15th Int. Workshop on Generalized Nets
Burgas, 16 October 2014, 17–22

Generalized nets with time dependent priorities

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Abstract: A new extension of the class of the ordinary Generalized nets is defined – Generalized nets with time dependent priorities (GNTDP). In a GNTDP, the priorities depend on the token and the current time. It is proved that the class of all GNTDP Σ_{TDP} is conservative extension of the class of all Generalized Nets Σ.
Keywords: Generalized Nets, Extension of generalized nets, Token, Priority, Time.
AMS Classification: 68Q85, 03E72.

1 Generalized nets with time dependent priorities

Generalized Nets (GNs) are extensions of Petri Nets [4]. They are defined in 1991 by Krasimir Atanasov [2] in a way that is principally different from the ways of defining the other types of Petri nets.

In this section, we give the formal definition of the concept of Generalized Net with Time Dependent Priorities (GNTDP), as a new extension of the concept of GN.

The basic builiding block of any GN is its *transition.*

Formally, every transition is described by a seven-tuple:

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively);

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will transfer from the transition's inputs to its outputs. Parameter r has the form of an IM:

$$r = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \ - \ \text{predicate}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array};$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the i-th input place to the j-th output place. When $r_{i,j}$ has truth-value "$true$", then a token from the i-th input place can be transferred to the j-th output place; otherwise, this is impossible;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \ge 0 - \ \text{natural number or } \infty) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array};$$

(f) $\Box$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places, and it is an expression constructed of variables and the Boolean connectives $\wedge$ and $\vee$ determining the following conditions:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,

$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – there must be at least one token in the set of places $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "$true$", the transition can become active, otherwise it cannot.

The Generalized Net is formally defined as the ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle,$$

where

(a) A is a set of transitions (see above);

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \to \mathcal{N}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to \mathcal{N}$, where

$$L = pr_1 A \cup pr_2 A$$

and obviously, L is the set of all GN-places;

(d) c is a function giving the capacities of the places, i.e., $c : L \rightarrow \mathcal{N}$;

(e) f is a function that calculates the truth values of the predicates of the transition's conditions;

(f) θ_1 is a function giving the next time-moment, for which a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3Z = t, t' \in [T, T + t^*]$ and $t \leq t'$; the value of this function is calculated at the moment when the transition terminates its functioning. Here and below pr_iX is the i-th projection of the n-dimensional set X.

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i.e., $\theta_2(t) = t'$, where $pr_4Z = t \in [T, T + t^*]$ and $t' \geq 0$; the value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(i) $\pi_{K,T}$ In the standard GNs $\pi_K : K \rightarrow N$. In the GNTDP $\pi_{K,T}$ is a function giving the priorities of the tokens by the token and the current time, i.e. $\pi_{K,T} : K \times [T, T + t^*] \rightarrow N$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the GN starts functioning; this moment is determined with respect to a fixed (global) time-scale;

(l) t^0 is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the GN functioning;

(n) X is a function which assigns initial characteristics to every token when it enters input place of the net;

(o) Φ is a characteristic function that assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition;

(p) b is a function giving the maximum number of characteristics witch a given token can receive, i.e., $b : K \rightarrow N$.

2 Example usage

The difference between standart GN and GNTDP is the function π_K. In the standart GN, the function returns the token priority for a given token.

Let us look at an example. We simulate a traffic on a crossroad. The token A in standart GN model, has only 1 priority during the simulation witch doesn't depend on time-step.

Let us have data for all cars on a crossroad. We will simulate the process of driving cars. In GN model each car is a token. All tokens are moved to the output places witch are different paths on the crossroad. If the car A has priority witch is higher than the other cars, it will passing through the croasrroad before others. It is not absolutely correct because if at the current time on the croassroad have an ambulance, the ambulance priority will be highest and the car A will have to wait.

Therefore the standart π_K definition is not enough and it is so far from data mining and changing GN models.

We can simulate the example above with different characteristic function, witch will set a new characteristic "time priority" by a token and current time.

Unfortunately if we create software realisation for each token characteristic, the compiler will reserves the proper amount of memory. On the other hand if we want to test only the prioritized criteria it would be a good idea if they are encapsulate or packing into a single component and all other data will be the same. So we introduce an GNTDP. In the definition, the priorities of the tokens is given by token on the current time-step.

In that case, each token has a different priority for each time-step. If we return to the example above, we will see that we can set the highest priority of the token A in some special moment when the patient deteriorated. With time token priority we can simulate all processes with previously known data with different token priority in different GN time-step.

3 Generalized nets with time dependent priorities as a conservative extension of the Generalized nets

Let Σ_{TDP} be the class of all GNTDP.

Theorem 1. *Every standard GN is a GNTDP, i.e the relation*

$$\Sigma_{TDP} \vdash \Sigma \tag{1}$$

holds (see [2, 3]).

Proof. This is so because every standard GN can be viewed as a GNTDP in which the token's priority is the same throughout, e.g. $\pi_{K,T}(k,t) = n$, $k \in K$ for all $t \in [T, T + t^*]$ and $n \in N$. Therefore, the class Σ_{TDP} is an extension of the class Σ of the standard GNs. □

We will prove the following theorem:

Theorem 2. *The class Σ_{TDP} is conservative extension of the class Σ, i.e.*

$$\Sigma \equiv \Sigma_{TDP} \tag{2}$$

Proof. To prove (2), because of (1), it is enough to show that the relation

$$\Sigma \vdash \Sigma_{TDP} \tag{3}$$

holds.

Let E be a given GNTDP

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle$$

We will construct a standard GN G with the same time, capacity and token components and the same transitions but with the token's priority function independent on the time $\pi_K : K \rightarrow N$. In the proof we will use the theorem for the completeness of the GN transitions which states that every GN net can be constructed only from the set of its transitions and the operations union, concatenation and iteration defined over transitions. Let the standard GN G be defined by

$$G = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K^*, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b^*\rangle\rangle$$

where

$$X^* = X \cup \{x_0^k \mid k \in K, x_0^k = \pi_{K,T}(k, \theta_K), x_0^k \in N\}.$$

Here x_0^k is the initial characteristic of token $k \in K$ and it is a token priority at the moment θ_K.

$$\Phi^* = \Phi \cup \phi_{\{k|k\in K\}},$$

where function $\phi_{\{k|k\in K\}}$ determines the characteristics of the token k in the form

$$\phi_{\{k|k\in K\}}(k, t) = "\{n \mid n = \pi_{K,T}(k, t), n \in N\}".$$

The characteristic function Φ^* assigns to each token the new characteristic of the token priority at the current time t, $t \in [T, T + t^*]$. Now the π_K function can return the priority characteristic of the token witch is x_0^k like initial value and the result from $\phi_{\{k|k\in K\}}$ at the moment $t \in [T, T + t^*]$.

$$b^*(K) = b(K) + 1$$

In the GN G, b^* is a function giving the maximum number of characteristics witch a given token can receive plus one for "time priority" characteristic.

We shall prove that both GNs E and G function equally. We will compare the functioning of one arbitrary transition Z of GNTDP E and its corresponding transition, for example Z' of G.

Obviously, these two transitions become active simultaneously and have equal duration of functioning and priorities. They have same input and output places. The corresponding places have equal priorities and capacities in both nets. The arcs have equal capacities.

Let us compare the transfer of two arbitrary corresponding tokens that enter simultaneously two corresponding input places of the two transitions. Let the token in GNTDP have some characteristics and the token in standard GN has same characteritics with one more "time priority", the capacities of the corresponding arcs of the transitions, the capacities of the corresponding output places and the predicates of the transition's condition are the same in both transitions. In step two of Algorithm for transition functioning for GNTDP and GN say that for each input place is created a list of tokens in the place ordered by their priority. In the transition Z(GNTDP) the priority is return by $\pi_{K,T}$. In the standard GN - transition Z' the token priority is the token characteristic "time priority" witch is result from $\phi_{\{k|k\in K\}}$ and it is the same with $\pi_{K,T}$ at the same moment $t \in [T, T + t^*]$.

Then either both of the tokens will be transferred to corresponding output places or stay in the input places. If they are transferred to two corresponding output places, they will receive the same characteristics because the characteristic functions of the two nets are the same except the "time priority" characteristics. Also the the maximum number of other characteristics are the same. Therefore, the two tokens behave in the same way. Seeing the two tokens were chosen arbitrarily we can assert that all other corresponding pairs of tokens have the same behaviour. Therefore the two transitions function equally. On the other hand, the pair of corresponding transitions was also chosen arbitrarily and from the Theorem for the completeness of the GN transitions (see [1, 3]) it follows that the two nets function equally which proves the validity of (3) and therefore the theorem is valid. □

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