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CONTRAST INTENSIFICATION OF SECOND TYPE GENERALISED INTUITIONISTIC FUZZY SETS

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ABSTRACT

In this paper, we have introduced 'second type generalised intuitionistic fuzzy sets' (STGIFS). Also we have defined the operations 'contrast intensification' and 'fuzzification' of STGIFS and verified results based on the properties of these operations.

KEY WORDS

Intuitionistic Fuzzy Sets (IFS), second type intuitionistic fuzzy sets (STIFS), second type generalised intuitionistic fuzzy sets (STGIFS), contrast intensification, fuzzification.

1. INTRODUCTION

In 1965, Lotfi A. Zadeh introduced fuzzy sets (FS) as generalisation of crisp sets. Later, many authors generalised the concept of fuzzy sets in different directions. One such important generalisation is done by Krassimir T. Atanassov in the form of 'intuitionistic fuzzy sets' (IFS) and 'intuitionistic fuzzy sets of second type' (IFSST). Following these concepts in [1] and [4], in this paper, we have defined STGIFS and verified various results on them.

In section 2, we recollect some preliminaries required for defining new concepts. In section 3, we deal with some operations on STGIFS and their properties and this paper is concluded with section 4.

2. PRELIMINARIES

Definition 2.1 (IFS)

Let X be a nonempty set. An IFS A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\} \quad (2.1)$$

where the fuzzy sets $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. The IFS (2.1) can also be written in the form A = $\langle x, \mu_A(x), \gamma_A(x)\rangle$ or simply A $=\langle \mu_A, \gamma_A\rangle$.

Note

An ordinary fuzzy set can also be written as $\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. ie., all the fuzzy sets are IFSs.

Definition 2.2 (STIFS)

Let X be a nonempty set. A STIFS A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$$

\where the fuzzy set $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ denote the membership and non-membership functions of A respectively, and

$$0 \leq [\mu_A(x)]^2 + [\gamma_A(x)]^2 \leq 1$$

for each $x \in X$.

Note

It is obvious that for all real numbers a, b $\in [0,1]$, if $0 \leq a + b \leq 1$, then $0 \leq a^2 + b^2 \leq 1$. Hence all the IFSs are IFSsST.

Definition 2.3 (GIFS)

Let X be a nonempty set. A GIFS A in X is defined as an object of the form

$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ where the fuzzy sets

$\mu_A(x) : X \to [0,1]$ and

$\gamma_A(x) : X \to [0,1]$

denote the membership and non-membership functions of A respectively, and satisfying the condition mini($\mu_A(x), \gamma_A(x)$) ≤ 0.5 for each $x \in X$.

Definition 2.4 (STGIFS)

Let X be a nonempty set. An STGIFS A in X is defined as an object of the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$$

where the fuzzy sets $\mu_A(x): X \rightarrow [0,1]$ and $\gamma_A(x): X \rightarrow [0,1]$ denote the membership and non-membership functions of A respectively, and satisfying the condition mini $([\mu_A(x)]^2, [\gamma_A(x)]^2) \leq 0.5$ for each $x \in X$.

Note

All GIFSs are STGIFSs but the converse is not true. The following diagram shows the relationship between IFS and its extensions.

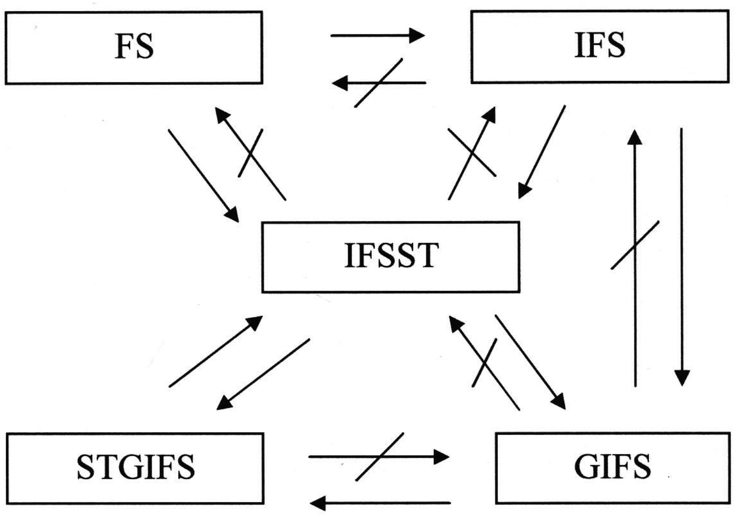

Fig 1. Relationship between intuitionistic fuzzy set and its extensions.

The above implications are verified through the following examples.

Example 2.5

Let X be a nonempty set. Define the membership function of a subset A of X as

$$\mu_A(x) = \begin{cases} x - 5, if 5 \leq x \leq 6 \\ -x + 7, if 6 < x \leq 7 \\ 0, elsewhere. \end{cases}$$

Also define the non-membership function of A as

$$\gamma_A(x) = \begin{cases} 6-x, if 5 \le x \le 6 \\ x-6, if 6 < x \le 7 \\ 1, elsewhere. \end{cases}$$

Now let X = {4.5, 5, 5.5, 6, 6.5, 7, 7.5}

Define

$$A = \{\langle 4.5,0,1\rangle, \langle 5,0,1\rangle, \langle 5.5,0.5,0.5\rangle, \langle 6,1,0\rangle, \langle 6.5,0.5,0.5\rangle, \langle 7,0,1\rangle\}$$

(i) $\{\langle x, \mu_A(x)\rangle : x \in X\}$ is a fuzzy set, since $0 \le \mu_A(x) \le 1$, for each $x \in X$.

(ii) Also, $\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ is an IFS, since

$0 \le \mu_A(x) + \gamma_A(x) \le 1$, for each $x \in X$.

(iii) $\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ is an IFSST, because

$0 \le \mu_A(x)^2 + \gamma_A(x)^2 \le 1$, for each $x \in X$.

(iv) $\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ is a GIFS, because

$\min(\mu_A(x), \gamma_A(x)) \le 0.5$, for each $x \in X$.

(v) A is also a STGIFS, since $\min(\mu_A(x)^2, \gamma_A(x)^2) \le 0.5$, for each $x \in X$.

The following example shows that some STGIFSs are also GIFSs.

Example 2.6

Consider $X = \{a, b, c, d\}$

Define $A = \{\langle a,0.8,0.4\rangle, \langle b,0.7,0.5\rangle, \langle c,0.3,0.7\rangle, \langle d,0,0.9\rangle\}$

Here A is a STGIFS as well as GIFS.

Note

It is not always true that all STGIFS are GIFSs. The following example illustrates this.

Example 2.7

Consider $X = \{a, b, c, d\}$

Define A = $\{\langle a,0.8,0.6\rangle,\langle b,0.6,0.5\rangle,\langle c,0.2,0.8\rangle,\langle d,0,0.9\rangle\}$

Here A is not a GIFS, because min ($\mu_A(a),\gamma_A(a)$) = min (0.8, 0.6) = 0.6 > 0.5

But A is a STGIFS, because min ($\mu_A{}^2(x,\gamma_A{}^2(x)$) $\leq$ 0.5, for each $x \in$ X.

3. SOME OPERATIONS ON STGIFS AND THEIR PROPERTIES

3.1 Preliminaries

Definition 3.1 (Proper GIFS)

If for a GIFS A, min ($\mu_A(x),\gamma_A(x)$) = 0.5, for each $x \in$ X, then A is said to be a proper GIFS.

Definition 3.2 (Contrast Intensification)

The contrast intensification of a STGIFS A of the Universe X, denoted by INTEN(A), defined by,

INTEN(A) = $\{\langle x,\mu_{INTEN}(x),\gamma_{INTEN}(x)\rangle : x \in X\}$ where

$$\mu_{INTEN}(x) = 1-(1-\mu_A(x)^8)^2 \text{ and}$$

$$\gamma_{INTEN}(x) = \left[1-(1-\gamma_A(x)^2)^2\right]^4.$$

Example 3.3

Consider the STGIFS A = $\{\langle 1,0.3,0.8\rangle,\langle 2,0.5,0.5\rangle,\langle 3,0.7,0.3\rangle,\langle 4,1,0.1\rangle\}$

$\therefore$ INTEN(A) = $\{\langle 1,0.00014,0.5739\rangle,\langle 2,0.0078,0.0366\rangle,\langle 3,0.1120,0.00087\rangle,\langle 4,1,0\rangle\}$

Note

(i) From the definition 3.2, it is seen that INTEN (A) = $2A^2$.

(ii) As the name suggests, contrast intensification intensifies the contrast of the STGIFS.

(iii) It is also to be noted that, in example 3.3, A is not an IFS, because $\mu_A(x)+\gamma_A(x) \notin [0,1]$ for 1, 4 $\in$ A.

Definition3.4

The fuzzification of a STGIFS A of the universe X, denoted by FUZZ(A), is defined by,

FUZZ(A) = $\{\langle x,\mu_{FUZZ}(x),\gamma_{FUZZ}(x)\rangle : x \in X\}$ where

$$\mu_{FUZZ}(x) = 1-(1-\mu_A(x)^2)^{\frac{1}{\sqrt{2}}} \text{ and}$$

$$\gamma_{FUZZ}(x) = \left[1-(1-\gamma_A(x)^2)^{\frac{1}{2}}\right]^{\frac{1}{\sqrt{2}}}.$$

Example 3.5

Consider the STGIFS A defined in example 3.3.

FUZZ (A) = $\{\langle 1,0.125,0.274\rangle, \langle 2,0.33,0.05\rangle, \langle 3,0.61,0.013\rangle, \langle 4,1,0\rangle\}$

Note

From example 3.3 and example 3.5, it is to be noted that INTEN (A) and FUZZ(A) are also STGIFSs.

Definition 3.6

Let A be a STGIFS of the universe X. Define the following operations on A:

(i) Necessity measure on A:

$$\Box A = \left\{\left\langle x, \mu_A(x), \sqrt{1-\mu_A(x)^2}\right\rangle : x \in X\right\}$$

(ii) Possibility measure on A:

$$\Diamond A = \left\{\left\langle x, \sqrt{1-\gamma_A(x)^2}, \gamma_A(x)\right\rangle : x \in X\right\}$$

(iii) The degree of non-determinacy (uncertainty) of an element $x \in X$ to the STGIFS A is defined by

$$\Pi_A(x) = \sqrt{1-\mu_A(x)^2-\gamma_A(x)^2}$$

Theorem 3.7

The degrees of uncertainty of all elements of $x \in X$ to STGIFS A are zero iff A is a crisp set.

Proof

The degrees of uncertainty of all elements of $x \in X$ to STGIFS A are zero $\Leftrightarrow$ $\sqrt{1-\mu_A(x)^2-\gamma_A(x)^2} = 0$

$\Leftrightarrow$ $[\mu_A(x)]^2 + [\gamma_A(x)]^2 = 1$

$\Leftrightarrow$ $\mu_A(x) = 0$ and $\gamma_A(x) = 1$ (or) $\mu_A(x) = 1$ and $\gamma_A(x) = 0$

$\Leftrightarrow$ A is a crisp set.

Proposition 3.8

(i) INTEN (A) $\subset$ A.

($A \subset B$ iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$, $\forall x \in X$).

(ii) □ (INTEN A) = INTEN(□A).

(iii) (Here, $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$, $\forall x \in X$).

(iii) ◊ (INTEN A) = INTEN (◊A).

(iv) $\prod_A(x) = 0$, then $\prod_{INTENA}(x) = 0$, $\forall x \in X$.

Proof

(i) INTEN (A) = $\left\{\left\langle x, 1-(1-\mu_A(x)^8)^2, \left[1-(1-\gamma_A(x)^2)^2\right]^4\right\rangle : x \in X\right\}$

A = $\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ such that

mini ($[\mu_A(x)]^2$, $[\gamma_A(x)]^2$)≤ 0.5 for each $x \in$ X.

Since $\mu_A(x): X \to [0,1]$ and $\gamma_A(x): X \to [0,1]$, we have

$1-(1-\mu_A(x)^8)^2 \le \mu_A(x)$ and $\left[1-(1-\gamma_A(x)^2)^2\right]^4 \ge \gamma_A(x)$ $\forall x \in X$.

Hence INTEN (A) $\subset$ A.

(ii) $$\Box A = \left\{\left\langle x, \mu_A(x), \sqrt{1-\mu_A(x)^2}\right\rangle : x \in X\right\}$$

$$\Box\,(\text{INTENA}) = \left\{\left\langle x, 1-(1-\mu_A(x)^8)^2, (1-\mu_A(x)^8)\sqrt{1-\mu_A(x)^{16}+2\mu_A(x)^8}\right\rangle : x \in X\right\} \quad (3.1)$$

$$\text{INTEN}(\Box A) = \left\{\left\langle x, 1-(1-\mu_A(x)^8)^2, (1-\mu_A(x)^2)^4\right\rangle : x \in X\right\} \quad (3.2)$$

From (3.1) and (3.2), it is seen that □(INTEN A) = INTEN(□A) only when $\mu_A(x)$ = 0 (or) $\mu_A(x)$ = 1.

(iii) $$\Diamond A = \left\{\left\langle x, \sqrt{1-\gamma_A(x)^2}, \gamma_A(x)\right\rangle : x \in X\right\}$$

$$\Diamond\,(\text{INTEN A}) = \left\{\left\langle x, \sqrt{1-(1-(1-\gamma_A(x)^2)^2)^8}, (1-(1-\gamma_A(x)^2)^2)^4\right\rangle : x \in X\right\} \quad (3.3)$$

$$\text{INTEN}(\Diamond A) = \left\{\left\langle x, \sqrt{1-\gamma_A(x)^2}, (1-(1-\gamma_A(x)^2)^2)^4\right\rangle : x \in X\right\} \quad (3.4)$$

From (3.3) and (3.4), it is seen that

◊ (INTEN A) = INTEN (◊A) only when $\gamma_A(x)$ = 0 (or) $\gamma_A(x)$ = 1.

(iv) $\prod_A(x) = 0$

$\Rightarrow \mu_A(x) = 0$ and $\gamma_A(x) = 1$ (or) $\mu_A(x) = 1$ and $\gamma_A(x) = 0$ (3.5)

Now, $\prod_{INTENA}(x) = \sqrt{1-(1-(1-\mu_A(x)^8)^2)^2-(1-(1-\gamma_A(x)^2)^2)^8}$ =0, by (3.5).

Hence proved.

4. Conclusion

In this paper, we defined the operation, contrast intensification of STGIFS and some of their properties are analyzed. Contrast intensification is one of the operations intended to enhance the image for pattern recognition and classification. Hence, our future work is to apply these properties in the field of image processing and pattern recognition with the help of the software MATLAB.

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