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Intuitionistic fuzzy ideal and intuitionistic fuzzy prime ideal in a ring

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Abstract: In this paper we further study the theory of Intuitionistic fuzzy ideals and intuitionistic fuzzy prime ideals. We have investigated these notions and shown a new result using the intuitionistic fuzzy points and a membership and nonmembership functions.

Keywords: Intuitionistic fuzzy ring, Intuitionistic fuzzy ideal, Intuitionistic fuzzy prime ideal, Intuitionistic fuzzy point.
AMS Classification: 03E72.

1 Introduction

In 1986 Atanassov introduced the notion of a intuitionistic fuzzy set as a generalization of Zadeh's fuzzy sets [15]. After the introduction of the notion of intuitionistic fuzzy subring by Hur, Kang and Song [4], many researchers have tried to generalize the notion of intuitionistic fuzzy subring. Marashdeh and Salleh [10] introduced the notion of intuitionistic fuzzy rings based on the notion of fuzzy space, Sharma [14] introduced the notion of translates of intuitionistic fuzzy subrings. The purpose of this paper is to improve the concept of intuitionistic fuzzy ideals of a ring given a new characterization using the intuitionistic fuzzy points and to show some results of fuzzy prime ideal.

2 Preliminaries

First we give the concept of intuitionistic fuzzy set defined by Atanassov as a generalization of the concept of fuzzy set given by Zadeh.

Definition 1. *[1, 2] The intuitionistic fuzzy sets (in shorts IFS) are defined on a non-empty set X as objects having the form*

$$A = \{\langle x, \mu(x), \nu(x)\rangle \mid x \in X\}$$

where the functions $\mu : X \to [0,1]$ and $\nu : X \to [0,1]$ denote the degrees of membership and of non-membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu(x) + \nu(x) \leq 1$ for all $x \in X$.

For the sake of simplicity, we shall use the symbol $\langle \mu, \nu \rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu(x), \nu(x)\rangle | x \in X\}$.

Definition 2. *[2] Let X be a nonempty set and let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be IFSs of X. Then*
$A \subset B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$,
$A = B$ iff $A \subset B$ and $B \subset A$,
$A^c = \langle \nu_A, \mu_A \rangle$,
$A \cap B = \langle \mu_A \wedge \mu_B, \nu_A \vee \nu_B \rangle$,
$A \cup B = \langle \mu_A \vee \mu_B, \nu_A \wedge \nu_B \rangle$,
$\Box A = \langle \mu_A, 1 - \mu_A \rangle$, $\quad \Diamond A = \langle 1 - \nu_A, \nu_A \rangle$.

Definition 3. *[3] Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point, written as $x_{(\alpha,\beta)}$ is defined to be an intuitionistic fuzzy subset of R, given by*

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta), \text{ if } x = y \\ (0, 1), \text{ if } x \neq y \end{cases}$$

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong in IFS $\langle \mu, \nu \rangle$ denoted by $x_{(\alpha,\beta)} \in \langle \mu, \nu \rangle$ if $\mu(x) \geq \alpha$ and $\nu(x) \leq \beta$ and we have for $x, y \in R$

$$x_{(t,s)} + y_{(\alpha,\beta)} = (x + y)_{(t\wedge\alpha, s\vee\beta)},$$
$$x_{(t,s)} y_{(\alpha,\beta)} = (xy)_{(t\wedge\alpha, s\vee\beta)}.$$

Definition 4. *[11] Let R be a ring. An intuitionistic fuzzy set $A = \{\langle x, \mu(x), \nu(x)\rangle | x \in R\}$ of R is said to be an intuitionistic fuzzy subring of R (in short, IFSR) of R if $\forall x, y \in R$*

i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$,

ii) $\nu(x - y) \leq \nu(x) \vee \nu(y)$,

iii) $\mu(xy) \geq \mu(x) \wedge \mu(y)$,

iv) $\nu(xy) \leq \nu(x) \vee \nu(y)$.

Definition 5. *[14] Let R be a ring. An intuitionistic fuzzy set $A = \{\langle x, \mu(x), \nu(x)\rangle | x \in R\}$ of R is said to be an intuitionistic fuzzy ideal of R (in short, IFI) of R if $\forall x, y \in R$*

i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$,

ii) $\nu(x - y) \leq \nu(x) \vee \nu(y)$,

iii) $\mu(xy) \geq \mu(x) \vee \mu(y)$,

iv) $\nu(xy) \leq \nu(x) \wedge \nu(y)$.

Definition 6. *[5] An intuitionistic fuzzy ideal $P = \langle \mu_P, \nu_P\rangle$ of a ring R, not necessarily non-constant, is called intuitionistic fuzzy prime ideal, if for any intuitionistic fuzzy ideals $A = \langle \mu_A, \nu_A\rangle$ and $B = \langle \mu_B, \nu_B\rangle$ of R the condition $AB \subset P$ implies that either $A \subset P$ or $B \subset P$.*

3 Main results

3.1 Intuitionistic fuzzy ideal

Let $\underline{R}$ be the subset of all intuitionistic fuzzy points of R, and let $\underline{A}$ denote the set of all intuitionistic fuzzy points contained in $A = \langle \mu_A, \nu_A\rangle$. That is, $\underline{A} = \{x_{(\alpha,\beta)} \in \underline{R} | \mu_A \geq \alpha$ and $\nu_A \leq \beta\}$

Theorem 1. *$A = \langle \mu, \nu\rangle$ is an intuitionistic fuzzy ideal of R if and only if:*

i) $\forall x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in \langle \mu_A, \nu_A\rangle,\ x_{(\alpha,\beta)} - y_{(\alpha',\beta')} \in \langle \mu, \nu\rangle$

ii) $\forall x_{(\alpha,\beta)} \in \underline{R},\ \forall y_{(\alpha',\beta')} \in \langle \mu, \nu\rangle,\ x_{(\alpha,\beta)} y_{(\alpha',\beta')} \in \langle \mu, \nu\rangle$.

Proof.
$\Rightarrow$)

Suppose that $\langle \mu, \nu\rangle$ is an intuitionistic fuzzy ideal, so we have for all $x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in \langle \mu, \nu\rangle$:

$$\mu(x - y) \geq \mu(x) \wedge \mu(y) \geq \alpha \wedge \alpha'$$

and

$$\nu(x - y) \leq \nu(x) \vee \nu(y) \leq \beta \vee \beta'.$$

Then,

$$x_{(\alpha,\beta)} - y_{(\alpha',\beta')} = (x - y)_{(\alpha\wedge\alpha',\beta\vee\beta')} \in \langle \mu, \nu\rangle,$$

and we have for $x_{(\alpha,\beta)} \in \underline{R}$ and $y_{(\alpha',\beta')} \in \langle \mu, \nu\rangle$:

$$\mu(xy) \geq \mu(x) \vee \mu(y) \geq \mu(y) \geq \alpha' \geq \alpha \wedge \alpha'$$

and

$$\nu(xy) \leq \nu(x) \wedge \nu(y) \leq \nu(y) \leq \alpha' \leq \alpha \vee \alpha',$$

hence, $(x.y)_{(\alpha\wedge\alpha',\beta\vee\beta')} = x_{(\alpha,\beta)} y_{(\alpha',\beta')} \in \langle \mu, \nu\rangle$.

$\Leftarrow$) Let $x, y \in R$. We have $x_{(\mu(x)\wedge\mu(y),\nu(x)\vee\nu(y))} \in \langle\mu,\nu\rangle$ and $y_{(\mu(x)\wedge\mu(y),\nu(x)\vee\nu(y))} \in \langle\mu,\nu\rangle$. Then, using the assumption we have

$$x_{(\mu(x)\wedge\mu(y),\nu(x)\vee\nu(y))} - y_{(\mu(x)\wedge\mu(y),\nu(x)\vee\nu(y))} \in \langle\mu,\nu\rangle.$$

Hence, $\mu(x-y) \geq \mu(x) \wedge \mu(y)$ and $\nu(x-y) \leq \nu(x) \vee \nu(y)$.

Now we will show that $\mu(xy) \geq \mu(x) \vee \mu(y)$ and $\nu(xy) \leq \nu(x) \wedge \nu(y)$.

Let $x, y \in R$, and suppose that $\mu(y) \geq \mu(x)$ and $\nu(x) \leq \nu(y)$ so for

$$\alpha = \alpha' = \mu(x) \vee \mu(y), \text{ and } \beta = \beta' = \nu(x) \wedge \nu(y),$$

we have

$$y_{(\alpha\vee\alpha',\beta\wedge\beta')} \in \langle\mu,\nu\rangle$$

since $x_{(\alpha\vee\alpha',\beta\wedge\beta')} \in \underline{R}$ implies that

$$x_{(\alpha\vee\alpha',\beta\wedge\beta')} \cdot y_{(\alpha\vee\alpha',\beta\wedge\beta')} \in \langle\mu,\nu\rangle.$$

Hence, $\mu(xy) \geq \mu(x) \vee \mu(y)$ and $\nu(xy) \leq \nu(x) \wedge \nu(y)$. The same is true if $\mu(x) \geq \mu(y)$ and $\nu(x) \leq \nu(y)$. □

3.2 Intuitionistic fuzzy prime ideal

Theorem 2. *[6] An intuitionistic fuzzy ideal $\langle\mu,\nu\rangle$ of R is an intuitionistic fuzzy prime ideal if and only if for any two intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\alpha',\beta')} \in \underline{R}$, $x_{(\alpha,\beta)} \cdot y_{(\alpha',\beta')} \in \langle\mu,\nu\rangle$, implies either $x_{(\alpha,\beta)} \in \langle\mu,\nu\rangle$ or $y_{(\alpha',\beta')} \in \langle\mu,\nu\rangle$.*

Theorem 3. *A subset $\langle\mu,\nu\rangle$ of R is said to be an intuitionistic fuzzy prime ideal if only if:*

i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$,

ii) $\nu(x-y) \leq \nu(x) \vee \nu(y)$,

iii) $\mu(xy) = \mu(x) \vee \mu(y)$,

iv) $\nu(xy) = \nu(x) \wedge \nu(y)$.

Proof. Let $\langle\mu,\nu\rangle$ be an intuitionistic fuzzy prime ideal. Suppose that $\mu(xy) > \mu(x) \vee \mu(y)$ and $\mu(x) \geq \mu(y)$, and suppose that $\nu(xy) < \nu(x) \wedge \nu(y)$ and $\nu(x) \leq \nu(y)$. Then $\mu(xy) > \mu(x) \geq \mu(y)$, and $\nu(xy) < \nu(x) \leq \nu(y)$ which implies that

$$x_{(\mu(xy),\nu(xy))} \notin \langle\mu,\nu\rangle \text{ and } y_{(\mu(xy),\nu(xy))} \notin \langle\mu,\nu\rangle.$$

Using the previous theorem, we have

$$x_{(\mu(xy),\nu(xy))} y_{(\mu(xy),\nu(xy))} \notin \langle\mu,\nu\rangle$$

which is absurd. Then,

$$\mu(xy) = \mu(x) \vee \mu(y) \text{ and } \nu(xy) = \nu(x) \wedge \nu(y)$$

Conversely, let $x_{(\alpha,\beta)}$, $y_{(\alpha',\beta')}$ be two intuitionistic fuzzy points of R, such that $x_{(\alpha,\beta)}y_{(\alpha',\beta')} \in \langle\mu,\nu\rangle$. Suppose that $x_{(\alpha,\beta)} \notin \langle\mu,\nu\rangle$ and $y_{(\alpha',\beta')} \notin \langle\mu,\nu\rangle$ for $\alpha = \alpha' = \mu(xy)$ and $\beta = \beta' = \nu(xy)$. We have $\mu(x) < \mu(xy)$ and $\mu(y) < \mu(xy)$ and $\nu(x) > \nu(xy)$ and $\nu(y) > \nu(xy)$. This implies that $x_{(\mu(xy),\nu(xy))} \notin \langle\mu,\nu\rangle$ and $y_{(\mu(xy),\nu(xy))} \notin \langle\mu,\nu\rangle$, which contradicts to $\langle\mu,\nu\rangle$ being an intuitionistic fuzzy prime ideal. □

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