Notes on Intuitionistic Fuzzy Sets
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A remark on intuitionistic fuzzy implications

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Abstract: In the paper an attempt is made at introducing a classification scheme for some of the intuitionistic fuzzy implications. This has allowed us to navigate the existing implications in a more consistent manner and has revealed a duplicate implication.
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1 Introduction

At present there are 190 proposed intuitionistic fuzzy implications defined in a series of papers [1–10, 12–17, 19–39, 41–46] most of which are collected in [11]. For alternative points of view on intuitionistic fuzzy implications we refer the interested reader to [40] and [47].

In order to make research into the existing 190 implications, we further focus our attention on the functions used to represent them (sg, $\overline{\text{sg}}$, $\min$, $\max$, $\cdot$) and we propose a sample classification based on their properties. This has allowed us to identify a duplicate implication, namely $\rightarrow_{40}$ which coincides with $\rightarrow_{173}$.

2 Preliminaries

Here we remind some of the basic definitions which will be used later.

Definition 1. *The functions sg and $\overline{sg}$ are defined for the real variable x as follows:*

$$sg(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x \le 0 \end{cases} \quad ; \quad \overline{sg}(x) = \begin{cases} 0, & \text{if } x > 0, \\ 1, & \text{if } x \le 0 \end{cases} \tag{1}$$

Remark 1. *From Definition 1 it is seen that the following equality holds:*

$$1 - sg(x) = \overline{sg}(x) \tag{2}$$

Next we recall the following:

Definition 2 (cf. [18]). *An intuitionistic fuzzy pair (IFP) is an ordered couple of real non-negative numbers $\langle a, b \rangle$, with the additional constraint:*

$$a + b \le 1. \tag{3}$$

If we denote the set of all IFPs by U_{IFP}, we can view an implication as a particular mapping of the kind (bound by additional constraints due to the Axioms that need to be satisfied):

$$I : U_{\text{IFP}} \times U_{\text{IFP}} \to U_{\text{IFP}}.$$

In other words all implications are of the form:

$$I(x, y) = \langle f(x, y), g(x, y) \rangle,$$

where $x \in U_{\text{IFP}}$, $y \in U_{\text{IFP}}$, $\langle f(x, y), g(x, y) \rangle \in U_{\text{IFP}}$. In our further considerations we suppose that everywhere $x = \langle a, b \rangle$ and $y = \langle c, d \rangle$.

In the 190 implications the most used arithmetical functions are $+, -$, sg, $\overline{\text{sg}}$, $\max, \min, \cdot$.

Let $a, b \in [0, 1]$, then the following property holds:

$$ab \le \min(a, b) \le \max(a, b) = 1 - \min(1 - a, 1 - b) \tag{4}$$

If $\langle a, b \rangle$ and $\langle c, d \rangle$ are IFPs, we have

$$ad \le (1 - b)(1 - c) \le \min(1 - b, 1 - c) = 1 - \max(b, c). \tag{5}$$

Let $a, b \in [0, 1]$, then the following property holds:

$$\text{sg}(a)\text{sg}(b) = \text{sg}(ab) = \min(\text{sg}(a), \text{sg}(b)) = \text{sg}(\min(a, b)). \tag{6}$$

3 Results

Using equations (2), (4), (5) and (6), we have established that a significant number of the 190 implications may be written as:

$$I(x,y) = \langle f(x,y), 1 - f(x,y)\rangle, \tag{7}$$

where $f(x,y) \in [0,1]$.

The list of all such implications is given below:

$\rightarrow_{12}, \rightarrow_{20}, \rightarrow_{22}, \rightarrow_{23}, \rightarrow_{32}, \rightarrow_{33}, \rightarrow_{34}, \rightarrow_{35}, \rightarrow_{37}, \rightarrow_{38}, \overset{*}{\rightarrow}_{40}, \rightarrow_{41}, \rightarrow_{42}, \rightarrow_{43}, \rightarrow_{48}, \rightarrow_{49}, \rightarrow_{50},$
$\rightarrow_{52}, \rightarrow_{55}, \rightarrow_{56}, \rightarrow_{57}, \rightarrow_{74}, \rightarrow_{76}, \rightarrow_{77}, \rightarrow_{85}, \rightarrow_{86}, \rightarrow_{87}, \rightarrow_{88}, \rightarrow_{86}, \rightarrow_{93}, \rightarrow_{94}, \rightarrow_{96}, \rightarrow_{97}, \rightarrow_{142},$
$\rightarrow_{143}, \rightarrow_{144}, \rightarrow_{145}, \rightarrow_{146}, \rightarrow_{147}, \rightarrow_{148}, \rightarrow_{149}, \rightarrow_{154,\lambda}, \rightarrow_{155,\lambda}, \rightarrow_{156,\lambda}, \rightarrow_{157,\lambda}, \rightarrow_{158,\gamma}, \rightarrow_{159,\lambda},$
$\rightarrow_{160,\gamma}, \rightarrow_{161,\gamma}, \rightarrow_{162,\alpha,\beta}, \rightarrow_{163,\alpha,\beta}, \rightarrow_{164,\alpha,\beta}, \rightarrow_{165,\alpha,\beta}, \rightarrow_{167}, \rightarrow_{168}, \rightarrow_{169}, \rightarrow_{170}, \rightarrow_{171}, \rightarrow_{172},$
$\overset{*}{\rightarrow}_{173}, \rightarrow_{174}, \rightarrow_{175}, \rightarrow_{177}, \rightarrow_{178}, \rightarrow_{179}, \rightarrow_{180}, \rightarrow_{180}, \rightarrow_{182}, \rightarrow_{183}, \rightarrow_{184}, \rightarrow_{185}, \rightarrow_{190}$

We have marked $\rightarrow_{40}$ and $\rightarrow_{173}$ by $*$ to denote the fact that they coincide.

Some of the remaining implications can be represented in the form

$$I(x,y) = \langle 1 - f(x,y), g(x,y)f(x,y)\rangle, \tag{8}$$

where $f(x,y), g(x,y) \in [0,1]$.

Namely,

$$\rightarrow_{2}, \rightarrow_{31}, \rightarrow_{47}, \rightarrow_{62}, \rightarrow_{83} .$$

One can easily observe that (7) may be treated as a particular case of (8) with the special choice of

$$g(x,y) = 1 \, \forall x, y \in U_{\text{IFP}}.$$

However, such approach while technically correct does not yield particularly useful information.

The rest of the implications have less tractable representations.

However, implications that satisfy (7) and/or (8) may be studied based on the properties of the functions $f(x,y)$ and $g(x,y)$, which allows for a more unified approach in treating them.

In the light of the above we can formulate the following

Open problem. *Can implications that do not satisfy* (7) *or* (8) *be categorized in suitable classes which can be described by a single formula?*

4 Conclusion

In the present paper we proposed a partial classification based on the representation of the existing implications. This allows not only to study implications which satisfy (7) and/or (8) in a unified manner, but also to introduce and study new implications with such property. It also helps in detecting duplicating or coinciding implications as was the case with implications $\rightarrow_{40}$ and $\rightarrow_{173}$.

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