regions affected by *Mikania micrantha*. The results show how the proposed method changes these values for different parameter settings and offers a new way to model uncertainty in spatial regions.

Keywords: Intuitionistic fuzzy sets, Topological operators, Controlled sets, Spatial objects modeling.

2020 Mathematics Subject Classification: 03E72.

Copyright © 2026 by the Authors. This is an Open Access paper distributed under the terms and conditions of the Creative Commons Attribution 4.0 International License (CC BY 4.0). https://creativecommons.org/licenses/by/4.0/

1 Introduction

The most fundamental topic for mathematical modeling of real-world problems is the determination of topological relationships. In addition to regions such as forests and rivers, many difficulties are encountered in modeling problems with uncertain boundaries such as fires, floods and epidemics in determining these relationships accurately and precisely. These uncertain situations that cannot be solved with crisp topology can be modeled with fuzzy topology. Fuzziness approach has numerous applications in modeling real-world problems and in decision making problems of engineering, health and social disciplines. Fuzzy topology concepts have been addressed theoretically and practically by many researchers. In the studies conducted in the following periods, it was seen that studying with only membership degrees was not sufficient. So, intuitionistic fuzzy topology concepts began to be used for real-world problems. Consequently, intuitionistic fuzzy topology began to be applied to real-world problems. Bustince *et al*. [4] investigated intuitionistic fuzzy generators and their application to intuitionistic fuzzy complementation, contributing to the theoretical development of intuitionistic fuzzy set representations. It was revealed by different researchers that the obtained results were closer to the truth. The most fundamental problem in the use of this concept in application areas is the determination of non-membership values. There are different approaches to defining this [12, 14]. In 2014, Çuvalcıoğlu presented a new approach to this problem. In this theory, the degree of non-membership of an element is related to the degree of membership of all other elements in the set. The theoretical properties of controlled sets have been studied by the author and their applications have been studied in different disciplines [7, 17, 19].

In this study, intuitionistic fuzzy set theory and pre-topological operators are used to examine spatial objects associated with *Mikania micrantha*. The membership degrees of the affected areas were calculated using the logarithmic function adopted from Shi and Liu [13]. In this application, only area magnitudes are considered; spatial adjacency and independently validated geographical boundary data are not included. Therefore, the results illustrate the effects of the selected membership and non-membership assignments on the resulting intuitionistic fuzzy representations and operator values, rather than demonstrating improved geographical accuracy. The resulting intuitionistic fuzzy representations are processed using the revised pre-interior and pre-closure operators, and the corresponding interior, closure, and operator-induced boundary values are examined. The main aim is to investigate how controlled-set-based non-membership assignment affects the intuitionistic fuzzy representation of spatial objects and the resulting operator values. The present study does not claim to introduce the operators for the first time; its contribution lies in examining the use of controlled sets for assigning non-membership degrees within the revised operator framework.

2 Preliminaries

Intuitionistic fuzzy set theory [2, 3] was introduced by Atanassov as an extension of Zadeh's fuzzy sets [20]. This concept was introduced by enlarging the truth value set to the lattice

$$L^* = \{(x_1, x_2): x_1 + x_2 \leq 1, x_1, x_2 \in [0,1]\}.$$

Definition 1. [2] *An intuitionistic fuzzy set (IFS) on a set* X *is an object*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where $\mu_A(x)$, $(\mu_A : X \to [0,1])$ *is the degree of membership of x in A,*$\nu_A(x)$, $(\nu_A : X \to [0,1])$ *is the degree of non- membership of x in A and* μ_A *and* ν_A *satisfy the following condition:*

$$\mu_A(x) + \nu_A(x) \leq 1, \textit{ for all } x \in X.$$

The class of intuitionistic fuzzy sets on X is denoted by IFS(X). The hesitation degree of x is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Some fundamental operations on intuitionistic fuzzy sets are given as follows.

Definition 2. [2] *An IFS A is said to be contained in an IFS B* $(A \sqsubseteq B)$ *if and only if, for all*

$$x \in X: \mu_A(x) \leq \mu_B(x) \textit{ and } \nu_A(x) \geq \nu_B(x),$$

$$A = B \Leftrightarrow A \sqsubseteq B \textit{ and } B \sqsubseteq A.$$

Definition 3. [2] L*et* $A \in$ IFS *and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ *then the complement of A is*

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}.$$

The intersection and the union of two IFSs A and B on X *are defined by*

$$A \sqcap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle : x \in X\},$$

$$A \sqcup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}\rangle : x \in X\}.$$

Some special intuitionistic fuzzy sets on X are defined as following:

$$O^* = \{\langle x, 0,1\rangle : x \in X\},$$

$$X^* = \{\langle x, 1,0\rangle : x \in X\}.$$

Atanassov defined topological operators and the extensions of these operators on intuitionistic fuzzy sets:

Definition 4. [3] *Let* $A \in IFS$ *and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$. *Then*

$$C(A) = \{\langle x, K, L\rangle : x \in X\}$$

where $K = sup_{y \in X}\mu_A(y)$, $L = \inf_{y \in X}\nu_A(y)$ *and*

$$I(A) = \{\langle x, k, \ell\rangle : x \in X\}$$

where $k = inf_{y \in X}\mu_A(y)$, $\ell = \sup_{y \in X}\nu_A(y)$.

Definition 5. [3] L*et* $A \in IFS$ *and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$. *Let K, L, k and* ℓ *be as above forms then,*

1. $C_\mu(A) = \{\langle x, K, \min(1 - K, \nu_A(x))\rangle : x \in X\}$;
2. $C_\nu(A) = \{\langle x, \mu_A(x), L\rangle : x \in X\}$;
3. $I_\mu(A) = \{\langle x, k, \nu_A(x)\rangle : x \in X\}$;
4. $I_\nu(A) = \{\langle x, \min(1 - \ell, \mu_A(x)), \ell\rangle : x \in X\}$.

Controlled set theory provides a systematic approach to determining non-membership degrees based on the membership degrees of elements in a set. In this approach, the non-membership degree of an element is related to the membership degrees of other elements in the same universe, subject to the condition that the sum of membership and non-membership degrees does not exceed one. The hesitation degree is then obtained as the remaining difference between one and the sum of these two degrees. Thus, controlled sets provide a consistent way to determine non-membership and hesitation degrees from the membership values of a given set.

In this study, controlled set theory is used to assign non-membership degrees to the regions affected by *Mikania micrantha*, complementing the membership degrees obtained from the logarithmic function.

The basic theoretical properties of the controlled set concept are as follows.

Definition 6. [6] *Let U be a universe, then* $\alpha: U \to I$ *is called* α*-set.*

From the definition, if *X* is a universe and μ is a fuzzy set on *X*, then *X* is a μ-set.

Definition 7. [6] *Let U be an* α*-set. The set U is called* α*-controlled set if*

$$\forall x \in U, \exists y \in U \ni 1 - \alpha(x) = \alpha(y).$$

If U is an α*-controlled set, then it is shown by* $U \in CS(\alpha)$.

Definition 8. [6] *Let* $U \in CS(\alpha)$ *and* $a \in U$. *The control set of* a *is*

$$\overline{a} = \{b \in E \ : \ 1 - \mu(a) = \mu(b)\}.$$

Definition 9. [6] *Let U be* α *-set. We define the following mapping on U such that*

$$\alpha^*(x) = \begin{cases} 1 - \alpha(x), & if\ x \in U_\alpha \\ \sup_y \alpha(y), & if\ y \in U \ni \alpha(x) < 1 - \alpha(y) \\ 0, & otherwise. \end{cases}$$

where $U_\alpha = \bigcup_{a \in U} \overline{a}$.

It is evident that α^* is a mapping from *U* to *I*. Moreover, it is readily observable that the sum of α^* and α is less than or equal to 1. From this properties, we can give the following definition.

Definition 10. [6] *Let U be an* α*-set. Then the set* $A = \{\langle x, \alpha(x), \alpha^*(x)\rangle : x \in U\}$ *is called* (α, α^*)*-controlled set.*

It can be easily seen that every (α, α^*)-controlled set is an intuitionistic fuzzy set. But the converse of this is not generally true.

In crisp sets and fuzzy sets theory, topological concepts such as open set, closed set, interior of a set, closure and boundary are needed in modeling real world problems. The definition and basic properties of intuitionistic fuzzy topology were introduced by Çoker [5]. Various studies on intuitionistic fuzzy topology have been extensively discussed [8–10, 16, 18].

Definition 11. [1, 11] *X is a pretopological space in respect to the pre-closure operator $\boldsymbol{c} : \boldsymbol{X} \to \boldsymbol{X}$, where $\boldsymbol{X}$ can be $\wp(X), FS(X)$ or $IFS(X)$ if and only if for any $A, B \in \boldsymbol{X}$ the following axioms are satisfied;*

1) $\boldsymbol{c}(\emptyset) = \emptyset$;
2) $A \subseteq \boldsymbol{c}(A)$;
3) $\boldsymbol{c}(A \cup B) = \boldsymbol{c}(A) \cup \boldsymbol{c}(B)$, *for all* $A, B \subset X$.

If in addition to above axioms, the operator $\boldsymbol{c}$ is idempotent, that is $\boldsymbol{c}(A) = \boldsymbol{c}\big(\boldsymbol{c}(A)\big)$, then $\boldsymbol{c}$ is called closure operator in $\boldsymbol{X}$.

Definition 12. [1] *For the pre-closure operator $\boldsymbol{c}$ defined on $\boldsymbol{X}$ we say that a set $A \in \boldsymbol{X}$ is closed if and only if $\boldsymbol{c}(A) = A$. Also,*

$$\tau^c = \{A : A \in X \,\&\, \boldsymbol{c}(A) = A\}$$

is the topology generated by the pre-closure operator $\boldsymbol{c}$. If $\boldsymbol{X}$ is $\wp(X)$, $FS(X)$ or $IFS(X)$, then τ is called crisp topology, fuzzy topology or intuitionistic fuzzy topology, respectively.

Definition 13. [1] *A pre-interior operator $\boldsymbol{i} : \boldsymbol{X} \to \boldsymbol{X}$ is a map which associates to each set $A \in \boldsymbol{X}$ a set $\boldsymbol{i}(A)$ such that;*

1) $\boldsymbol{i}(\boldsymbol{X}) = \boldsymbol{X}$;
2) $\boldsymbol{i}(A) \subseteq A$;
3) $\boldsymbol{i}(A \cap B) = \boldsymbol{i}(A) \cap \boldsymbol{i}(B)$, *for all* $A, B \subset X$.

If in addition to above axioms the operator $\boldsymbol{i}$ is idempotent, that is $\boldsymbol{i}(A) = \boldsymbol{i}(\boldsymbol{i}(A))$, then $\boldsymbol{i}$ is called interior operator in $\boldsymbol{X}$. $\boldsymbol{X}$ can be $\wp(X), FS(X)$ or $IFS(X)$.

Definition 14. [1] *For the pre-interior operator $\boldsymbol{i}$ defined on $\boldsymbol{X}$ we say that a set $A \in X$ is open if and only if $\boldsymbol{i}(A) = A$. Also, $\tau_i = \{A : A \in X \,\&\, \boldsymbol{i}(A) = A\}$ is the topology generated by the pre-interior operator $\boldsymbol{i}$. If X is $\wp(X), FS(X)$ or $IFS(X)$ then τ is called crisp topology, fuzzy topology or intuitionistic fuzzy topology, respectively.*

If $\boldsymbol{i}$ is pre-interior operator, then $\boldsymbol{c}(A) = \neg\boldsymbol{i}(\neg A)$ is its corresponding pre-closure. That is $\big(\boldsymbol{c}(A), \neg\boldsymbol{i}(\neg A)\big)$ is a pair of conjugate pre-closure/pre-interior operators.

Proposition 1. [1] *If $\boldsymbol{i}$ and $\boldsymbol{c}$ is a conjugate pair of pre-interior and pre-closure operators in $\boldsymbol{X}$, then $\tau^c = \{\neg A : A \in \tau_i\}$ and $\tau_i = \{\neg B : B \in \tau^c\}$.*

The pre-topological operators to be studied in this article were defined by Tarsuslu (Yılmaz) [15]. The definitions of this pre-interior and pre-closure operators have been revised to ensure that the axioms hold. The following definitions and propositions demonstrate the mathematical validity of these revised operators under the specified parameter conditions. Due to the redefinition, it is also necessary to show that the operators are elements of IFS(X).

Definition 15. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1], \gamma \neq 1$ the topological operator $I_{\alpha,\beta}^{\gamma,\omega} : IFS(X) \to IFS(X)$ is defined as follows:*

$$\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \begin{cases} 0, & 0 \le \mu_A(x) < \alpha\gamma(1-\beta) \\ (1-\beta)\mu_A(x), & \alpha\gamma(1-\beta) \le \mu_A(x) < \alpha\gamma \\ (1-\beta)\alpha\gamma + \dfrac{\mu_A(x)-\alpha\gamma}{1-\gamma}(1-(1-\beta)\gamma), & \alpha\gamma \le \mu_A(x) < \alpha \\ \mu_A(x), & \alpha \le \mu_A(x) \le 1 \end{cases}$$

and

$$\nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \begin{cases} \min\left\{\beta, max\left\{\nu_A(x), \omega(1-\beta)\left(1-\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\right)\right\}\right\} & 0 \le \nu_A(x) \le \beta\omega \\ \nu_A(x), & \beta\omega < \nu_A(x) < \beta \end{cases}$$

Proposition 2. *For every $A \in IFS(X)$, the above defined $I_{\alpha,\beta}^{\gamma,\omega}(A)$ is indeed a valid element of IFS(X).*

Proof. We first show that $0 \le \mu_{I(A)}(x) \le \mu_A(x) \le 1$. Then,

$$\begin{aligned} \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) - \mu_A(x) &= \frac{\beta\gamma[\mu_A(x)-\alpha]}{1-\gamma} \\ &\le 0, \end{aligned}$$

because $\mu_A(x) < \alpha$. Hence it is holds for every $x \in X$. It follows that

$$\nu_A(x) \le 1 - \mu_A(x) \le 1 - \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x).$$

Firstly, if $0 \le \nu_A(x) \le \omega\beta$, then

$$\nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \min\left\{\beta, \max\left\{\nu_A(x), \omega(1-\beta)\left[1-\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\right]\right\}\right\}.$$

And $\nu_A(x) \le 1 - \mu_{I(A)}(x)$. Also, since $0 < \omega(1-\beta) < 1$,

$$\omega(1-\beta)\left[1-\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\right] \le 1 - \mu_{I(A)}(x).$$

Thus both arguments of the maximum are at most $1 - \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)$. And therefore

$$0 \le \nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \le 1 - \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x).$$

If $\omega\beta < \nu_A(x) \le 1$, then $\nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \nu_A(x)$. Consequently,

$$0 \le \mu_{I(A)}(x) + \nu_{I(A)}(x) \le 1. \qquad \square$$

Lemma 1. *The membership function satisfies $0 \le \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \le \mu_A(x)$, is nondecreasing as a function of $\mu_A(x)$, and preserves the minimum operation:*

$$\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A\sqcap B)}(x) = \min\{\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x), \mu_{I_{\alpha,\beta}^{\gamma,\omega}(B)}(x)\}.$$

Proposition 3. *Let X be a set and* $A \in IFS(X)$. *For* $\alpha, \beta, \gamma, \omega \in [0,1]$, $\beta \geq \frac{1}{2}, \gamma \neq 1$ *the operator* $I_{\alpha,\beta}^{\gamma,\omega}(A)$ *is a pre-interior operator in* $IFS(X)$.

Proof. **(P1)** $I_{\alpha,\beta}^{\gamma,\omega}(X^*) = X^*$**.** For the universal intuitionistic fuzzy set X^*,

$$\mu_{X^*}(x) = 1, \qquad \nu_{X^*}(x) = 0.$$

The non-membership function gives

$$\nu_{I(X^*)}(x) = \min\{\beta, \max\{0,0\}\} = 0.$$

(P2) $I_{\alpha,\beta}^{\gamma,\omega}(A) \sqsubseteq A$**.** The lemma gives

$$\mu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \leq \mu_A(x).$$

If $0 \leq \nu_A(x) \leq \omega\beta$, then the maximum in the definition of $\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x)$ is at least $\nu_A(x)$. Since $\nu_A(x) \leq \omega\beta < \beta$, taking the minimum with β does not reduce the result below $\nu_A(x)$. If $\nu_A(x) > \omega\beta$, then $\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \nu_A(x)$. Thus

$$\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \geq \nu_A(x),$$

which proves $\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A) \sqsubseteq A$.

(P3) $\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(\mathrm{A} \sqcap \mathrm{B}) = \mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(\mathrm{A}) \sqcap \mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(\mathrm{B})$**.** The membership equality follows from the lemma. To prove the non-membership equality, set

$$h_A = \omega(1-\beta)\left[1 - \mu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\right]$$
$$h_B = \omega(1-\beta)\left[1 - \mu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(B)}(x)\right]$$

By the membership equality,

$$\omega(1-\beta)\left[1 - \mu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A\sqcap B)}(x)\right] = \max\{h_A, h_B\}.$$

Since $\beta \geq \frac{1}{2}$,

$$h_A, h_B \leq \omega(1-\beta) \leq \omega\beta.$$

If $\max\{\nu_A(x), \nu_B(x)\} \leq \omega\beta$,

$$\begin{aligned} \nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A\sqcap B)}(x) &= \max\{\nu_A(x), \nu_B(x), h_A, h_B\} \\ &= \max\{\max\{\nu_A(x), h_A\}, \max\{\nu_B(x), h_B\}\} \\ &= \max\{\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x), \nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(B)}(x)\}. \end{aligned}$$

If $\max\{\nu_A(x), \nu_B(x)\} > \omega\beta$, assume without loss of generality that $\nu_A(x) > \omega\beta$. Then $\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \nu_A(x)$. If $\nu_B(x) > \omega\beta$, then $\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(B)}(x) = \nu_B(x)$. If $\nu_B(x) \leq \omega\beta$, (6) gives

$$\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(B)}(x) = \max\{\nu_B(x), h_B\} \leq \omega\beta < \nu_A(x).$$

Therefore,

$$\max\{\nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(A)}(x), \nu_{\mathrm{I}_{\alpha,\beta}^{\gamma,\omega}(B)}(x)\} = \max\{\nu_A(x), \nu_B(x)\}.$$

Because $\max\{\nu_A(x), \nu_B(x)\} > \omega\beta$, the second branch applies to $I(A \sqcap B)$, so

$$\nu_{I^{\gamma,\omega}_{\alpha,\beta}(A\sqcap B)}(x) = \max\{\nu_A(x), \nu_B(x)\}.$$

Thus the non-membership equality holds in both cases. □

Definition 16. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1]$, the topological operator $C^{\gamma,\omega}_{\alpha,\beta} : IFS(X) \to IFS(X)$ is defined as follows:*

$$\mu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) = \begin{cases} \min\left\{\beta, \max\left\{\mu_A(x), \omega(1-\beta)\left[1 - \nu_{C^{\gamma,\omega}_{\alpha,\beta}}(x)\right]\right\}\right\}, & 0 \le \mu_A(x) \le \omega\beta, \\ \mu_A(x), & \omega\beta < \mu_A(x) \le 1. \end{cases}$$

and

$$\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) = \begin{cases} 0, & 0 \le \nu_A(x) < \alpha\gamma(1-\beta), \\ (1-\beta)\nu_A(x), & \alpha\gamma(1-\beta) \le \nu_A(x) < \alpha\gamma, \\ (1-\beta)\alpha\gamma + \dfrac{\nu_A(x) - \alpha\gamma}{1-\gamma}[1-(1-\beta)\gamma], & \alpha\gamma \le \nu_A(x) < \alpha, \\ \nu_A(x), & \alpha \le \nu_A(x) \le 1, \end{cases}$$

Proposition 4. *For every $A \in IFS(X)$, the above defined $C^{\gamma,\omega}_{\alpha,\beta}(A)$ is indeed a valid element of IFS(X).*

Proposition 5. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1], \beta \ge \frac{1}{2}$, $\gamma \ne 1$ the operator $C^{\gamma,\omega}_{\alpha,\beta}(A)$ is a pre-closure operator in $IFS(X)$.*

Theorem 1. *Let X be a set and $A \in IFS(X)$, then $C^{\gamma,\omega}_{\alpha,\beta}(A) = \neg I^{\gamma,\omega}_{\alpha,\beta}(\neg A)$.*

Proof. Following the pretopological framework for intuitionistic fuzzy sets described by Marinov and Atanassov [11], the proposed pre-interior and pre-closure operators satisfy the corresponding pretopological axioms, thereby establishing a pretopological structure on the intuitionistic fuzzy set. Based on these operators, an operator-induced boundary is defined to investigate the boundary characteristics of intuitionistic fuzzy sets. This formulation provides the mathematical basis for examining boundary-related structures in the spatial application considered in this study.

The boundary of set A is $\partial A = C^{\gamma,\omega}_{\alpha,\beta}(A) \sqcap \left(\neg\, I^{\gamma,\omega}_{\alpha,\beta}(A)\right)$ by the intuitionistic fuzzy boundary definition given in Malek [10]. □

3 Intuitionistic fuzzy topological operators via controlled sets

Shi and Liu investigated *Mikania micrantha*, an invasive plant found in the Hong Kong countryside that can cause the death of host trees [13]. They used fuzzy interior and closure operators to analyze the affected regions. In the present study, controlled set theory is used to determine the non-membership degrees of the affected regions. The resulting intuitionistic fuzzy representations are then examined using intuitionistic fuzzy pre-interior and pre-closure operators.

To maintain consistency with the fuzzification approach of Shi and Liu [13], we use the same logarithmic function to determine membership degrees from area measurements. The main difference in the present study is the use of controlled set theory to assign non-membership degrees.

The methodology used by Shi and Liu is described in the following steps:

Methodology

(i) Aerial photographs of the studied area are viewed as fuzzy spaces.

(ii) Areas affected by *Mikania micrantha* in the aerial photo are viewed as fuzzy sets in each of fuzzy spaces (see Table 1).

(iii) The membership values of affected areas are calculated.

(iv) The fuzzy value is defined by

$$\mu_A(x) = \begin{cases} \dfrac{\log(\text{Area of certain affected area})}{\log(\text{Total area of affected area})}, & \text{if } \dfrac{\log(\bullet)}{\log(*)} > 0 \\ 0, & \text{otherwise} \end{cases}$$

which is a well-defined mapping from the interval [0,1) to the interval [0,1].

Table 1. Size of each area affected by *Mikania Micrantha*

ID	Area	ID	Area	ID	Area
1	7.97	**19**	60.50	**37**	195.46
2	8.17	**20**	61.97	**38**	265.48
3	8.80	**21**	63.86	**39**	293.35
4	10.23	**22**	73.64	**40**	312.60
5	10.37	**23**	73.83	**41**	315.02
6	15.32	**24**	76.67	**42**	343.49
7	17.09	**25**	77.10	**43**	349.76
8	17.52	**26**	82.68	**44**	388.28
9	24.75	**27**	85.58	**45**	401.55
10	25.12	**28**	87.16	**46**	403.61
11	28.00	**29**	93.80	**47**	498.05
12	31.69	**30**	104.38	**48**	564.57
13	31.92	**31**	105.35	**49**	629.68
14	36.75	**32**	135.05	**50**	774.58
15	37.83	**33**	142.95	**51**	786.10
16	42.46	**34**	155.60	**52**	855.94
17	53.36	**35**	184.86	**53**	1014.44
18	57.10	**36**	192.21		
				Total	10713.60
				Average	202.14

In this study, the $\mu^*(x) = \nu(x)$ values were obtained by using controlled set to calculate the non-membership degrees of regions affected by *Mikania micrantha*. Thus, we can view the studied area as an intuitionistic fuzzy space and consider the areas of influence as intuitionistic fuzzy sets. The aim is to examine how the proposed intuitionistic fuzzy operators represent the uncertainty associated with the affected regions of this invasive plant. For this purpose, we studied with intuitionistic fuzzy pre-interior and intuitionistic fuzzy pre-closure operators which are the open and closed sets, respectively.

In Table 2 and Table 3 give the pre-interior, pre-closure and operator-induced boundary values for each region under two choices of parameters α, β, γ and ω. The results obtained are in close relation with the membership degrees of the elements of the pretopological space examined. Appropriate values can be worked out according to the needs of the region.

Table 2. $\alpha = 0.3$, $\beta = 0.7$, $\gamma = 0.5$, $\omega = 0.4$

ID	Area	μ_{Area}	$\mu^*_{Area} = \nu_{Area}$	$\mu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\mu_{C^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{C^{\gamma,\omega}_{\alpha,\beta}}$	μ_{∂}	ν_{∂}
1	7.97	0.223691	0.745974	0.170275	0.745974	0.223691	0.745974	0.223691	0.745974
2	8.17	0.226362	0.745974	0.174815	0.745974	0.226362	0.745974	0.226362	0.745974
3	8.80	0.234367	0.745974	0.188424	0.745974	0.234367	0.745974	0.234367	0.745974
4	10.23	0.250594	0.745974	0.216010	0.745974	0.250594	0.745974	0.250594	0.745974
5	10.37	0.252058	0.745974	0.218499	0.745974	0.252058	0.745974	0.252058	0.745974
6	15.32	0.294114	0.694582	0.289994	0.694582	0.294114	0.694582	0.294114	0.694582
7	17.09	0.305896	0.682819	0.305896	0.682819	0.305896	0.682819	0.305896	0.682819
8	17.52	0.308574	0.682819	0.308574	0.682819	0.308574	0.682819	0.308574	0.682819
9	24.75	0.345806	0.646651	0.345806	0.646651	0.345806	0.646651	0.345806	0.646651
10	25.12	0.347405	0.646651	0.347405	0.646651	0.347405	0.646651	0.347405	0.646651
11	28.00	0.359102	0.631219	0.359102	0.631219	0.359102	0.631219	0.359102	0.631219
12	31.69	0.372443	0.619945	0.372443	0.619945	0.372443	0.619945	0.372443	0.619945
13	31.92	0.373223	0.619945	0.373223	0.619945	0.373223	0.619945	0.373223	0.619945
14	36.75	0.388408	0.601506	0.388408	0.601506	0.388408	0.601506	0.388408	0.601506
15	37.83	0.391529	0.601506	0.391529	0.601506	0.391529	0.601506	0.391529	0.601506
16	42.46	0.403972	0.568510	0.403972	0.568510	0.403972	0.568510	0.403972	0.568510
17	53.36	0.428596	0.568510	0.428596	0.568510	0.428596	0.568510	0.428596	0.568510
18	57.10	0.435897	0.562501	0.435897	0.562501	0.435897	0.562501	0.435897	0.562501
19	60.50	0.442130	0.543932	0.442130	0.543932	0.442130	0.543932	0.442130	0.543932
20	61.97	0.444717	0.543932	0.444717	0.543932	0.444717	0.543932	0.444717	0.543932
21	63.86	0.447955	0.543932	0.447955	0.543932	0.447955	0.543932	0.447955	0.543932
22	73.64	0.463311	0.534794	0.463311	0.534794	0.463311	0.534794	0.463311	0.534794
23	73.83	0.463589	0.534794	0.463589	0.534794	0.463589	0.534794	0.463589	0.534794
24	76.67	0.467657	0.528667	0.467657	0.528667	0.467657	0.528667	0.467657	0.528667
25	77.10	0.468259	0.528667	0.468259	0.528667	0.468259	0.528667	0.468259	0.528667
26	82.68	0.475789	0.501903	0.475789	0.501903	0.475789	0.501903	0.475789	0.501903
27	85.58	0.479505	0.501903	0.479505	0.501903	0.479505	0.501903	0.479505	0.501903
28	87.16	0.481476	0.501903	0.481476	0.501903	0.481476	0.501903	0.481476	0.501903
29	93.80	0.489388	0.501903	0.489388	0.501903	0.489388	0.501903	0.489388	0.501903
30	104.38	0.500906	0.489388	0.500906	0.489388	0.500906	0.489388	0.489388	0.500906
31	105.35	0.501903	0.489388	0.501903	0.489388	0.501903	0.489388	0.489388	0.501903
32	135.05	0.528667	0.468259	0.528667	0.468259	0.528667	0.468259	0.468259	0.528667

Contd.

ID	Area	μ_{Area}	$\mu^{*}_{Area} = \nu_{Area}$	$\mu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\mu_{C^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{C^{\gamma,\omega}_{\alpha,\beta}}$	μ_{∂}	ν_{∂}
33	142.95	0.534794	0.534794	0.534794	0.463589	0.534794	0.463589	0.463589	0.534794
34	155.60	0.543932	0.543932	0.543932	0.447955	0.543932	0.447955	0.447955	0.543932
35	184.86	0.562501	0.562501	0.562501	0.435897	0.562501	0.435897	0.435897	0.562501
36	192.21	0.566703	0.566703	0.566703	0.428596	0.566703	0.428596	0.428596	0.566703
37	195.46	0.568510	0.568510	0.568510	0.428596	0.568510	0.428596	0.428596	0.568510
38	265.48	0.601506	0.601506	0.601506	0.391529	0.601506	0.391529	0.391529	0.601506
39	293.35	0.612265	0.612265	0.612265	0.373223	0.612265	0.373223	0.373223	0.612265
40	312.60	0.619114	0.619114	0.619114	0.373223	0.619114	0.373223	0.373223	0.619114
41	315.02	0.619945	0.619945	0.619945	0.373223	0.619945	0.373223	0.373223	0.619945
42	343.49	0.629269	0.629269	0.629269	0.359102	0.629269	0.359102	0.359102	0.629269
43	349.76	0.631219	0.631219	0.631219	0.359102	0.631219	0.359102	0.359102	0.631219
44	388.28	0.642478	0.642478	0.642478	0.347405	0.642478	0.347405	0.347405	0.642478
45	401.55	0.646100	0.646100	0.646100	0.347405	0.646100	0.347405	0.347405	0.646100
46	403.61	0.646651	0.646651	0.646651	0.347405	0.646651	0.347405	0.347405	0.646651
47	498.05	0.669309	0.669309	0.669309	0.308574	0.669309	0.308574	0.308574	0.669309
48	564.57	0.682819	0.682819	0.682819	0.308574	0.682819	0.308574	0.308574	0.682819
49	629.68	0.694582	0.694582	0.694582	0.294114	0.694582	0.289994	0.294114	0.694582
50	774.58	0.716902	0.716902	0.716902	0.252058	0.716902	0.218499	0.252058	0.716902
51	786.1	0.718492	0.718492	0.718492	0.252058	0.718492	0.218499	0.252058	0.718492
52	855.94	0.727665	0.727665	0.727665	0.252058	0.727665	0.218499	0.252058	0.727665
53	1014.44	0.745974	0.745974	0.745974	0.252058	0.745974	0.218499	0.252058	0.745974

Table 3. $\alpha = 0.6$, $\beta = 0.5$, $\gamma = 0.7$, $\omega = 0.3$

ID	Area	μ_{Area}	$\mu^{*}_{Area} = \nu_{Area}$	$\mu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\mu_{C^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{C^{\gamma,\omega}_{\alpha,\beta}}$	μ_{∂}	ν_{∂}
1	7.97	0.223691	0.745974	0.111846	0.745974	0.223691	0.745974	0.223691	0.745974
2	8.17	0.226362	0.745974	0.113181	0.745974	0.226362	0.745974	0.226362	0.745974
3	8.80	0.234367	0.745974	0.117184	0.745974	0.234367	0.745974	0.234367	0.745974
4	10.23	0.250594	0.745974	0.125297	0.745974	0.250594	0.745974	0.250594	0.745974
5	10.37	0.252058	0.745974	0.126029	0.745974	0.252058	0.745974	0.252058	0.745974
6	15.32	0.294114	0.694582	0.147057	0.694582	0.294114	0.694582	0.294114	0.694582
7	17.09	0.305896	0.682819	0.152948	0.682819	0.305896	0.682819	0.305896	0.682819
8	17.52	0.308574	0.682819	0.154287	0.682819	0.308574	0.682819	0.308574	0.682819
9	24.75	0.345806	0.646651	0.172903	0.646651	0.345806	0.646651	0.345806	0.646651
10	25.12	0.347405	0.646651	0.173703	0.646651	0.347405	0.646651	0.347405	0.646651
11	28.00	0.359102	0.631219	0.179551	0.631219	0.359102	0.631219	0.359102	0.631219
12	31.69	0.372443	0.619945	0.186222	0.619945	0.372443	0.619945	0.372443	0.619945
13	31.92	0.373223	0.619945	0.186612	0.619945	0.373223	0.619945	0.373223	0.619945
14	36.75	0.388408	0.601506	0.194204	0.601506	0.388408	0.601506	0.388408	0.601506
15	37.83	0.391529	0.601506	0.195765	0.601506	0.391529	0.601506	0.391529	0.601506
16	42.46	0.403972	0.568510	0.201986	0.56851	0.403972	0.531772	0.403972	0.531772
17	53.36	0.428596	0.568510	0.228625	0.56851	0.428596	0.531772	0.428596	0.531772
18	57.10	0.435897	0.562501	0.244444	0.562501	0.435897	0.518752	0.435897	0.518752
19	60.50	0.442130	0.543932	0.257948	0.543932	0.44213	0.478519	0.44213	0.478519
20	61.97	0.444717	0.543932	0.263554	0.543932	0.444717	0.478519	0.444717	0.478519
21	63.86	0.447955	0.543932	0.270569	0.543932	0.447955	0.478519	0.447955	0.478519

Contd.

ID	Area	μ_{Area}	$\mu^{*}{}_{Area} = \nu_{Area}$	$\mu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\mu_{C^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{C^{\gamma,\omega}_{\alpha,\beta}}$	μ_{∂}	ν_{∂}
22	73.64	0.463311	0.534794	0.303841	0.534794	0.463311	0.458720	0.463311	0.45872
23	73.83	0.463589	0.534794	0.304443	0.534794	0.463589	0.458720	0.463589	0.45872
24	76.67	0.467657	0.528667	0.313257	0.528667	0.467657	0.445445	0.467657	0.445445
25	77.10	0.468259	0.528667	0.314561	0.528667	0.468259	0.445445	0.468259	0.445445
26	82.68	0.475789	0.501903	0.330876	0.501903	0.475789	0.387457	0.475789	0.387457
27	85.58	0.479505	0.501903	0.338928	0.501903	0.479505	0.387457	0.479505	0.387457
28	87.16	0.481476	0.501903	0.343198	0.501903	0.481476	0.387457	0.481476	0.387457
29	93.80	0.489388	0.501903	0.360341	0.501903	0.489388	0.387457	0.489388	0.387457
30	104.38	0.500906	0.489388	0.385296	0.489388	0.500906	0.360341	0.489388	0.385296
31	105.35	0.501903	0.489388	0.387457	0.489388	0.501903	0.360341	0.489388	0.387457
32	135.05	0.528667	0.468259	0.445445	0.468259	0.528667	0.314561	0.468259	0.445445
33	142.95	0.534794	0.463589	0.458720	0.463589	0.534794	0.304443	0.463589	0.458720
34	155.60	0.543932	0.447955	0.478519	0.447955	0.543932	0.270569	0.447955	0.478519
35	184.86	0.562501	0.435897	0.518752	0.435897	0.562501	0.244444	0.435897	0.518752
36	192.21	0.566703	0.428596	0.527857	0.428596	0.566703	0.228625	0.428596	0.527857
37	195.46	0.568510	0.428596	0.531772	0.428596	0.568510	0.228625	0.428596	0.531772
38	265.48	0.601506	0.391529	0.601506	0.391529	0.601506	0.195765	0.391529	0.601506
39	293.35	0.612265	0.373223	0.612265	0.373223	0.612265	0.186612	0.373223	0.612265
40	312.60	0.619114	0.373223	0.619114	0.373223	0.619114	0.186612	0.373223	0.619114
41	315.02	0.619945	0.373223	0.619945	0.373223	0.619945	0.186612	0.373223	0.619945
42	343.49	0.629269	0.359102	0.629269	0.359102	0.629269	0.179551	0.359102	0.629269
43	349.76	0.631219	0.359102	0.631219	0.359102	0.631219	0.179551	0.359102	0.631219
44	388.28	0.642478	0.347405	0.642478	0.347405	0.642478	0.173703	0.347405	0.642478
45	401.55	0.646100	0.347405	0.646100	0.347405	0.646100	0.173703	0.347405	0.646100
46	403.61	0.646651	0.347405	0.646651	0.347405	0.646651	0.173703	0.347405	0.646651
47	498.05	0.669309	0.308574	0.669309	0.308574	0.669309	0.154287	0.308574	0.669309
48	564.57	0.682819	0.308574	0.682819	0.308574	0.682819	0.154287	0.308574	0.682819
49	629.68	0.694582	0.294114	0.694582	0.294114	0.694582	0.147057	0.294114	0.694582
50	774.58	0.716902	0.252058	0.716902	0.252058	0.716902	0.126029	0.252058	0.716902
51	786.10	0.718492	0.252058	0.718492	0.252058	0.718492	0.126029	0.252058	0.718492
52	855.94	0.727665	0.252058	0.727665	0.252058	0.727665	0.126029	0.252058	0.727665
53	1014.44	0.745974	0.252058	0.745974	0.252058	0.745974	0.126029	0.252058	0.745974

4 Conclusion

In this study, intuitionistic fuzzy pre-interior and pre-closure operators were applied to model spatial regions affected by *Mikania micrantha* using controlled set-based intuitionistic fuzzy representations. The membership degrees were determined from the areas of the affected regions, while the non-membership degrees were assigned using a controlled set-based approach. The resulting intuitionistic fuzzy values were then processed using the proposed operators to obtain the corresponding interior, closure, and operator-induced boundary values.

The results obtained for the examined parameter configurations demonstrate how the proposed operators transform the intuitionistic fuzzy representations of the spatial regions. In comparison with the study of Shi and Liu [13], the results did not exhibit the same pattern of zero-valued interior and closure degrees reported in their analysis. This difference should be interpreted in the context of the distinct fuzzy representations and operator formulations

employed in the two studies, rather than as direct evidence of the superiority of one approach over the other.

The main contribution of this study is the integration of controlled set-based non-membership assignment with intuitionistic fuzzy pre-interior and pre-closure operators in a spatial modeling application. This framework provides an alternative way to represent uncertainty in the affected regions and to examine the resulting interior, closure, and boundary characteristics. Further studies involving independently validated spatial boundaries and additional datasets may help assess the practical accuracy and applicability of the proposed framework.

References

[1] Arkhangel'skii, A. V., & Pontryagin, L. S. (Eds.). (1990). *General Topology I: Basic Concepts and Constructions, Dimension Theory*. Encyclopaedia of Mathematical Sciences, Vol. 17. Springer-Verlag, Berlin, Heidelberg.

[2] Atanassov, K. T. (1983). Intuitionistic fuzzy sets. In: *VII ITKR Session, Sofia, June, 1983* (Deposed in Centr. Sci.-Techn. Library of the Bulg. Acad. of Sci., 1697/84) (in Bulgarian). Reprinted: *Int. J. Bioautomation*, 2016, 20 (S1), S1–S6.

[3] Atanassov, K. T. (2012). *On Intuitionistic Fuzzy Sets Theory*. Studies in Fuzziness and Soft Computing, Vol. 283. Springer, Berlin, Heidelberg.

[4] Bustince, H., Kacprzyk, J., & Mohedano, V. (2000). Intuitionistic fuzzy generators: Application to intuitionistic fuzzy complementation. *Fuzzy Sets and Systems*, 114(3), 485–504.

[5] Çoker, D. (1997). An introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets and Systems*, 88, 81–89.

[6] Çuvalcıoğlu, G. (2014). Some properties of controlled set theory. *Notes on Intuitionistic Fuzzy Sets*, 20(2), 37–42.

[7] Çuvalcıoğlu, G., & Varol, S. U. (2021). Decision making process for serving restaurants using intuitionistic fuzzy set theory via controlled sets. *Journal of Universal Mathematics*, 4(2), 296–325.

[8] Kutlu, F., & Bilgin, T. (2015). Temporal intuitionistic fuzzy topology in Šostak's sense. *Notes on Intuitionistic Fuzzy Sets*, 21(2), 63–70.

[9] Lupiañez, F. G. (2004). Intuitionistic fuzzy topological operators and topology. *International Journal of Pure and Applied Mathematics*, 17(1), 35–40.

[10] Malek, M. R. (2004). Spatial object modeling in intuitionistic fuzzy topological spaces. In: *Tsumoto, S., Słowiński, R., Komorowski, J., & Grzymała-Busse, J.W. (Eds.). Rough Sets and Current Trends in Computing. RSCTC 2004*. Lecture Notes in Computer Science, 3066, pp. 427–434. Springer, Berlin, Heidelberg.

[11] Marinov, E., & Atanassov, K. T. (2020). Partially continuous pretopological and topological operators for intuitionistic fuzzy sets. *Iranian Journal of Fuzzy Systems*, 17(2), 1–15.

[12] Radhika, C., & Parvathi, R. (2016) Intuitionistic fuzzification functions. *Global Journal of Pure and Applied Mathematics*, 12(2), 1211–1227.

[13] Shi, W., & Liu, K. (2007). A fuzzy topology for computing the interior, boundary, and exterior of spatial objects quantitatively in GIS. *Computers & Geosciences*, 33, 898–915.

[14] Sugeno, M. (1974). *Theory of Fuzzy Integrals and Its Applications.* PhD thesis, Tokyo Institute of Technology, Tokyo, Japan.

[15] Tarsuslu (Yılmaz), S. (2023). A study on intuitionistic fuzzy topological operators. *Italian Journal of Pure and Applied Mathematics*, 49, 863–875.

[16] Tarsuslu (Yılmaz), S. (2023). Application of intuitionistic fuzzy topological operators in spatial objects modeling. *Bitlis Eren Üniversitesi Fen Bilimleri Dergisi*, 12(2), 524–530.

[17] Tarsuslu (Yılmaz), S., & Çuvalcıoğlu, G. (2024). An application of controlled sets in medical diagnosis. In: *Melin, P., Castillo, O. (Eds.). New Directions on Hybrid Intelligent Systems Based on Neural Networks, Fuzzy Logic, and Optimization Algorithms*. Studies in Computational Intelligence, Vol. 1146, pp. 9–17. Springer, Cham.

[18] Tuğrul, F. (2021). Connectedness in Temporal Intuitionistic Fuzzy Topology in Chang's Sense. *Journal of Universal Mathematics*, 4(2), 157–165.

[19] Tuğrul, F., & Çitil, M. (2021). A new perspective on evaluation system in education with ıntuitionistic fuzzy logic and PROMETHEE algorithm. *Journal of Universal Mathematics*, 4(1), 13–24.

[20] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.