Notes on Intuitionistic Fuzzy Sets
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A novel, simple, and effective intuitionistic fuzzy classifier

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Abstract: We present a novel classifier based on Atanassov's intuitionistic fuzzy sets (IFSs, for short). Using benchmark data from *UCI Machine Learning Repository*, we demonstrate that the proposed classifier achieves very good results.
Keywords: Intuitionistic fuzzy sets, Classification, a novel classifier.
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1 Introducton

Classifiers for elements or objects represented in terms of Intuitionistic Fuzzy Sets (IFSs) are particularly important because this type of representation reflects the way human beings perceive and interpret real-world objects and phenomena. Human perception is rarely based on complete, precise, and unambiguous information. Instead, it involves positive, negative assessments, and situations in which available knowledge is insufficient to make a definite judgement. The IFS

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framework captures these three aspects via membership, non-membership, and hesitation degrees, respectively. Consequently, IFSs provide a more comprehensive representation of real world situations than approaches that consider only the degree of membership or non-memberhip. Developing classification methods for IFS-represented data is therefore essential, as it enables objects to be distinguished and assigned to appropriate classes while accounting for uncertainty, incomplete information, and hesitation inherent in the available knowledge. Such classifiers can support more realistic and reliable decision-making in situations where conventional classification approaches may be insufficient.

This paper presents a novel classification framework designed to provide a systematic and reliable approach to categorizing elements described in terms of Intuitionistic Fuzzy Sets (IFSs) and belonging to one of two classes. The proposed classifier is based on a clear structure and refined classification criteria. Its aim is to improve the recognition of elements belonging to the class of interest (which, in this paper is always a smaller class) , achieve high classification accuracy, and provide a foundation for further research, practical applications, and future refinement.

To evaluate the effectiveness of the proposed classifier, we apply it to several benchmark datasets from the *UCI Machine Learning Repository* (https://archive.ics.uci.edu/datasets). The experimental results demonstrate that the proposed method can improve both the recognition of the individual class we are interested in, and classification accuracy, compared with various classification algorithms implemented in WEKA.

2 A brief introduction to intuitionistic fuzzy sets

In 1965, Zadeh proposed fuzzy sets [46], in which each element of a universe X is assigned a degree of membership. A fuzzy set A' in X is given by

$$A' = \{\langle x, \mu_{A'}(x)\rangle \mid x \in X\}, \tag{1}$$

where $\mu_{A'} : X \to [0, 1]$ is the membership function of A'.

One generalization of this concept is Atanassov's intuitionistic fuzzy set (IFS) [1–3]. An IFS A in X is defined by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{2}$$

where $\mu_A, \nu_A : X \to [0, 1]$ are the membership and non-membership functions, respectively. For each $x \in X$, the corresponding degrees satisfy

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{3}$$

More on assigning membership and non-membership degrees from data is given by Szmidt and Baldwin in [11].

Every fuzzy set A' can be represented as an IFS by taking the non-membership degree of each element to be the complement of its membership degree. This gives:

$$A = \{\langle x, \mu_{A'}(x), 1 - \mu_{A'}(x)\rangle | x \in X\}. \tag{4}$$

For an IFS A, the quantity

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

is called the *hesitation margin* of x with respect to A. It expresses the lack of knowledge about whether x belongs to A (cf. Atanassov [2]). By (3), $0 \leq \pi_A(x) \leq 1$ for every $x \in X$. We specifically pay attention to the hesitation margin, as it will be highly relevant for the applications discussed herewith.

The hesitation margin further plays an important role in the study of distances between IFSs [12, 13, 16, 23, 25], as well as in measures of entropy [17, 27] and similarity [28, 40].

The hesitation margin is also of key importance in information processing [10] and in a range of applications, including image processing [6], the classification of imbalanced and overlapping classes [43–45], and classification using intuitionistic fuzzy decision trees [5]. Further applications comprise attribute selection [37, 38, 41], ranking of alternatives [39, 42], multiagent decision making, negotiations, voting, and group decision making [4, 7, 14, 15, 18–20, 23, 24, 26, 29, 36], as well as genetic algorithms [8]. The hesitation margin also plays a specific role in the formulation of the Hausdorff distance for IFSs [31] and for the distinction between IFSs and interval-valued fuzzy sets [35].

In the present paper, explicit three-term representation of IFSs is used, rendering account of the membership degree μ, the non-membership degree ν, and the hesitation margin π. Numerours applications (see [10, 16, 17, 21, 22, 29–34, 36]) have shown the practical advantages of this three-term approach.

3 Classifier accuracy and error

In binary classification, a classifier's *accuracy* is the proportion of instances it classifies correctly, while its *error rate* is the proportion it assigns to the wrong class. These overall measures can be insufficient when one class is much smaller than the other or when the two types of misclassification, i.e., false positives and false negatives, carry different costs. In such cases, we use the confusion matrix (Table 1) as an additional metrics to examine the error rates for the two classes separately.

Table 1. The Confusion Matrix

	Tested Legal	**Tested Illegal**
Actual Legal	TP	FN
Actual Illegal	FP	TN

The abbreviations in the above table have the following meaning:

- TP – the number of correctly classified legal points,
- FN – the number of incorrectly classified legal points as illegals,

- FP – the number of incorrectly classified illegal points as legals,
- TN – the number of correctly classified illegal points,

and the following formulas for accuracy, and the four class-specific rates will be used throughout the paper:

$$ACC = \frac{\text{correctly classified points}}{\text{total points}} = \frac{TP + TN}{TP + TN + FP + FN}; \tag{6}$$

$$TPR = \frac{\text{legals correctly classified}}{\text{total legals}} = \frac{TP}{TP + FN} \tag{7}$$

$$FNR = \frac{\text{legals incorrectly classified}}{\text{total legals}} = 1 - TPR = \frac{FN}{TP + FN} \tag{8}$$

$$FPR = \frac{\text{illegals incorrectly classified}}{\text{total illegals}} = \frac{FP}{FP + TN} \tag{9}$$

$$TNR = \frac{\text{illegals correctly classified}}{\text{total illegals}} = 1 - FPR = \frac{TN}{TN + FP} \tag{10}$$

4 A novel intuitionistic fuzzy classifier

Assume that we have a dataset of intuitionistic fuzzy instances x_i, $i = 1, \ldots, N$, belonging to one of two classes, labeled "1" and "0". Each instance is described by K attributes, denoted by A_k, where $k = 1, \ldots, K$, and has a known class label "1" or "0", which allows us to verify whether the classification we propose is correct.

The objective is to correctly classify these instances. For each instance we introduce the evaluation measure $M_{i,j}$, defined in (11).

$$M_{i,j} = \pi_{i,j}(\mu_{i,j} - \nu_{i,j}) \tag{11}$$

where M takes values

$$M \in [-0.25, 0.25] \tag{12}$$

$$M > 0 \quad \text{if} \quad \mu > \nu \quad \text{and} \quad \pi > 0 \tag{13}$$

$$M = 0 \quad \text{if} \quad \mu = \nu \quad \text{or} \quad \pi = 0 \tag{14}$$

$$M < 0 \quad \text{if} \quad \mu < \nu \quad \text{and} \quad \pi > 0 \tag{15}$$

Given the values $M_{i,j}$, for a fixed threshold X, where X takes integer values from 1 to K, an instance is classified as belonging to class '1' if $M(i, j) \geq 0$ for at least X attributes. Otherwise, the instance is classified as belonging to class '0', that is, if fewer than X attributes satisfy the condition $M_{i,j} \geq 0$.

Remark 1. For $\pi > 0$, the sign of M in (11) is determined by $(\mu - \nu)$. When $\pi = 0$ (14) the score is equal to zero, and the element is classified as belonging to a smaller class, '1', since $M \geq 0$ even when $\nu > \mu$. This convention ensures that elements belonging to class '1' are identified as effectively as possible. The results obtained (cf. Section 5) confirm that our approach is justified.

Example 1. Consider an element, characterized by eight attributes, belonging to the class of interest, namely, Class '1'. Each attribute is described by $[\mu, \nu, \pi]$:

Attribute 1: $[0.014, 0.233, 0.753]$, $M : -0.054$
Attribute 2: $[0.809, 0.169, 0.021]$, $M : 0.627$
Attribute 3: $[0.049, 0.14, 0.811]$, $M : -0.017$
Attribute 4: $[0.122, 0.072, 0.806]$, $M : 0.010$
Attribute 5: $[0.479, 0.292, 0.229]$, $M : 0.144$
Attribute 6: $[0.148, 0.216, 0.636]$, $M : -0.025$
Attribute 7: $[0.198, 0.385, 0.417]$, $M : -0.109$
Attribute 8: $[0.226, 0.133, 0.641]$, $M : 0.033$

We can observe that, for four attributes ($X = 4$) the value of M is greater than 0. This means that for 4 attributes (2, 4, 5 and 8) the evaluation measure M classifies this element as belonging to class '1'. The same conclusion is reached for this element when $X = 1$, and $X = 2$, and $X = 3$. However, for $X = 5$ or 6 or 7 or 8, the classification of this element is incorrect, since the measure M is not positive or equal to zero for all five, or all six, or all seven or all eight attributes. The same procedure is applied to each element. Once all the results have been obtained, we can calculate the classification errors from (6)–(10).

5 Results

In this section, we will present results for benchmark datasets from the *UCI Repository* to demonstrate the behavior of our approach. In all figures we report TPR (7) for the smaller class, which we aim to detect as good as possible while maintaining good Accuracy (6).

5.1 Results for dataset 'Heart'

Source: `https://archive.ics.uci.edu/dataset/95/spect+heart`

Number of instances: 267 (55+212)

Number of attributes: 22 features (binary) + 1 target

Task type: Classification – predicting the presence of heart disease 55 cases.

Figure 1 shows Accuracy, and TPR (concerning the smaller class, namely, 55 elements) for the 'Heart' dataset as a function of the the number of attributes for which $M \geq 0$ and which correctly classify a case as belonging to class 1. Figure 2 shows histogram.

In Table 2 we have results for the evaluation measure M examined for 18, and 19 attributes. For 19 attributes the Accuracy is equal to 81.7% and TPR equal to 70.9%.

Table 2. Results for dataset 'Heart'

M(.)	ACC	TPR	FNR	FPR	TNR
18	76	78.2	21.8	24.5	75.5
19	81.7	70.9	29.1	15.6	84.4

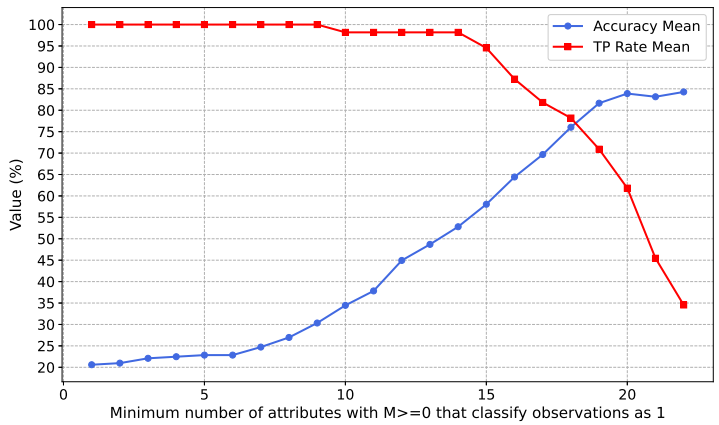


Figure 1. 'Heart'– the values of Accuracy and TPR

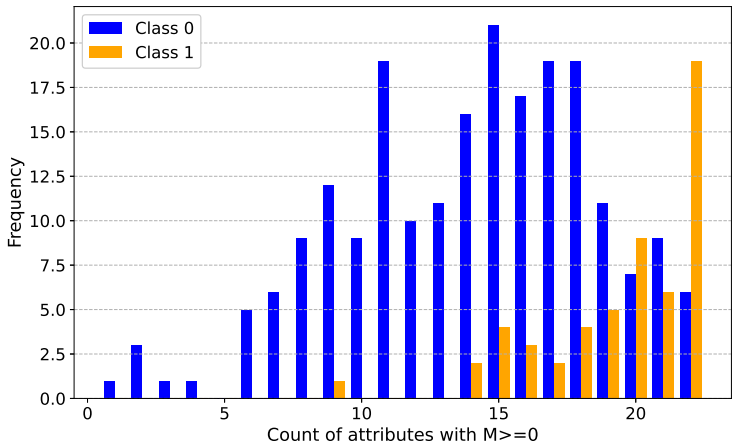


Figure 2. 'Heart' – histogram

The best classification results for the 'Heart' dataset obtained from WEKA are achieved with the LMT (Logistic Model for Tree). The Accuracy is 82.9%, but the model shows poor recognition of 55 patients with heart disease, with a TPR of only 47.8%.

The best result of TPR was achieved by k-NN with $k = 5$, equal to $70,2\%$ and the Accuracy equal to 80.4%

5.2 Results for dataset 'Hepatitis'

https://archive.ics.uci.edu/dataset/46/hepatitis

We use the official cleaned version of the data (cases without missing values).

Number of instances: 80 (13 + 67).

Number of attributes: 19 (excluding the class).

Task type: Classification (binary: live vs die).

Classes Die: 13, Live: 67.

In Table 3, we present the results obtained using the evaluation measure M for 10, and 11 attributes. For 10 attributes, the accuracy is equal to 73.8% and the TPR is 100%. For 11 attributes, the accuracy is equal to 78.8% and the TPR is 92.3%.

Table 3. Results for dataset ‘Hepatitis’

M	ACC	TPR	FNR	FPR	TNR
10	73.8	100	0	31.3	68.7
11	78.8	92.3	7.7	23.9	76.1

The highest accuracy among the classifiers evaluated in WEKA was achieved by the Random Forest classifier, with an accuracy 87.9% and the TPR of $= 41.5\%$. The highest TPR among the WEKA classifiers was obtained by the k-NN algorithm with $k = 1$, reaching $= 55.7\%$ with the accuracy of 82.3%. In both cases, the TPR values obtained using WEKA classifiers are realtively low. In contrast, the evaluation measure M achieves substantially better recognition of the class of interest. Figure 3 shows Accuracy and TPR for the ‘Hepatitis’ dataset as a function of the the number of attributes for which $M \geq 0$ and which correctly classify a case as belonging to class 1. Figure 4 shows histogram.

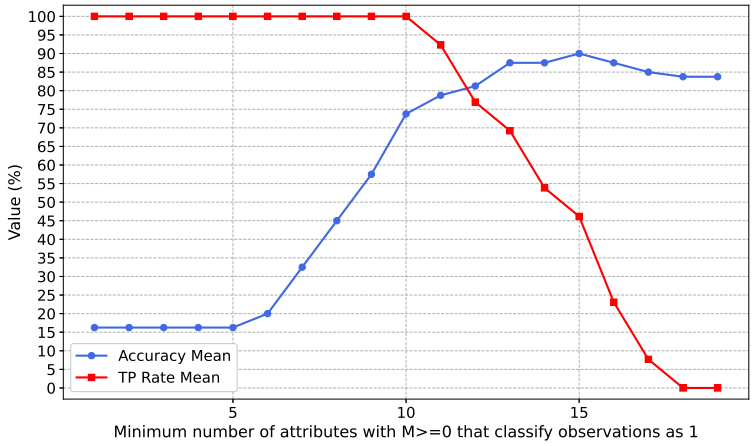


Figure 3. ‘Hepatitis’ – Accuracy and TPR

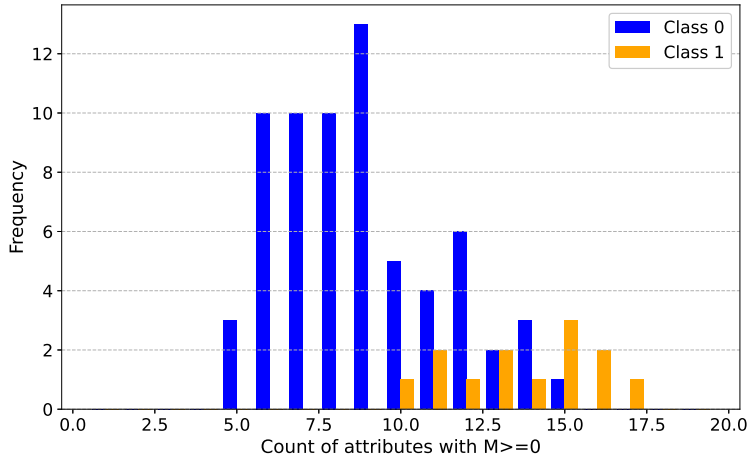


Figure 4. ‘Hepatitis’ – histogram

5.3 Results for dataset ‘Ionosphere’

‘Ionosphere’ is a dataset with two classes (class proportion: $225 : 126$), 351 instances, and 35 attributes (source: `https://archive.ics.uci.edu/ml/datasets/ionosphere`). The 35-th attribute points out to a class which can be “good” or “bad” according to radar returns from the ionosphere.

In Table 4, we present the results obtained using the evaluation measure M for 5, and 6 attributes. For 5 attributes, the accuracy is equal to 84.3% and the TPR is 93.7%. For 6 attributes, the accuracy is equal to 85.5% and the TPR is 89.7%.

Table 4. Results for dataset ‘Ionosphere’

M	ACC	TPR	FNR	FPR	TNR
5	84.3	93.7	6.3	20.9	79.1
6	85.5	89.7	10.3	16.9	83.1

The best results among the algorithms in WEKA are achieved by Random Forest, with an accuracy of 93.3%. The detection rate of the minority class in this case is 87%. No other algorithm obtained better results.

With the evaluation measure M, we achieve slightly lower accuracy but better detection of the minority class “1”, which is the class of particular interest to us.

Figure 5 shows Accuracy and TPR for the ‘Ionosphere’ dataset as a function of the the number of attributes for which $M \geq 0$ and which correctly classify a case as belonging to class 1. Figure 6 shows histogram.

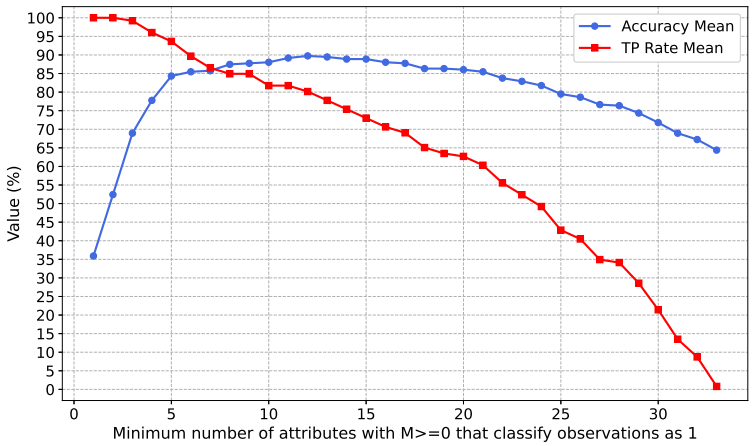


Figure 5. ‘Ionosphere’ – Accuracy and TPR

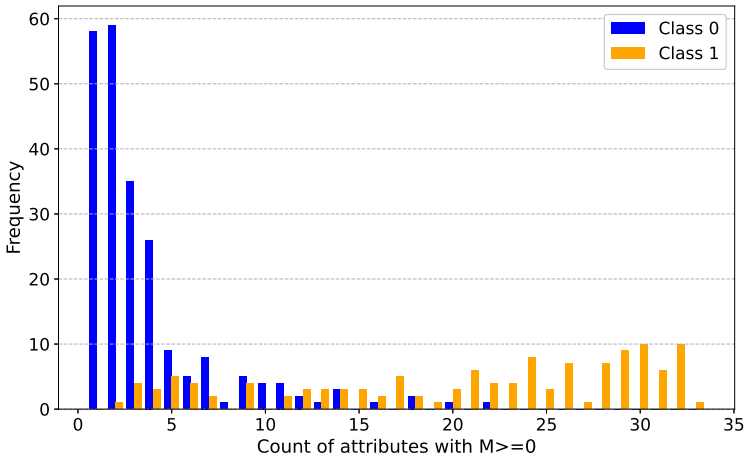


Figure 6. 'Ionosphere' – histogram

5.4 Results for dataset Diabetes Pima

Source: `https://www.kaggle.com/datasets/uciml/pima-indians-diabetes-database`

Number of instances: 768.

Number of attributes: 8 input + 1 class (so 9 total).

Classification: diabetes vs no diabetes.

Diabetic cases: 268, Non-diabetic cases: 500.

In Table 5, we present the results obtained using the evaluation measure M for 4, and 5 attributes. For 4 attributes, the accuracy is equal to 66% and the TPR is 77.6%. For 5 attributes, the accuracy is equal to 71.7% and the TPR is 61.2%.

Table 5. Results for dataset 'Diabetes Pima'

M	ACC	TPR	FNR	FPR	TNR
4	66	77.6	22.4	40.2	59.8
5	71.7	61.2	38.8	22.6	77.4

Among the classifiers available in WEKA, the Functions.Logistic algorithm achieves the highest accuracy of 77.2%, with a True Positive Rate (TPR) of 56.3% for the diabetes class.

The Random Forest classifier achieves an accuracy of 75.9% and the highest TPR of 60.6%.

Since our primary goal is to maximize the detection of the minority class, the evaluation measure M provides a slight improvement in the TPR, increasing it to 61.2% compared with 60.6% achieved by Random Forest.

Figure 7 shows Accuracy and TPR for the 'Ionosphere' dataset as a function of the the number of attributes for which $M \geq 0$ and which correctly classify a case as belonging to class 1. Figure 8 shows histogram.

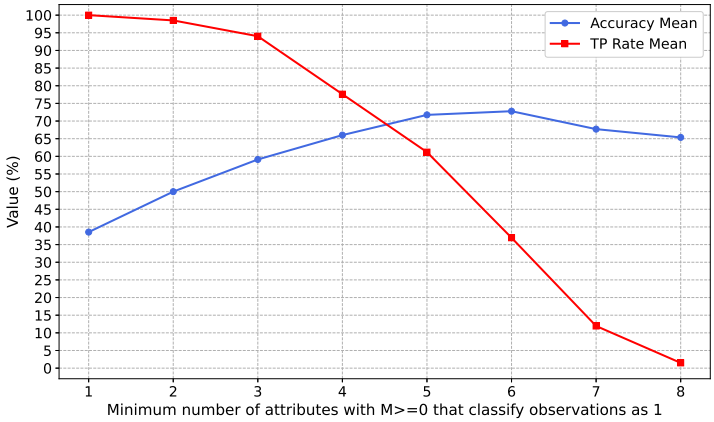


Figure 7. 'Pima' – Accuracy and TPR

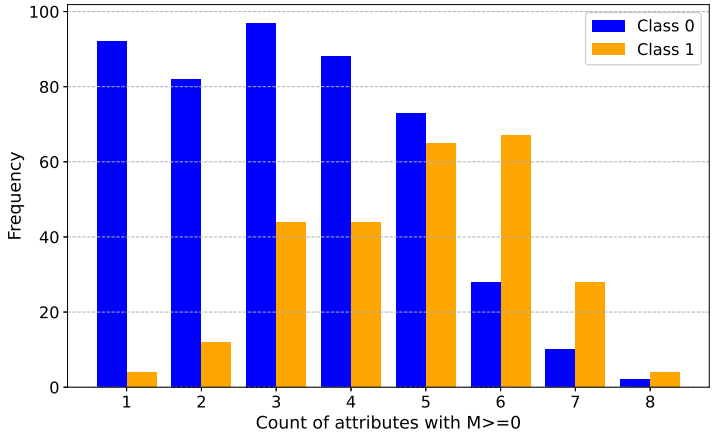


Figure 8. 'Pima' – histogram

5.5 Results for dataset 'Sonar'

Source: `https://doi.org/10.24432/C5T01Q`

Number of instances: 208.

Number of attributes: 60 + 1 class attribute.

Classification – distinguishing between sonar signals bounced off a metal cylinder (mine) vs a rock.

Mines (metal cylinder): 111 instances,

In Table 6, we present the results obtained using the evaluation measure M for 27, and 28 attributes. For 27 attributes, the accuracy is equal to 86.5% and the TPR is 96.9%. For 28 attributes, the accuracy is equal to 88% and the TPR is 93.8%.

Table 6. Results for dataset 'Sonar'

M	ACC	TPR	FNR	FPR	TNR
27	86.5	96.9	3.1	22.5	77.5
28	88	93.8	6.2	17.1	82.9

For the Sonar dataset, the best accuracy was obtained using the lazy.IBk classifier from WEKA, reaching 85.4%, with a True Positive Rate (TPR) of 80.2%.

The evaluation measure M yields better results, particularly in terms of detecting the class of interest.

Figure 9 shows Accuracy and TPR for the 'Sonar' dataset as a function of the the number of attributes for which $M \geq 0$ and which correctly classify a case as belonging to the smaller class 1. Figure 10 shows histogram.

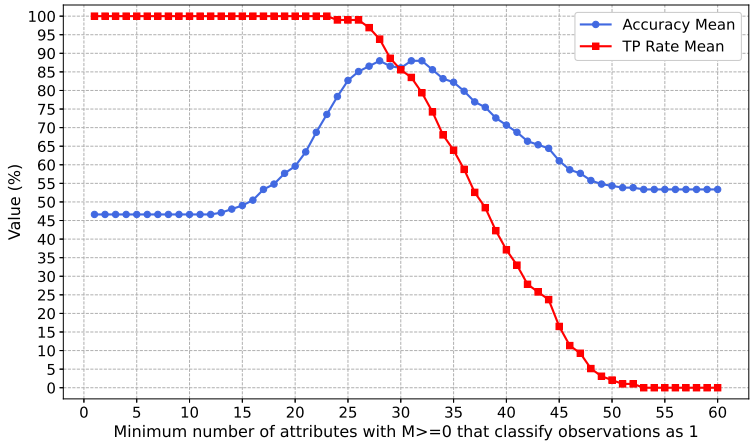


Figure 9. 'Sonar' – Accuracy and TPR

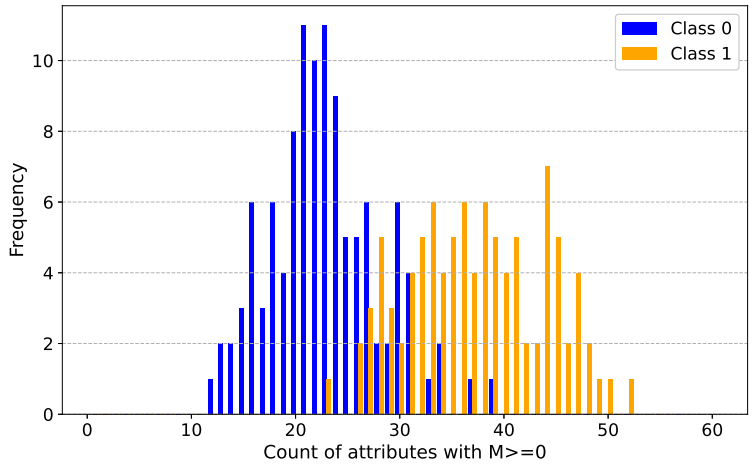


Figure 10. 'Sonar' – histogram

5.6 Results for Quinlan's example

We will present results for the famous Quinlan's example [9], the so-called 'Saturday Morning' (Table 7).

The method of expressing the example in terms of IFSs is given in [30]. The result is given in Table 8.

We examine TPR with respect to the elements of the smaller class, denoted by the letter N in Tables 7 and 8.

Table 7. The 'Saturday Morning' data from [9]

No.	Attributes				Class
	Outlook	Temperature	Humidity	Windy	
1	sunny	hot	high	false	N
2	sunny	hot	high	true	N
3	overcast	hot	high	false	P
4	rain	mild	high	false	P
5	rain	cool	normal	false	P
6	rain	cool	normal	true	N
7	overcast	cool	normal	true	P
8	sunny	mild	high	false	N
9	sunny	cool	normal	false	P
10	rain	mild	normal	false	P
11	sunny	mild	normal	true	P
12	overcast	mild	high	true	P
13	overcast	hot	normal	false	P
14	rain	mild	high	true	N

Table 8. The 'Saturday Morning' data in terms of A-IFSs

No.	Attributes				Class
	Outlook	Temperature	Humidity	Windy	
1	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	$(0.2, 0, 0.8)$	N
2	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	N
3	$(1, 0, 0)$	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	$(0.2, 0, 0.8)$	P
4	$(0.2, 0.11, 0.69)$	$(0, 0, 1)$	$(0, 0.33, 0.67)$	$(0.2, 0, 0.8)$	P
5	$(0.2, 0.11, 0.69)$	$(0.4, 0.11, 0.49)$	$(0.6, 0, 0.4)$	$(0.2, 0, 0.8)$	P
6	$(0.2, 0.11, 0.69)$	$(0.4, 0.11, 0.49)$	$(0.6, 0, 0.4)$	$(0, 0.33, 0.67)$	N
7	$(1, 0, 0)$	$(0.4, 0.11, 0.49)$	$(0.6, 0, 0.4)$	$(0, 0.33, 0.67)$	P
8	$(0, 0.33, 0.67)$	$(0, 0, 1)$	$(0, 0.33, 0.67)$	$(0.2, 0, 0.8)$	N
9	$(0, 0.33, 0.67)$	$(0.4, 0.11, 0.49)$	$(0.6, 0, 0.4)$	$(0.2, 0, 0.8)$	P
10	$(0.2, 0.11, 0.69)$	$(0, 0, 1)$	$(0.6, 0, 0.4)$	$(0.2, 0, 0.8)$	P
11	$(0, 0.33, 0.67)$	$(0, 0, 1)$	$(0.6, 0, 0.4)$	$(0, 0.33, 0.67)$	P
12	$(1, 0, 0)$	$(0, 0, 1)$	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	P
13	$(1, 0, 0)$	$(0, 0.33, 0.67)$	$(0.6, 0, 0.4)$	$(0.2, 0, 0.8)$	P
14	$(0.2, 0.11, 0.69)$	$(0, 0, 1)$	$(0, 0.33, 0.67)$	$(0, 0.33, 0.67)$	N

Table 9. Results for dataset 'Saturday Morning'

M	ACC	TPR	FNR	FPR	TNR
3	78.6	80	20	22.2	77.8

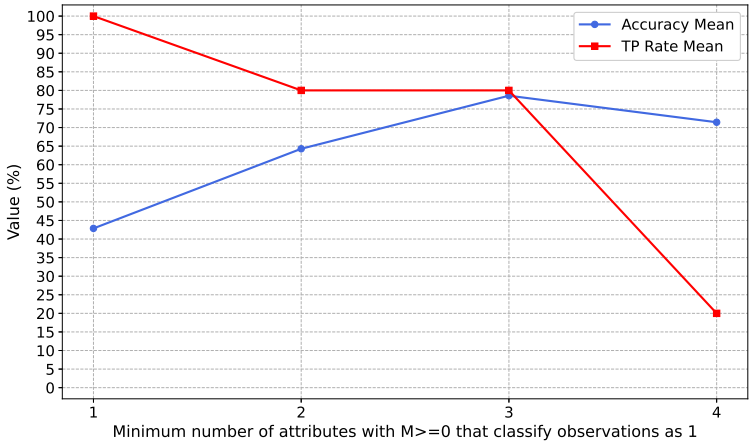


Figure 11. 'Saturday Morning' – Accuracy and TPR

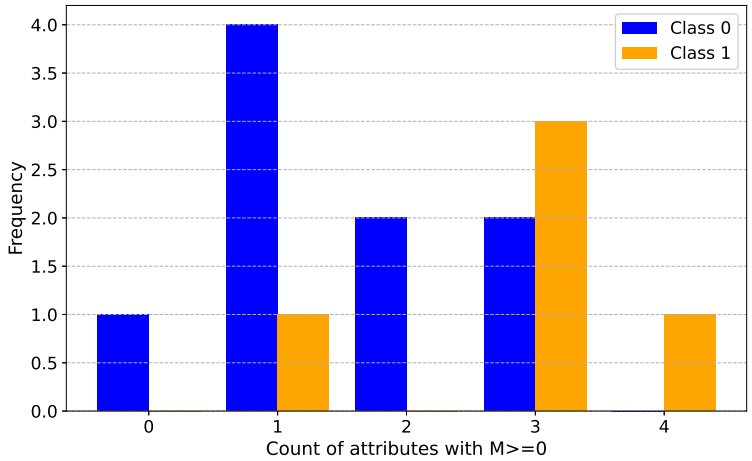


Figure 12. 'Saturday Morning' – histogram

From WEKA we obtained the best result for function.Logistic: accuracy $= 66\%$, TPR $= 59\%$, TNR $= 70.5\%$.

Using the evaluation measure M, we achieved better results in terms of overall accuracy (78.6%), the detection of the minority class (80%), and the detection of majority class (77.8%).

6 Conclusions

We proposed a novel classifier based on the representation of classified elements using Intuitionistic Fuzzy Sets (IFSs). Each element is described by its degree of membership, degree of non-membership, and hesitation margin. The aim of the classifier is to detect the smaller class as effectively as possible while maintaining high classification accuracy.

The proposed classifier achieves good results in detecting the smaller and more difficult to identify class. All the cases examined follow the patterns illustrated in the Figures 1, 3, 5, 7, 9, 11.

When the criterion for assigning an element to the smaller class is based on a small number

of attributes X satisfying the specified conditions $M \geq 0$, all elements from the smaller class are correctly classified. However, elements from the larger class may also satisfy these conditions. As a result, they are incorrectly assigned to the smaller class. This leads to a high TPR but low accuracy.

As the number of attributes X satisfying the conditions $M \geq 0$ increases, fewer elements from larger class are incorrectly classified. However, some elements from the smaller class are also misclassified. Consequently, the TPR decreases, while accuracy increases.

At a certain point, the two curves (TPR and Accuracy) intersect. This point indicates a potentially optimal number of attributes for the classifier. If X increases further, the TPR continues to decrease, while Accuracy increases. However, since our main objective is to detect the smaller class as effectively as possible, increasing X beyond the intersection point is not desirable.

In all the cases examined, the proposed classifier achieved better results than the 12 best classifiers available in WEKA. The differences varied depending on the dataset. In some cases, the improvement was smaller, while in others, it was substantial.

The proposed classifier is also easy to implement. Moreover, it offers computational efficiency, simplicity of computation, and high explainability.

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