Issues in IFSs and GNs, Vol. 12, 2015/2016, 1–19

On Intuitionistic Fuzzy Implications

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Abstract: The list of all intuitionistic fuzzy implications, introduced by the moment is given. Some properties of these implications are studied.
Keywords and phrases: Intuitionistic fuzzy set, Intuitionistic fuzzy implication.
2000 Mathematics Subject Classification: 03E72.

1 Introduction

In classical logic (see, e.g., [27]), to each proposition (sentence) we juxtapose its truth value: truth – denoted by 1, or falsity – denoted by 0. In the case of fuzzy logic [28], this truth value is a real number in the interval $[0, 1]$ and it may be called "truth degree". In the intuitionistic fuzzy case, we add one more value – "falsity degree" – which is in interval $[0, 1]$ as well. Thus, to the proposition p, two real numbers, $\mu(p)$ and $\nu(p)$, are assigned with the following constraint:

$$\mu(p) + \nu(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositions, that here and below we will denote by $\mathcal{S}$, in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

Hence the function $V : S \to [0, 1] \times [0, 1]$ gives the truth and falsity degrees of all elements of $\mathcal{S}$.

We assume that the evaluation function V assigns to the logical truth T

$$V(T) = \langle 1, 0 \rangle,$$

and to the logical falsity F

$$V(F) = \langle 0, 1 \rangle.$$

Let for every real number x:

$$\mathrm{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ \\ 0 & \text{if } x \le 0 \end{cases}, \qquad \overline{sg}(x) = \begin{cases} 0 & \text{if } x > 0 \\ \\ 1 & \text{if } x \le 0 \end{cases}.$$

In a series of papers, 185 different implications were defined and some of their basic properties were studied. In some of these publications, some misprints in the formulas were found during the last years. Here, we give the full list of the corrected intuitionistic fuzzy implications.

The first 10 implications, described in [5] are intuitionistic fuzzy analogues of existing fuzzy implications in literature (see, e.g., [26]). The next five implications were introduced in author's publications [7, 13, 14, 15]. Two of them are given in papers of B. Kolev [13] ($\to_{13}$) and of T. Trifonov ($\to_{14}$) [14, 15] and the author. From the so constructed 15 implications, using the formula $\neg A = A \to F$, five negations are formulated – the standard (classical) negation $\neg_1$ and four others $\neg_2, ..., \neg_5$, where F is defined above and $\to$ is each one of these 15 implications [4]. By formulas

$$A \to B = \neg A \vee B \quad \text{and} \quad A \to B = \neg A \vee \neg\neg B,$$

these 5 negations generate eight new implications. Each of these implications generates four new implications, by formulas using intuitionistic fuzzy (standard, classical) modal operators. The intuitionistic fuzzy set analogues of these implications are given in [6]. Last author's implications are published in [1, 2, 3, 9, 10, 11].

In [16, 17, 18, 19], L. Atanassova introduced 11 new implications ($\to_{139}$, ..., $\to_{149}$). P. Dworniczak generalized them in [23, 24, 25] (as implications $\to_{150}$, ..., $\to_{152}$) and L. Atanassova modified Dworniczak's implications in [20, 21, 22] (as implications $\to_{154}, ..., \to_{165}$).

2 List of the Intuitionistic Fuzzy Implications

All existing at the moment implications are given in Table 1. In it, we keep the numeration from [6].

Table 1: List of the intuitionistic fuzzy implications

$\rightarrow_1$	$\langle \max(b, \min(a, c)), \min(a, d)\rangle$
$\rightarrow_2$	$\langle \overline{sg}(a-c), d\mathrm{sg}(a-c)\rangle$
$\rightarrow_3$	$\langle 1-(1-c)\mathrm{sg}(a-c)), d\mathrm{sg}(a-c)\rangle$
$\rightarrow_4$	$\langle \max(b, c), \min(a, d)\rangle$
$\rightarrow_5$	$\langle \min(1, b+c), \max(0, a+d-1)\rangle$
$\rightarrow_6$	$\langle b+ac, ad\rangle$
$\rightarrow_7$	$\langle \min(\max(b, c), \max(a, b), \max(c, d)),$ $\max(\min(a, d), \min(a, b), \min(c, d))\rangle$
$\rightarrow_8$	$\langle 1-(1-\min(b, c))\mathrm{sg}(a-c), \max(a, d)\mathrm{sg}(a-c), \mathrm{sg}(d-b)\rangle$
$\rightarrow_9$	$\langle b+a^2c, ab+a^2d\rangle$
$\rightarrow_{10}$	$\langle c\overline{sg}(1-a)+\mathrm{sg}(1-a)(\overline{sg}(1-c)+b\mathrm{sg}(1-c)),$ $d\overline{sg}(1-a)+a\mathrm{sg}(1-a)\mathrm{sg}(1-c)\rangle$
$\rightarrow_{11}$	$\langle 1-(1-c)\mathrm{sg}(a-c), d\mathrm{sg}(a-c)\mathrm{sg}(d-b)\rangle$
$\rightarrow_{12}$	$\langle \max(b, c), 1-\max(b, c)\rangle$
$\rightarrow_{13}$	$\langle b+c-bc, ad\rangle$
$\rightarrow_{14}$	$\langle 1-(1-c)\mathrm{sg}(a-c)-d\overline{sg}(a-c)\mathrm{sg}(d-b), d\mathrm{sg}(d-b)\rangle$
$\rightarrow_{15}$	$\langle 1-(1-\min(b, c))\mathrm{sg}(\mathrm{sg}(a-c)+\mathrm{sg}(d-b))$ $-\min(b, c)\mathrm{sg}(a-c)\mathrm{sg}(d-b),\ 1-(1-\max(a, d))\mathrm{sg}(\overline{sg}(a-c)$ $+\overline{sg}(d-b))-\max(a, d)\overline{sg}(a-c)\overline{sg}(d-b)\rangle$
$\rightarrow_{16}$	$\langle \max(\overline{sg}(a), c), \min(\mathrm{sg}(a), d)\rangle$
$\rightarrow_{17}$	$\langle \max(b, c), \min(ab+a^2, d)\rangle$
$\rightarrow_{18}$	$\langle \max(b, c), \min(1-b, d)\rangle$
$\rightarrow_{19}$	$\langle \max(1-\mathrm{sg}(\mathrm{sg}(a)+\mathrm{sg}(1-b)), c), \min(\mathrm{sg}(1-b), d)\rangle$
$\rightarrow_{20}$	$\langle \max(\overline{sg}(a), \mathrm{sg}(c)), \min(\mathrm{sg}(a), \overline{sg}(c))\rangle$
$\rightarrow_{21}$	$\langle \max(b, c(c+d)), \min(a(a+b), d(c^2+d+cd))\rangle$
$\rightarrow_{22}$	$\langle \max(b, 1-d), 1-\max(b, 1-d)\rangle$
$\rightarrow_{23}$	$\langle 1-\min(\mathrm{sg}(1-b), \overline{sg}(1-d)), \min(\mathrm{sg}(1-b), \overline{sg}(1-d))\rangle$
$\rightarrow_{24}$	$\langle \overline{sg}(a-c)\overline{sg}(d-b), \mathrm{sg}(a-c)\mathrm{sg}(d-b)\rangle$
$\rightarrow_{25}$	$\langle \max(b, \overline{sg}(a)\overline{sg}(1-b)), c\overline{sg}(d)\overline{sg}(1-c)), \min(a, d)\rangle$
$\rightarrow_{26}$	$\langle \max(\overline{sg}(1-b), c), \min(\mathrm{sg}(a), d)\rangle$
$\rightarrow_{27}$	$\langle \max(\overline{sg}(1-b), \mathrm{sg}(c)), \min(\mathrm{sg}(a), \overline{sg}(1-d))\rangle$

$\rightarrow_{28}$	$\langle \max(\overline{sg}(1-b), c), \min(a,d)\rangle$
$\rightarrow_{29}$	$\langle \max(\overline{sg}(1-b), \overline{sg}(1-c)), \min(a, \overline{sg}(1-d))\rangle$
$\rightarrow_{30}$	$\langle \max(1-a, \min(a, 1-d)), \min(a,d)\rangle$
$\rightarrow_{31}$	$\langle \overline{sg}(a+d-1), d\mathrm{sg}(a+d-1)\rangle$
$\rightarrow_{32}$	$\langle 1-d\mathrm{sg}(a+d-1), d\mathrm{sg}(a+d-1)\rangle$
$\rightarrow_{33}$	$\langle 1-\min(a,d), \min(a,d)\rangle$
$\rightarrow_{34}$	$\langle \min(1, 2-a-d), \max(0, a+d-1)\rangle$
$\rightarrow_{35}$	$\langle 1-ad, ad\rangle$
$\rightarrow_{36}$	$\langle \min(1-\min(a,d), \max(a,1-a), \max(1-d,d)),$ $\max(\min(a,d), \min(a,1-a), \min(1-d,d))\rangle$
$\rightarrow_{37}$	$\langle 1-\max(a,d)\mathrm{sg}(a+d-1), \max(a,d)\mathrm{sg}(a+d-1)\rangle$
$\rightarrow_{38}$	$\langle 1-a+a^2(1-d), a(1-a)+a^2d\rangle$
$\rightarrow_{39}$	$\langle (1-d)\overline{sg}(1-a)+\mathrm{sg}(1-a)(\overline{sg}(d)+(1-a)\mathrm{sg}(d)),$ $d\overline{sg}(1-a)+a\mathrm{sg}(1-a)\mathrm{sg}(d)\rangle$
$\rightarrow_{40}$	$\langle 1-\mathrm{sg}(a+d-1), 1-\overline{sg}(a+d-1)\rangle$
$\rightarrow_{41}$	$\langle \max(\overline{sg}(a), 1-d), \min(\mathrm{sg}(a), d)\rangle$
$\rightarrow_{42}$	$\langle \max(\overline{sg}(a), \mathrm{sg}(1-d)), \min(\mathrm{sg}(a), \overline{sg}(1-d))\rangle$
$\rightarrow_{43}$	$\langle \max(\overline{sg}(a), 1-d), \min(\mathrm{sg}(a), d)\rangle$
$\rightarrow_{44}$	$\langle \max(\overline{sg}(a), 1-d), \min(a,d)\rangle$
$\rightarrow_{45}$	$\langle \max(\overline{sg}(a), \overline{sg}(d)), \min(a, \overline{sg}(1-d))\rangle$
$\rightarrow_{46}$	$\langle \max(b, \min(1-b, c)), 1-\max(b,c)\rangle$
$\rightarrow_{47}$	$\langle \overline{sg}(1-b-c), (1-c)\mathrm{sg}(1-b-c)\rangle$
$\rightarrow_{48}$	$\langle 1-(1-c)\mathrm{sg}(1-b-c), (1-c)\mathrm{sg}(1-b-c)\rangle$
$\rightarrow_{49}$	$\langle \min(1, b+c), \max(0, 1-b-c)\rangle$
$\rightarrow_{50}$	$\langle b+c-bc, 1-b-c+bc\rangle$
$\rightarrow_{51}$	$\langle \min(\max(b,c), \max(1-b,b), \max(c,1-c)),$ $\max(1-\max(b,c), \min(1-b,b), \min(c,1-c))\rangle$
$\rightarrow_{52}$	$\langle 1-(1-\min(b,c))\mathrm{sg}(1-b-c), 1-\min(b,c)\mathrm{sg}(1-b-c)\rangle$
$\rightarrow_{53}$	$\langle b+(1-b)^2c, (1-b)b+(1-b)^2(1-c)\rangle$
$\rightarrow_{54}$	$\langle c\overline{sg}(b)+\mathrm{sg}(b)(\overline{sg}(1-c)+b\mathrm{sg}(1-c)),$ $(1-c)\overline{sg}(b)+(1-b)\mathrm{sg}(b)\mathrm{sg}(1-c)\rangle$
$\rightarrow_{55}$	$\langle 1-\mathrm{sg}(1-b-c), 1-\overline{sg}(1-b-c)\rangle$
$\rightarrow_{56}$	$\langle \max(\overline{sg}(1-b), c), \min(\mathrm{sg}(1-b), 1-c)\rangle$
$\rightarrow_{57}$	$\langle \max(\overline{sg}(1-b), \mathrm{sg}(c)), \min(\mathrm{sg}(1-b), \overline{sg}(c))\rangle$
$\rightarrow_{58}$	$\langle \max(\overline{sg}(1-b), \overline{sg}(1-c)), 1-\max(b,c)\rangle$
$\rightarrow_{59}$	$\langle \max(\overline{sg}(1-b), c), 1-\max(b,c)\rangle$
$\rightarrow_{60}$	$\langle \max(\overline{sg}(1-b), \overline{sg}(1-c)), \min(1-b, \overline{sg}(c))\rangle$

$\rightarrow_{61}$	$\langle \max(c, \min(b, d)), \min(a, d) \rangle$
$\rightarrow_{62}$	$\langle \overline{sg}(d - b), a\mathrm{sg}(d - b) \rangle$
$\rightarrow_{63}$	$\langle 1 - (1 - b)\mathrm{sg}(d - b), a\mathrm{sg}(d - b) \rangle$
$\rightarrow_{64}$	$\langle c + bd, ad \rangle$
$\rightarrow_{65}$	$\langle 1 - (1 - \min(b, c))\mathrm{sg}(d - b), \max(a, d)\mathrm{sg}(d - b)\mathrm{sg}(a - c) \rangle$
$\rightarrow_{66}$	$\langle c + d^2 b, bd + d^2 a \rangle$
$\rightarrow_{67}$	$\langle b\overline{sg}(1 - d) + \mathrm{sg}(1 - d)(\overline{sg}(1 - b) + c\mathrm{sg}(1 - b)),$ $a\overline{sg}(1 - d) + d\mathrm{sg}(1 - d)\mathrm{sg}(1 - b) \rangle$
$\rightarrow_{68}$	$\langle 1 - (1 - b)\mathrm{sg}(d - b), a\mathrm{sg}(d - b)\mathrm{sg}(a - c) \rangle$
$\rightarrow_{69}$	$\langle 1 - (1 - b)\mathrm{sg}(d - b) - a\overline{sg}(d - b)\mathrm{sg}(a - c), a\mathrm{sg}(a - c) \rangle$
$\rightarrow_{70}$	$\langle \max(\overline{sg}(d), b), \min(\mathrm{sg}(d), a) \rangle$
$\rightarrow_{71}$	$\langle \max(b, c), \min(cd + d^2, a) \rangle$
$\rightarrow_{72}$	$\langle \max(b, c), \min(1 - c, a) \rangle$
$\rightarrow_{73}$	$\langle \max(1 - \max(\mathrm{sg}(d), \mathrm{sg}(1 - c)), b), \min(\mathrm{sg}(1 - c), a) \rangle$
$\rightarrow_{74}$	$\langle \max(\mathrm{sg}(b), \overline{sg}(d)), \min(\overline{sg}(b), \mathrm{sg}(d)) \rangle$
$\rightarrow_{75}$	$\langle \max(c, b(a + b)), \min(d(c + d), a(b^2 + a) + ab) \rangle$
$\rightarrow_{76}$	$\langle \max(c, 1 - a), \min(1 - c, a) \rangle$
$\rightarrow_{77}$	$\langle (1 - \min(\overline{sg}(1 - a), \mathrm{sg}(1 - c))), \min(\overline{sg}(1 - a), \mathrm{sg}(1 - c)) \rangle$
$\rightarrow_{78}$	$\langle \max(\overline{sg}(1 - c), b), \min(\mathrm{sg}(d), a) \rangle$
$\rightarrow_{79}$	$\langle \max(\overline{sg}(1 - c), \mathrm{sg}(b)), \min(\mathrm{sg}(d), \overline{sg}(1 - a)) \rangle$
$\rightarrow_{80}$	$\langle \max(\overline{sg}(1 - c), b), \min(d, a) \rangle$
$\rightarrow_{81}$	$\langle \max(\overline{sg}(1 - b), \overline{sg}(1 - c)), \min(d, \overline{sg}(1 - a)) \rangle$
$\rightarrow_{82}$	$\langle \max(1 - d, \min(d, 1 - a)), \min(d, a) \rangle$
$\rightarrow_{83}$	$\langle \overline{sg}(a + d - 1), a\mathrm{sg}(a + d - 1) \rangle$
$\rightarrow_{84}$	$\langle 1 - a\mathrm{sg}(a + d - 1), a\mathrm{sg}(a + d - 1) \rangle$
$\rightarrow_{85}$	$\langle 1 - d + d^2(1 - a), d(1 - d) + d^2 \rangle$
$\rightarrow_{86}$	$\langle (1 - a)\overline{sg}(1 - d) + \mathrm{sg}(1 - d)(\overline{sg}(a) + (1 - d)\mathrm{sg}(d)),$ $a\overline{sg}(1 - d) + d\mathrm{sg}(1 - d)\mathrm{sg}(a) \rangle$
$\rightarrow_{87}$	$\langle \max(\overline{sg}(d), 1 - a), \min(\mathrm{sg}(d), a) \rangle$
$\rightarrow_{88}$	$\langle \max(\overline{sg}(d), \mathrm{sg}(1 - a)), \min(\mathrm{sg}(d), \overline{sg}(1 - a)) \rangle$
$\rightarrow_{89}$	$\langle \max(\overline{sg}(d), 1 - a), \min(d, a) \rangle$
$\rightarrow_{90}$	$\langle \max(\overline{sg}(a), \overline{sg}(d)), \min(d, \overline{sg}(1 - a)) \rangle$
$\rightarrow_{91}$	$\langle \max(c, \min(1 - c, b)), 1 - \max(b, c) \rangle$
$\rightarrow_{92}$	$\langle \overline{sg}(1 - b - c), \min(1 - b, \mathrm{sg}(1 - b - c)) \rangle$
$\rightarrow_{93}$	$\langle (1 - \min(1 - b, \mathrm{sg}(1 - b - c)), \min(1 - b, \mathrm{sg}(1 - b - c)) \rangle$
$\rightarrow_{94}$	$\langle c + (1 - c)^2 b, (1 - c)c + (1 - c)^2(1 - b) \rangle$

$\rightarrow_{95}$	$\langle \min(b, \overline{sg}(c)) + \mathrm{sg}(c)(\overline{sg}(1-b) + \min(c, \mathrm{sg}(1-b))),$ $\min(1-b, \overline{sg}(c)) + \min(1-c, \mathrm{sg}(c), \mathrm{sg}(1-b))\rangle$
$\rightarrow_{96}$	$\langle \max(\overline{sg}(1-c), b), \min(\mathrm{sg}(1-b), 1-c)\rangle$
$\rightarrow_{97}$	$\langle \max(\overline{sg}(1-c), \mathrm{sg}(b)), \min(\mathrm{sg}(1-c), \overline{sg}(b))\rangle$
$\rightarrow_{98}$	$\langle \max(\overline{sg}(1-c), b), 1 - \max(b, c)\rangle$
$\rightarrow_{99}$	$\langle \max(\overline{sg}(1-c), \overline{sg}(1-b)), \min(1-c, \overline{sg}(b))\rangle$
$\rightarrow_{100}$	$\langle \max(\min(b, \mathrm{sg}(a)), c), \min(a, \mathrm{sg}(b), d)\rangle$
$\rightarrow_{101}$	$\langle \max(\min(b, \mathrm{sg}(a)), \min(c, \mathrm{sg}(d))), \min(a, \mathrm{sg}(b), \mathrm{sg}(c), d)\rangle$
$\rightarrow_{102}$	$\langle \max(b, \min(c, \mathrm{sg}(d))), \min(a, \mathrm{sg}(c), d)\rangle$
$\rightarrow_{103}$	$\langle \max(\min(1-a, \mathrm{sg}(a)), 1-d), \min(a, \mathrm{sg}(1-a), d)\rangle$
$\rightarrow_{104}$	$\langle \max(\min(1-a, \mathrm{sg}(a)), \min(1-d, \mathrm{sg}(d))),$ $\min(a, \mathrm{sg}(1-a), d, \mathrm{sg}(1-d))\rangle$
$\rightarrow_{105}$	$\langle \max(1-a, \min(1-d, \mathrm{sg}(d))), \min(a, d, \mathrm{sg}(1-d))\rangle$
$\rightarrow_{106}$	$\langle \max(\min(b, \mathrm{sg}(1-b)), c), \min(1-b, \mathrm{sg}(b), 1-c)\rangle$
$\rightarrow_{107}$	$\langle \max(\min(b, \mathrm{sg}(1-b)), \min(c, \mathrm{sg}(1-c))),$ $\min(1-b, \mathrm{sg}(b), 1-c, \mathrm{sg}(c))\rangle$
$\rightarrow_{108}$	$\langle \max(b, \min(c, \mathrm{sg}(1-c))), \min(1-b, 1-c, \mathrm{sg}(c))\rangle$
$\rightarrow_{109}$	$\langle b + \min(\overline{sg}(1-a), c), ab + \min(\overline{sg}(1-a), d))\rangle$
$\rightarrow_{110}$	$\langle \max(b, c), \min(ab + \overline{sg}(1-a), d)\rangle$
$\rightarrow_{111}$	$\langle \max(b, cd + \overline{sg}(1-c)), \min(ab + \overline{sg}(1-a),$ $d(cd + \overline{sg}(1-c)) + \overline{sg}(1-d))\rangle$
$\rightarrow_{112}$	$\langle b + c - bc, ab + \overline{sg}(1-a)d\rangle$
$\rightarrow_{113}$	$\langle b + cd - b(cd + \overline{sg}(1-c)),$ $(ab + \overline{sg}(1-a))(d(cd + \overline{sg}(1-c)) + \overline{sg}(1-d))\rangle$
$\rightarrow_{114}$	$\langle 1 - a + \min(\overline{sg}(1-a), 1-d), a(1-a) + \min(\overline{sg}(1-a), d)\rangle$
$\rightarrow_{115}$	$\langle 1 - \min(a, d), \min(a(1-a) + \overline{sg}(1-a), d)\rangle$
$\rightarrow_{116}$	$\langle \max(1-a, (1-d)d + \overline{sg}(d)),$ $\min(a(1-a) + \overline{sg}(1-a), d((1-d)d + \overline{sg}(d)) + \overline{sg}(1-d))\rangle$
$\rightarrow_{117}$	$\langle 1 - a - d + ad, (a(1-a) + \overline{sg}(1-a))d\rangle$
$\rightarrow_{118}$	$\langle 1 - a + (1-d)d - (1-a)((1-d)d + \overline{sg}(d)),$ $(a(1-a) + \overline{sg}(1-a))d((1-d)d + \overline{sg}(d)) + \overline{sg}(1-d)\rangle$
$\rightarrow_{119}$	$\langle b + \min(\overline{sg}(b), c), (1-b)b + \min(\overline{sg}(b), 1-c)\rangle$
$\rightarrow_{120}$	$\langle \max(b, c), \min((1-b)b + \overline{sg}(b), 1-c)\rangle$
$\rightarrow_{121}$	$\langle \max(b, c(1-c) + \overline{sg}(1-c)),$ $\min((1-b)b + \overline{sg}(b), (1-c)(c(1-c) + \overline{sg}(1-c))) + \overline{sg}(c)\rangle$
$\rightarrow_{122}$	$\langle b + c - bc, ((1-c)b + \overline{sg}(b))(1-c)\rangle$
$\rightarrow_{123}$	$\langle b + c(1-c) - (b(c(1-c) + \overline{sg}(1-c))),$ $((1-b)b + \overline{sg}(b))(((1-c)(c(1-c) + \overline{sg}(1-c))) + \overline{sg}(c))\rangle$

$\rightarrow_{124}$	$\langle c+\min(\overline{sg}(1-d),b), cd+\min(\overline{sg}(1-d),a)\rangle$
$\rightarrow_{125}$	$\langle \max(b,c), \min(cd+\overline{sg}(1-d),a)\rangle$
$\rightarrow_{126}$	$\langle \max(c, ab+\overline{sg}(1-b)),$ $\min(cd+\overline{sg}(1-d), a(ab+\overline{sg}(1-b))+\overline{sg}(1-a))\rangle$
$\rightarrow_{127}$	$\langle b+c-bc, (cd+\overline{sg}(1-d))a\rangle$
$\rightarrow_{128}$	$\langle c+ab-c(ab+\overline{sg}(1-b)),$ $(cd+\overline{sg}(1-d))(a(ab+\overline{sg}(1-b))+\overline{sg}(1-a))\rangle$
$\rightarrow_{129}$	$\langle 1-d+\min(\overline{sg}(1-d),1-a), d(1-d)+\min(\overline{sg}(1-d),a)\rangle$
$\rightarrow_{130}$	$\langle 1-\min(d,a), \min(d(1-d)+\overline{sg}(1-d),a)\rangle$
$\rightarrow_{131}$	$\langle \max(1-d, (1-a)a+\overline{sg}(a)),$ $\min(d(1-d)+\overline{sg}(1-d), a((1-a)a+\overline{sg}(a))+\overline{sg}(1-a))\rangle$
$\rightarrow_{132}$	$\langle 1-ad, (d(1-d)+\overline{sg}(1-d))a\rangle$
$\rightarrow_{133}$	$\langle 1-d+(1-a)a-(1-d)((1-a)a+\overline{sg}(a)),$ $(d(1-d)+\overline{sg}(1-d))(a((1-a)a+\overline{sg}(a))+\overline{sg}(1-a))\rangle$
$\rightarrow_{134}$	$\langle c+\min(\overline{sg}(c),b), (1-c)c+\min(\overline{sg}(c),(1-b))\rangle$
$\rightarrow_{135}$	$\langle \max(b,c), \min((1-c)c+\overline{sg}(c),1-b)\rangle$
$\rightarrow_{136}$	$\langle \max(c, b(1-b)+\overline{sg}(1-b)),$ $\min((1-c)c+\overline{sg}(c), (1-b)(b(1-b)+\overline{sg}(1-b))+\overline{sg}(b))\rangle$
$\rightarrow_{137}$	$\langle b+c-bc, ((1-c)c+\overline{sg}(c))(1-b)\rangle$
$\rightarrow_{138}$	$\langle c+b(1-b)-c(b(1-b)+\overline{sg}(1-b)),$ $((1-c)c+\overline{sg}(c))((1-b)(b(1-b)+\overline{sg}(1-b))+\overline{sg}(b))\rangle$
$\rightarrow_{139}$	$\langle \frac{b+c}{2}, \frac{a+d}{2}\rangle$
$\rightarrow_{140}$	$\langle \frac{b+c+\min(b,c)}{3}, \frac{a+d+\max(a,d)}{3}\rangle$
$\rightarrow_{141}$	$\langle \frac{b+c+\max(b,c)}{3}, \frac{a+d+\min(a,d)}{3}\rangle$
$\rightarrow_{142}$	$\langle \frac{3-a-d-\max(a,d)}{3}, \frac{a+d+\max(a,d)}{3}\rangle$
$\rightarrow_{143}$	$\langle \frac{1-a+c+\min(1-a,c)}{3}, \frac{2+a-c-\min(1-a,c)}{3}\rangle$
$\rightarrow_{144}$	$\langle \frac{1+b-d+\min(b,1-d)}{3}, \frac{2-b+d-\min(b,1-d)}{3}\rangle$
$\rightarrow_{145}$	$\langle \frac{b+c+\min(b,c)}{3}, \frac{3-b-c-\min(b,c)}{3}\rangle$
$\rightarrow_{146}$	$\langle \frac{3-a-d-\min(a,d)}{3}, \frac{a+d+\min(a,d)}{3}\rangle$
$\rightarrow_{147}$	$\langle \frac{1-a+c+\max(1-a,c)}{3}, \frac{2+a-c-\max(1-a,c)}{3}\rangle$
$\rightarrow_{148}$	$\langle \frac{1+b-d+\max(b,1-d)}{3}, \frac{2-b+d-\max(b,1-d)}{3}\rangle$
$\rightarrow_{149}$	$\langle \frac{b+c+\max(b,c)}{3}, \frac{3-b-c-\max(b,c)}{3}\rangle$
$\rightarrow_{150,\lambda}$	$\langle \frac{b+c+\lambda-1}{2\lambda}, \frac{a+d+\lambda-1}{2\lambda}\rangle$, where $\lambda \geq 1$
$\rightarrow_{151,\gamma}$	$\langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1}\rangle$, where $\gamma \geq 1$

$\rightarrow_{152,\alpha,\beta}$	$\langle\frac{b+c+\alpha-1}{\alpha+\beta}, \frac{a+d+\beta-1}{\alpha+\beta}\rangle$, where $\alpha \geq 1, \beta \in [1,\alpha]$
$\rightarrow_{153,\varepsilon,\eta}$	$\langle\min(1,\max(c,b+\varepsilon)),\max(0,\min(d,a-\eta))\rangle$, where $\varepsilon,\eta \in [0,1]$ and $\varepsilon \leq \eta < 1$
$\rightarrow_{154,\lambda}$	$\langle\frac{-a+c+\lambda}{2\lambda}, \frac{a-c+\lambda}{2\lambda}\rangle$, where $\lambda \geq 1$
$\rightarrow_{155,\lambda}$	$\langle\frac{1-a-d+\lambda}{2\lambda}, \frac{a+d+\lambda-1}{2\lambda}\rangle$, where $\lambda \geq 1$
$\rightarrow_{156,\lambda}$	$\langle\frac{b+c+\lambda-1}{2\lambda}, \frac{1-b-c+\lambda}{2\lambda}\rangle$, where $\lambda \geq 1$
$\rightarrow_{157,\lambda}$	$\langle\frac{b-d+\lambda}{2\lambda}, \frac{-b+d+\lambda}{2\lambda}\rangle$, where $\lambda \geq 1$
$\rightarrow_{158,\gamma}$	$\langle\frac{1-a+c+\gamma}{2\gamma+1}, \frac{a-c+\gamma}{2\gamma+1}\rangle$, where $\gamma \geq 1$
$\rightarrow_{159,\gamma}$	$\langle\frac{2-a-d+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1}\rangle$, where $\gamma \geq 1$
$\rightarrow_{160,\gamma}$	$\langle\frac{b-d+\gamma+1}{2\gamma+1}, \frac{-b+d+\gamma}{2\gamma+1}\rangle$, where $\gamma \geq 1$
$\rightarrow_{161,\gamma}$	$\langle\frac{b+c+\gamma}{2\gamma+1}, \frac{1-b-c+\gamma}{2\gamma+1}\rangle$, where $\gamma \geq 1$
$\rightarrow_{162,\alpha,\beta}$	$\langle\frac{-a+c+\alpha}{\alpha+\beta}, \frac{a-c+\beta}{\alpha+\beta}\rangle$, where $\alpha \geq 1, \beta \in [1,\alpha]$
$\rightarrow_{163,\alpha,\beta}$	$\langle\frac{1-a-d+\alpha}{\alpha+\beta}, \frac{a+d+\beta-1}{\alpha+\beta}\rangle$, where $\alpha \geq 1, \beta \in [1,\alpha]$
$\rightarrow_{164,\alpha,\beta}$	$\langle\frac{b-d+\alpha}{\alpha+\beta}, \frac{-b+d+\beta}{\alpha+\beta}\rangle$, where $\alpha \geq 1, \beta \in [1,\alpha]$
$\rightarrow_{165,\alpha,\beta}$	$\langle\frac{b+c+\alpha-1}{\alpha+\beta}, \frac{1-b-c+\beta}{\alpha+\beta}\rangle$, where $\alpha \geq 1, \beta \in [1,\alpha]$
$\rightarrow_{166}$	$\langle\max(b,\min(a,c)),\min(a,\max(b,d))\rangle$
$\rightarrow_{167}$	$\langle\max(1-a,\min(a,c)),\min(a,1-\min(a,c))\rangle$
$\rightarrow_{168}$	$\langle\max(1-a,\min(a,1-d)),1-\max(1-a,\min(a,1-d))\rangle$
$\rightarrow_{169}$	$\langle\max(b,\min(1-b,c)),1-\max(b,\min(1-b,c))\rangle$
$\rightarrow_{170}$	$\langle\max(b,\min(1-b,1-d)),1-\max(b,\min(1-b,1-d))\rangle$
$\rightarrow_{171}$	$\langle\overline{sg}(\max(a,d)-\max(b,c)),\mathrm{sg}(\max(a,d)-\max(b,c))\rangle$
$\rightarrow_{172}$	$\langle\overline{sg}(a-c),\mathrm{sg}(a-c)\rangle$
$\rightarrow_{173}$	$\langle\overline{sg}(a+d-1),\mathrm{sg}(a+d-1)\rangle$
$\rightarrow_{174}$	$\langle\overline{sg}(1-b-c),\mathrm{sg}(1-b-c)\rangle$
$\rightarrow_{175}$	$\langle\overline{sg}(d-b),\mathrm{sg}(d-b)\rangle$
$\rightarrow_{176}$	$\langle\overline{sg}(a-c)+\mathrm{sg}(a-c)\max(b,c),\mathrm{sg}(a-c)\min(a,d)\rangle$
$\rightarrow_{177}$	$\langle\overline{sg}(a-c)+\mathrm{sg}(a-c)\max(1-a,c),\mathrm{sg}(a-c)\min(a,1-c)\rangle$
$\rightarrow_{178}$	$\langle\overline{sg}(a-1+d)+\mathrm{sg}(a-1+d)(1-\min(a,d)),$ $\mathrm{sg}(a-1+d)\min(a,d)\rangle$
$\rightarrow_{179}$	$\langle\overline{sg}(1-b-c)+\mathrm{sg}(1-b-c)\max(b,c),$ $\mathrm{sg}(1-b-c)(1-\max(b,c))\rangle$
$\rightarrow_{180}$	$\langle\overline{sg}(d-b)+\mathrm{sg}(d-b)\max(b,1-d),\mathrm{sg}(d-b)\min(1-b,d)\rangle$
$\rightarrow_{181}$	$\langle 1-\mathrm{sg}(a).(1-c),d.\mathrm{sg}(a)\rangle$
$\rightarrow_{182}$	$\langle 1-\mathrm{sg}(a).(1-c),(1-c).\mathrm{sg}(a)\rangle$

$\rightarrow_{183}$	$\langle 1 - \mathrm{sg}(a).d, d.\mathrm{sg}(a)\rangle$
$\rightarrow_{184}$	$\langle 1 - \mathrm{sg}(1-b).d, d.\mathrm{sg}(1-b)\rangle$
$\rightarrow_{185}$	$\langle 1 - \mathrm{sg}(1-b).(1-c), (1-c).\mathrm{sg}(1-b)\rangle$

3 The Intuitionistic Fuzzy Implications and the Basic Logical Constants

By direct check we see that the following equalities hold.

$$\langle 0,1\rangle \rightarrow_i \langle 0,1\rangle = \begin{cases} \langle 1,0\rangle, & \text{for } i = 1, ..., 99, 102, 105, 108, ..., 127, \\ & 129, ..., 132, 134, ..., 137, 153, 166, ..., \\ & 180 \\ \langle 0,0\rangle, & \text{for } i = 100, 101, 103, 104, 106, 107, \\ & 128, 133, 138 \\ \langle \frac{1}{2}, \frac{1}{2}\rangle, & \text{for } i = 139, \\ \langle \frac{1}{3}, \frac{2}{3}\rangle, & \text{for } i = 140, 142, ..., 145, \\ \langle \frac{2}{3}, \frac{1}{3}\rangle, & \text{for } i = 141, 146, ..., 149, \\ \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 150, 154, ..., 157, \\ \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 151, 158, ..., 161, \\ \langle \frac{\alpha+1}{\alpha+\beta}, \frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 152, 162, ..., 165. \end{cases}$$

$$\langle 0,1\rangle \rightarrow_i \langle 1,0\rangle = \begin{cases} \langle 1,0\rangle, & \text{for } i = 1, ..., 100, 102, 103, 105, 106, \\ & 108, ..., 112, 114, ..., 117, ..., 121, 123, \\ & ..., 126, 128, ..., 132, 134, ..., 137, 139, \\ & ..., 149, 153, 166, ..., 180 \\ \langle 0,0\rangle, & \text{for } i = 101, 104, 107, 113, 118, 122, \\ & 127, 133, 138 \\ \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 150, 154, ..., 157 \\ \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 151, 158, ..., 161, \\ \langle \frac{\alpha+1}{\alpha+\beta}, \frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 152, 162, ..., 165. \end{cases}$$

$$\langle 0,1\rangle \to_i \langle 0,0\rangle = \begin{cases} \langle 1,0\rangle, & \text{for } i = 1,...,6,8,...,60,62,63,65,67, \\ & ...,99,102,103,105,108,...,117,119, \\ & ...,123,125,...,127,129,...,132,134, \\ & ...,137,142,144,146,148,153,166, \\ & ...,180 \\ \langle 0,0\rangle, & \text{for } i = 7,61,64,66,100,101,104, \\ & 106,107,118,124,128,133,138 \\ \langle \frac{1}{2},0\rangle, & \text{for } i = 139, \\ \langle \frac{1}{3},0\rangle, & \text{for } i = 140, \\ \langle \frac{2}{3},0\rangle, & \text{for } i = 141, \\ \langle \frac{1}{3},\frac{2}{3}\rangle, & \text{for } i = 143,145, \\ \langle \frac{2}{3},\frac{1}{3}\rangle, & \text{for } i = 147,149, \\ \langle \frac{1}{2},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 150, \\ \langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 151, \\ \langle \frac{\alpha}{\alpha+\beta},\frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 152, \\ \langle \frac{1}{2},\frac{1}{2}\rangle, & \text{for } i = 154,156 \\ \langle \frac{\lambda+1}{2\lambda},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 155,157, \\ \langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 158,161, \\ \langle \frac{\gamma+2}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 159,160, \\ \langle \frac{\alpha}{\alpha+\beta},\frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 162,165 \\ \langle \frac{\alpha+1}{\alpha+\beta},\frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 163,164. \end{cases}$$

$$
\langle 0,0\rangle \to_i \langle 0,1\rangle = \begin{cases}
\langle 0,0\rangle, & \text{for } i = 1,4,...,7,9,10,13,17,21,24,\\
 & ...,29,61,...,73,75,78,...,81,100,...,\\
 & 104,106,...,113,124,...,128,133,\\
 & 166,169\\
\langle 1,0\rangle, & \text{for } i = 2,3,8,11,16,20,20,...,45,76,\\
 & 77,82,...,90,105,114,...,118,129,...,\\
 & 132,167,168,172,173,176,...,178\\
\langle 0,1\rangle, & \text{for } i = 12,14,15,18,19,22,23,46,...,\\
 & 60,74,91,...,99,119,...,123,134,...,\\
 & 138,144,145,148,149,170,171,174,\\
 & 175,179,180\\
\langle 0,\frac{1}{2}\rangle, & \text{for } i = 139,154,155\\
\langle 0,\frac{2}{3}\rangle, & \text{for } i = 140,\\
\langle 0,\frac{1}{3}\rangle, & \text{for } i = 141,\\
\langle \frac{1}{3},\frac{2}{3}\rangle, & \text{for } i = 142,143,\\
\langle \frac{2}{3},\frac{1}{3}\rangle, & \text{for } i = 146,147,\\
\langle \frac{1}{2},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 150,\\
\langle \frac{\lambda-1}{2\lambda},\frac{1}{2}\rangle, & \text{for } i = 150\\
\langle \frac{\gamma}{2\gamma+1},\frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 151\\
\langle \frac{\alpha-1}{\alpha+\beta},\frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 152\\
\langle \varepsilon,0\rangle, & \text{for } i = 153,\\
\langle \frac{\lambda-1}{2\lambda},\frac{\lambda+1}{2\lambda}\rangle, & \text{for } i = 156,157\\
\langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 158,159,\\
\langle \frac{\gamma}{2\gamma+1},\frac{\gamma+1}{2\gamma+1}\rangle, & \text{for } i = 160,161,\\
\langle \frac{\alpha}{\alpha+\beta},\frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 162,163\\
\langle \frac{\alpha-1}{\alpha+\beta},\frac{\beta+1}{\alpha+\beta}\rangle, & \text{for } i = 163,164.
\end{cases}
$$

$$
\langle 0,0\rangle \to_i \langle 0,0\rangle = \begin{cases}
\langle 0,0\rangle, & \text{for } i = 1,4,...,7,9,10,13,17,...,19, \\
& 21,25,...,29,61,64,66,67,71,...,73, \\
& 75,78,...,81,100,...,102,104,106, \\
& ...,113,118,124,...,128,133, \\
& 139,...,141,166,169 \\
\langle 1,0\rangle, & \text{for } i = 2,3,8,11,14,...,16,20,22,..., \\
& 24,30,...,45,62,63,65,68,...,70,74, \\
& 76,77,82,...,90,103,105,114,..., \\
& 117,129,...,132,142,146,167,168, \\
& 170,...,173,175,...,178,180 \\
\langle 0,1\rangle, & \text{for } i = 12,46,...,60,91,...,99,119, \\
& ...,123,134,...,138,145,174,179 \\
\langle \frac{1}{3},\frac{2}{3}\rangle, & \text{for } i = 143,144 \\
\langle \frac{2}{3},\frac{1}{3}\rangle, & \text{for } i = 147,148,149 \\
\langle \frac{\lambda-1}{2\lambda},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 150 \\
\langle \frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 151 \\
\langle \frac{\alpha-1}{\alpha+\beta},\frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 152 \\
\langle \varepsilon,0\rangle, & \text{for } i = 153 \\
\langle \frac{1}{2},\frac{1}{2}\rangle, & \text{for } i = 154,157 \\
\langle \frac{\lambda+1}{2\lambda},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 155 \\
\langle \frac{\lambda-1}{2\lambda},\frac{\lambda+1}{2\lambda}\rangle, & \text{for } i = 156 \\
\langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 158,160 \\
\langle \frac{\gamma+2}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 159 \\
\langle \frac{\gamma}{2\gamma+1},\frac{\gamma+1}{2\gamma+1}\rangle, & \text{for } i = 161 \\
\langle \frac{\alpha}{\alpha+\beta},\frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 162,164 \\
\langle \frac{\alpha+1}{\alpha+\beta},\frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 163 \\
\langle \frac{\alpha-1}{\alpha+\beta},\frac{\beta+1}{\alpha+\beta}\rangle, & \text{for } i = 165.
\end{cases}
$$

$$\langle 0,0\rangle \to_i \langle 1,0\rangle = \begin{cases} \langle 0,0\rangle, & \text{for } i = 1,6,7,9,101,102,104,107, \\ & ...,109,113,118,123,133,166 \\ \langle 1,0\rangle, & \text{for } i = 2,...,5,8,10,...,100,103,105, \\ & 106,110,...,112,114,...,117,119,..., \\ & 122,124,...,132,134,...,138,142, \\ & 143,146,147,153,167,...,180 \\ \langle \frac{1}{2},0\rangle, & \text{for } i = 139 \\ \langle \frac{1}{3},0\rangle, & \text{for } i = 140 \\ \langle \frac{2}{3},0\rangle, & \text{for } i = 141 \\ \langle \frac{1}{3},\frac{2}{3}\rangle, & \text{for } i = 144,145 \\ \langle \frac{2}{3},\frac{1}{3}\rangle, & \text{for } i = 146,149 \\ \langle \frac{1}{2},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 150 \\ \langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 151 \\ \langle \frac{\alpha}{\alpha+\beta},\frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 152,164,165 \\ \langle \frac{\lambda+1}{2\lambda},\frac{\lambda-1}{2\lambda}\rangle, & \text{for } i = 154,155 \\ \langle \frac{1}{2},\frac{1}{2}\rangle, & \text{for } i = 156,157 \\ \langle \frac{\gamma+2}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}\rangle, & \text{for } i = 158,159 \\ \langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 160,161 \\ \langle \frac{\alpha+1}{\alpha+\beta},\frac{\beta-1}{\alpha+\beta}\rangle, & \text{for } i = 162,163. \end{cases}$$

$$\langle 1,0\rangle \to_i \langle 0,1\rangle = \begin{cases} \langle 0,1\rangle, & \text{for } i = 1,...,99,109,...,149,166,..., \\ & 168,170,...,180 \\ \langle 0,0\rangle, & \text{for } i = 100,...,108,169 \\ \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle, & \text{for } i = 150,154,...,157 \\ \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle, & \text{for } i = 151,158,...,161, \\ \langle \frac{\alpha-1}{\alpha+\beta}, \frac{\beta+1}{\alpha+\beta}\rangle, & \text{for } i = 152,162,...,165 \\ \langle \varepsilon, 1-\eta\rangle, & \text{for } i = 153. \end{cases}$$

$$\langle 1,0\rangle \to_i \langle 1,0\rangle = \begin{cases} \langle 1,0\rangle, & \text{for } i = 1,...,100,103,106,109,..., \\ & 112,114,...,117,119,...,122,124, \\ & ...,138,153,166,...,180 \\ \langle 0,0\rangle, & \text{for } i = 101,102,104,105,107,108, \\ & 113,123 \\ \langle 0, \frac{1}{2}\rangle, & \text{for } i = 139,150,155,...,157 \\ \langle \frac{1}{3}, \frac{2}{3}\rangle, & \text{for } i = 140,142,...,145 \\ \langle \frac{2}{3}, \frac{1}{3}\rangle, & \text{for } i = 141,146,...,149 \\ \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 151,158,...,161, \\ \langle \frac{\alpha}{\alpha+\beta}, \frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 152,162,...,165. \end{cases}$$

$$\langle 1,0\rangle \to_i \langle 0,0\rangle = \begin{cases} \langle 0,0\rangle, & \text{for } i = 1,...,11,13,14,16,...,19,21,\\ & 24,...,29,61,64,66,67,71,75,78,...,\\ & 81,100,...,102,104,...,113,118,124,\\ & ...,128,166,167,169,176 \\ \langle 0,1\rangle, & \text{for } i = 12,15,20,46,...,60,69,72,73,\\ & 76,77,91,...,99,119,...,123,134,...,\\ & 138,143,145,147,149,171,172,174,\\ & 177 \\ \langle 1,0\rangle, & \text{for } i = 22,23,30,...,45,62,63,65,68,\\ & 70,74,82,...,90,103,114,...,117,129,\\ & ...,133,168,170,173,175,179,180 \\ \langle 0,\frac{1}{2}\rangle, & \text{for } i = 139 \\ \langle 0,\frac{2}{3}\rangle, & \text{for } i = 140 \\ \langle 0,\frac{1}{3}\rangle, & \text{for } i = 141 \\ \langle \frac{1}{3},\frac{2}{3}\rangle, & \text{for } i = 142,144 \\ \langle \frac{2}{3},\frac{1}{3}\rangle, & \text{for } i = 146,148 \\ \langle \frac{\lambda-1}{2\lambda},\frac{1}{2}\rangle, & \text{for } i = 150 \\ \langle \frac{\gamma}{2\gamma+1},\frac{\gamma+1}{2\gamma+1}\rangle, & \text{for } i = 151 \\ \langle \frac{\alpha-1}{\alpha+\beta},\frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 152 \\ \langle \varepsilon,0\rangle, & \text{for } i = 153 \\ \langle \frac{\lambda-1}{2\lambda},\frac{\lambda+1}{2\lambda}\rangle, & \text{for } i = 154,156 \\ \langle \frac{1}{2},\frac{1}{2}\rangle, & \text{for } i = 155,157 \\ \langle \frac{\gamma}{2\gamma+1},\frac{\gamma+1}{2\gamma+1}\rangle, & \text{for } i = 158,161 \\ \langle \frac{\gamma+1}{2\gamma+1},\frac{\gamma}{2\gamma+1}\rangle, & \text{for } i = 159,160 \\ \langle \frac{\alpha-1}{\alpha+\beta},\frac{\beta+1}{\alpha+\beta}\rangle, & \text{for } i = 162,165 \\ \langle \frac{\alpha}{\alpha+\beta},\frac{\beta}{\alpha+\beta}\rangle, & \text{for } i = 163,164. \end{cases}$$

It is interesting to mention that the well-known axiom of the classical logic $A \rightarrow A$ is an IFT for implications $\rightarrow_1$, ..., $\rightarrow_9$, $\rightarrow_{11}$, ..., $\rightarrow_{15}$, $\rightarrow_{17}$, $\rightarrow_{18}$, $\rightarrow_{20}$, ..., $\rightarrow_{24}$, $\rightarrow_{27}$, ..., $\rightarrow_{38}$, $\rightarrow_{40}$, $\rightarrow_{42}$, $\rightarrow_{44}$, ..., $\rightarrow_{53}$, $\rightarrow_{55}$, $\rightarrow_{57}$, $\rightarrow_{59}$, ..., $\rightarrow_{66}$, $\rightarrow_{68}$, $\rightarrow_{69}$, $\rightarrow_{71}$, $\rightarrow_{72}$, $\rightarrow_{74}$, ..., $\rightarrow_{77}$, $\rightarrow_{79}$, ..., $\rightarrow_{85}$, $\rightarrow_{88}$, ..., $\rightarrow_{94}$, $\rightarrow_{97}$, ..., $\rightarrow_{139}$, $\rightarrow_{141}$, $\rightarrow_{146}$, ..., $\rightarrow_{170}$, $\rightarrow_{176}$, ..., $\rightarrow_{185}$, while it is just a tautology for implications $\rightarrow_2$, $\rightarrow_3$, $\rightarrow_5$, $\rightarrow_8$, $\rightarrow_{11}$, $\rightarrow_{14}$, $\rightarrow_{15}$, $\rightarrow_{20}$, $\rightarrow_{23}$, $\rightarrow_{24}$, $\rightarrow_{27}$, $\rightarrow_{31}$, $\rightarrow_{32}$, $\rightarrow_{34}$, $\rightarrow_{37}$, $\rightarrow_{40}$, $\rightarrow_{42}$, $\rightarrow_{47}$, ..., $\rightarrow_{49}$, $\rightarrow_{52}$, $\rightarrow_{55}$, $\rightarrow_{57}$, $\rightarrow_{62}$, $\rightarrow_{63}$, $\rightarrow_{65}$, $\rightarrow_{68}$, $\rightarrow_{69}$, $\rightarrow_{74}$, $\rightarrow_{77}$, $\rightarrow_{79}$, $\rightarrow_{83}$, $\rightarrow_{84}$, $\rightarrow_{88}$, $\rightarrow_{92}$, $\rightarrow_{93}$, $\rightarrow_{97}$, $\rightarrow_{176}$, ..., $\rightarrow_{185}$.

The intuitionistic fuzzy implications that satisfy the following equalities

$$\langle 0,1\rangle \rightarrow_i \langle 0,1\rangle = \langle 1,0\rangle,$$

$$\langle 0,1\rangle \rightarrow_i \langle 1,0\rangle = \langle 1,0\rangle,$$

$$\langle 1,0\rangle \rightarrow_i \langle 0,1\rangle = \langle 0,1\rangle,$$

$$\langle 1,0\rangle \rightarrow_i \langle 1,0\rangle = \langle 1,0\rangle,$$

as standard tautologies, will be called "implications of (fully) tautological type" (T-implications), while the implications that satisfy these equalities as IFTs, will be called "implications from IFT type" (I-implications), and the rest are incorrect and will be denoted as N-implications.

Acknowledgements

The author is thankful for the support provided by the Bulgarian National Science Fund under Grant Ref. No. DFNI-I-02-5 "InterCriteria Analysis: A New Approach to Decision Making".

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