17th Int. Conf. on IFSs, Sofia, 1–2 November 2013
Notes on Intuitionistic Fuzzy Sets
Vol. 19, 2013, No. 3, 99–110

Intuitionistic fuzzy generalized nets with characteristics of the places of Types 1 and 3

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Abstract: Two new extensions of the class Σ are defined - Intuitionistic fuzzy generalized nets with characteristics of the places of type 1 (IFGNCP1) and type 3 (IFGNCP3). The general algorithm for transition functioning in these nets is presented. It is proved that the functioning and the results of the work of every net from the two new types can be described by a standard GN.
Keywords: Generalized nets, Intuitionistic fuzzy generalized nets.
AMS Classification: 03E72.

1 Introduction

In Generalized nets with characteristics of the places (GNCP) (see [2]) the places can receive characteristics and keep information about the flow of the tokens into the net. Previously, the idea of assigning characteristics to the places has been known from the Intuitionistic fuzzy generalized nets of type 2 (IFGN2). Again in [2] it is proved that the class of all GNCP Σ_{CP} is conservative extension of the class Σ of all GNs. Formally, GNCP E is denoted by

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle$$

All components of E with the exception of the function Ψ are the same as in the standard GNs (see [3,4]). The function Ψ assigns characteristics to the places - the number of tokens from each type in the place. This function can be extended to give the time moments when the tokens enter the place or other information related to the work of the net. The algorithms for transition functioning are basically the same as in the standard GNs and we shall not describe them. The only difference is that now the function Ψ assigns characteristics to the output places when a transfer has occurred.

The connection between GNCP on one hand, and IFGN1 and IFGN2 on the other, is studied in [1]. It is shown how we can represent the work of GNCP in terms of IFGN1 and IFGN2. The two new types of GNs defined in this paper combine the features of GNCP and IFGN1 and IFGN3.

2 Intuitionistic fuzzy generalized nets with characteristics of the places of Type 1

In the definition of GNCP the function f which calculates the truth values of the predicates of the transition's conditions has values from the set $\{0, 1\}$. In IFGN1 the function f calculates the truth values of the predicates in the form $\langle \mu_{i,j}, \nu_{i,j} \rangle$ where $\mu_{i,j}$ and $\nu_{i,j}$ are the degrees of validity and non-validity of the predicate $r_{i,j}$ and they are real numbers from the interval $[0, 1]$ such that $\mu_{i,j} + \nu_{i,j} \leq 1$. The connection between GNCP and IFGN1 is discussed in details in [1]. It is proved that the functioning and the result of work of every GNCP can be represented by an IFGN1. However, the constructive proof presented there shows that it is not practical to try to represent a net from one of the types with a net from the other type. Especially if we want to keep record of the number of the tokens in some of the places, to avoid using a GNCP we need to use additional places and tokens which may not be practical for large nets. Therefore, if the modelled process requires intuitionistic fuzzy evaluation of the predicates and characteristics of the places, we will need a new type of GN. In analogy with IFGN1 we shall call such nets Intuitionistic fuzzy generalized nets with characteristics of the places of type 1 (IFGNCP1). Every transition of an IFGNCP1 has the same components as the transitions of the ordinary GN(see Fig. 1), i.e. it is described by a seven-tuple :

$$Z = \langle L', L'', t_1, t_2, r, M, \Box \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively). For the transition in Fig. 1 these are

$$L' = \{l'_1, l'_2, \ldots, l'_m\}$$

and

$$L'' = \{l''_1, l''_2, \ldots, l''_n\}$$

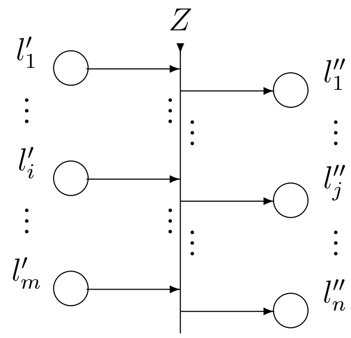

Figure 1.

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will be transferred from the transition's inputs to its outputs. Parameter r has the form of an Index Matrix (IM, see [3, 4]):

$$r = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \;\; - \text{ predicate}) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array} ;$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the i-th input place to the j-th output place.

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \geq 0 - \text{ natural number or } \infty) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array} ;$$

(f) $\square$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places, and it is an expression constructed of variables and the Boolean connectives $\wedge$ and $\vee$ determining the following conditions:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$	$-$	every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,
$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$	$-$	there must be at least one token in the set of places $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

An IFGNCP1 has the form

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle$$

where

(a) A is the set of transitions as described above;

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \to \mathcal{N}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to \mathcal{N}$, where

$$L = pr_1 A \cup pr_2 A$$

and obviously, L is the set of all places of the net;

(d) c is a function giving the capacities of the places, i.e., $c : L \to \mathcal{N}$;

(e) f is a function that calculates the truth values of the predicates of the transition's conditions. As in IFGN1 here the function f gives intuitionistic fuzzy evaluation of the predicates in the form $\langle \mu_{i,j}, \nu_{i,j} \rangle$ where $\mu_{i,j}$ and $\nu_{i,j}$ are the degrees of validity and non-validity of the predicate $r_{i,j}$. They are real numbers in $[0, 1]$ and satisfy the inequality $\mu_{i,j} + \nu_{i,j} \leq 1$;

(f) θ_1 is a function giving the next time-moment for which a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3Z = t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning;

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i.e., $\theta_2(t) = t'$, where $pr_4Z = t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the net's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \to \mathcal{N}$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the IFGNCP1 starts functioning. This moment is determined with respect to a fixed (global) time-scale;

(l) t^0 is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the functioning of the net;

(n) X is a function which assigns initial characteristics to every token when it enters input place of the net. If $\alpha \in K$, then it enters the net with initial characteristic x_0^α;

(p) Φ is a characteristic function that assigns two-valued current characteristic to every token when it makes the transfer from input to output place. The first value is a standard characteristic in the sense of the ordinary GNs. The second is an ordered couple of real numbers $\langle \mu_{i,j}, \nu_{i,j} \rangle$ such that $0 \leq \mu_{i,j} \leq 1$, $0 \leq \nu_{i,j} \leq 1$ and $\mu_{i,j} + \nu_{i,j} \leq 1$. Here $\mu_{i,j}$ is the degree of truth and $\nu_{i,j}$ is the degree of falsity of the corresponding predicate;

(q) Ψ is a characteristic function that assigns a two-valued characteristic to the place when a token enters the place. The first value is the same as in GNCP - the number of tokens from each type in the places, the time moments when the tokens entered the place etc. The second is an ordered couple of real numbers $\langle \mu_{i,j}, \nu_{i,j} \rangle$ such that $0 \leq \mu_{i,j} \leq 1$, $0 \leq \nu_{i,j} \leq 1$ and $\mu_{i,j} + \nu_{i,j} \leq 1$. Here $\mu_{i,j}$ is the degree of truth and $\nu_{i,j}$ is the degree of falsity of the predicate corresponding to the input place from which the tokens are transferred.

(r) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \to \mathcal{N}$.

All other components except f, Φ and Ψ are the same as in the standard GNs. As in IFGN1 here the function f gives intuitionistic fuzzy evaluation of the predicates in the form $\langle \mu_{i,j}, \nu_{i,j} \rangle$ where $\mu_{i,j}$ and $\nu_{i,j}$ are the degrees of validity and non-validity of the predicate. The characteristic function Φ which in the case of ordinary GN assigns characteristics to every token when it makes the transfer from input to output place now adds to these characteristics the degrees of validity and non-validity of the predicate corresponding to the input and output places. In this way when the token finishes its transfer in the net we can determine the degrees of validity and non-validity of the transfer. The function Ψ assigns characteristics to the places as in the GNCP and also the degrees of truth and falsity of every predicate corresponding to the input places from which tokens have been transferred to the current output place. These characteristics can be used to obtain intuitionistic fuzzy evaluation of each place.

The general algorithm for transition functioning of IFGNCP1 is based on the algorithm for transition functioning in IFGN1 (see [5]). We will denote this algorithm by **(Algorithm A')**.

Algorithm A'

(A'01) The non-empty input and the non-full output places are arranged in descending order by their priority. The tokens in every input place are divided into two groups $P_1(l)$ and $P_2(l)$. The first group consists of the tokens that can be transferred to the output places of the transition. The second group consists of the tokens that cannot be transferred at the current time moment.

(A'02) For every input place the tokens from the first group are arranged according to their priorities.

(A'03) An empty index matrix R which corresponds to the index matrix of the predicates r is generated. A value "$\langle 0, 1\rangle$" is asigned to all elements of R which:

- are in a row corresponding to empty input place;
- are in a column corresponding to full output place;
- are placed in a position (i, j) for which the current capacity of the arc between the i-th input and j-th output place is 0;

(A'04) The places are passed sequentially by order of their priorities starting with the place with the highest priority for which transfer has not occurred on the current time step and which has at least one token. For the token with highest priority from the first group we determine if it can split or not. The predicates in the row corresponding to the current input place are checked. If the token cannot split the checking of the predicates stops with the first predicate whose truth value is different from "$\langle 0, 1\rangle$". If the token can split, the truth values of all predicates in the row for which the elements of R are not equal to "$\langle 0, 1\rangle$" are evaluated.

(A'05) Depending on the execution of the operator for permission or prohibition of tokens' splitting, the token from **(A'04)** is transferred either to all permitted output places or to the place with the highest priority. The transfer depends on one of the following conditions:

C1 $\mu(r_{i,j}) = 1, \nu(r_{i,j}) = 0$ (the case of ordinary GN)

C2 $\mu(r_{i,j}) > \frac{1}{2}$ $(> \nu(r_{i,j}))$

C3 $\mu(r_{i,j}) \geq \frac{1}{2}$ $(\geq \nu(r_{i,j}))$

C4 $\mu(r_{i,j}) > \nu(r_{i,j})$

C5 $\mu(r_{i,j}) \geq \nu(r_{i,j})$

C6 $\mu(r_{i,j}) > 0$

C7 $\nu(r_{i,j}) < 1$, i.e. at least $\pi(r_{i,j}) > 0$, where $\pi(r_{i,j}) = 1 - \mu(r_{i,j}) - \nu(r_{i,j})$ is the degree of uncertainty(indeterminancy) and $f(r_{i,j}) = \langle \mu(r_{i,j}), \nu(r_{i,j}) \rangle$.

The condition for transfer of the tokens which will be used is determined for every transition before the firing of the net. If a token cannot be transferred at the current time step, it is moved to the second group of the corresponding input place. The tokens which have been transferred are moved into the second group of the output places. The tokens which have entered the input place after the activation of the transition are moved to the second group too.

(A'06) The values of the characteristic functions Φ and Ψ for the output places (one or more) in which tokens have entered according to **(A'05)** are calculated. The token obtains the next characteristic in the form:

"value of Φ for the current token, $\mu_{i,j}$, $\nu_{i,j}$"

The output place obtains characteristic in the form:

"value of Ψ for the respective place, $\mu_{i,j}$,$\nu_{i,j}$"

(A'07) Put values "$\langle 0, 1 \rangle$" in all rows of R for which the corresponding input place is already empty. Put values "$\langle 0, 1 \rangle$" in all columns of R for which the corresponding output place is already full as a result of the transfer of the tokens on step **(A'05)**. Put values "$\langle 0, 1 \rangle$" in places of R for which the capacity of the arc between the corresponding input and output place becomes 0 as a result of the transfer of the tokens.

(A'08) The current number of tokens in all input places for the current transition decrements with 1 for each token that has gone out of them at this time step. If the current number of tokens for a given input place is zero, the elements of the corresponding row of the index matrix R are assigned the value "$\langle 0, 1 \rangle$".

(A'09) The capacities of all output places in which a token determined at Step (A'04) has entered decrement with 1. If the maximum number of tokens for a given output place is reached, the elements of the corresponding column of the index matrix R are assigned value "$\langle 0, 1 \rangle$".

(A'10) The capacities of all arcs through which a token has passed decrement with 1. If the capacity of an arc has reached 0, to the element from the index matrix R that corresponds to this arc is assigned the value "$\langle 0, 1 \rangle$".

(A'11) If there are still tokens in the input places that can be transferred, and not all output places have reached their capacity, and there are arcs with non-zero capacities, then the algorithm proceeds to Step **(A'12)** otherwise it proceeds to Step **(A'14)**.

(A'12) The current model time t is increased with t^0.

(A'13) Is the current time moment equal to or greater than $t_1 + t_2$? If the answer to the question is no, return to Step **(A'04)**, otherwise go to Step **(A'14)**.

(A'14) Termination of the transition's functioning.

The general algorithm for functioning of IFGNCP1 is the same as the algorithm for functioning of GN denoted by **(Algorithm B')** (see [4]).

Let $\Sigma_{IFGNCP1}$ be the class of all IFGNCP1. The following theorem shows how $\Sigma_{IFGNCP1}$ is related to the class Σ of all GNs.

Theorem 1 *For every IFGNCP1 E there exists an ordinary GN that describes the functioning and the results of work of E.*

Proof. Let the given IFGNCP1 E have the components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle$$

For every transition $Z \in pr_1 pr_1 E$ (see Fig.1) with components

$$Z = \langle L', L'', t_1, t_2, r, M, \Box\rangle,$$

we construct a corresponding transition $Z^* = \langle L'^*, L''^*, t_1, t_2, r^*, M^*, \Box^*\rangle$ (see Fig.2)

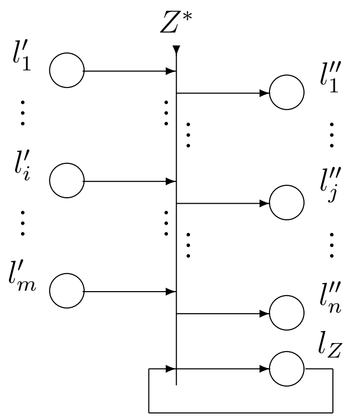

Figure 2.

Z^* is obtained from Z by the addition of a new place l_Z which is input and output for the transition.

$$L'^* = L' \cup \{l_Z\}$$
$$L''^* = L'' \cup \{l_Z\}$$

We use the same notation for the corresponding places in Z and Z^* to avoid complicating the notation. Both transitions become active at the same time and have equal durations of their active states. In place l_Z a token α_Z will loop and keep the characteristics of the output places of Z. The rest of the components of Z^* are defined as follows. If

$$r = pr_5 Z = [L', L'', \{r_{l'_i, l''_j}\}]$$

is the IM of the transition's condition, then

$$r^* = pr_5 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r^*_{l'_i, l''_j}\}]$$

where

$$(\forall l'_i \in L')(\forall l''_j \in L'')(r^*_{l'_i, l''_j} = r_{l'_i, l''_j})$$

$$(\forall l'_i \in L')(\forall l''_j \in L'')(r^*_{l'_i, l_Z} = r^*_{l_Z, l''_j} = \text{“}false\text{”}),$$
$$r^*_{l_Z, l_Z} = \text{“}true\text{”};$$

If

$$M = pr_6 Z = [L', L'', \{m_{l'_i, l''_j}\}]$$

has the form of an IM, then

$$M^* = pr_6 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m^*_{l'_i, l''_j}\}],$$

where

$$(\forall l'_i \in L')(\forall l''_j \in L'')(m^*_{l'_i, l''_j} = m_{l'_i, l''_j}),$$
$$(\forall l'_i \in L')(\forall l''_j \in L'')(m^*_{l'_i, l_Z} = m^*_{l_Z, l''_j} = 0),$$
$$m^*_{l_Z, l_Z} = 1$$

Let A^* be the set of all transitions obtained after reapeating the above procedure for all transitions of E. We will construct a standard GN G and prove that it represents the functioning and the results of work of E. Let G have the form:

$$G = \langle\langle A^*, \pi^*_A, \pi^*_L, c^*, f^*, \theta_1, \theta_2\rangle, \langle K^*, \pi^*_K, \theta^*_K\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b^*\rangle\rangle$$

A^* is the set of transitions described above.

$$(\forall Z^*_i \in A^*)(\pi^*_A(Z^*_i) = \pi_A(Z_i))$$

$$\pi^*_L = \pi_L \cup \pi_{\{l_Z | Z \in A\}},$$

where function $\pi_{\{l_Z | Z \in A\}}$ determines the priorities of the new places that are elements of set $\{l_Z | Z \in A\}$ and the priorities of the places l_Z for every transition $Z \in A$ are the minimal among the priorities of all other places of the transition Z.

$$c^* = c \cup c_{\{l_Z | Z \in A\}},$$

where function $c_{\{l_Z | Z \in A\}}$ satisfies the equality

$$c_{\{l_Z | Z \in A\}}(l_Z) = 1,$$

The transfer of tokens in E is determined by the conditions $C1, \cdots, C7$ from step **(A'05)** of the algorithm for functioning of the transitions in IFGNCP1. In order to preserve this conditions in G where the function f^* should assign to the predicates values from the set $\{0, 1\}$, for the condition $C1$ we define f^* in the following way:

$\mathbf{C1}^*$ $f^*(r_{i,j}) = \lfloor pr_1 f(r_{i,j}) \rfloor$.

where $\lfloor x \rfloor$ is the floor function which maps a real number x to the largest integer smaller or equal to x.

Similarly, in the other cases we define $f^*(r_{i,j})$ as follows:

$\mathbf{C2}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

$\mathbf{C3}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

C4* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

C5* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

C6* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > 0 \\ 0, & \text{otherwise} \end{cases}$

C7* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \nu(r_{i,j}) < 1 \\ 0, & \text{otherwise} \end{cases}$

$$K^* = (\bigcup_{l \in Q^I} K_l) \bigcup \{\alpha_Z | Z \in A\},$$

where K_l is the set of tokens that enter the net E through place l. The set of the tokens of G consists of all tokens of E and all additional tokens in the l_Z-places.

$$\pi_K^* = \pi_K \cup \pi_{\{l_Z | Z \in A\}}$$

where the function $\pi_{\{l_Z|Z\in A\}}$ determines the priorities of the α_Z tokens. The α_Z tokens stay in the l_Z places during the entire period of functioning of the net and no other tokens can enter the l_Z places. We can consider that the α_Z tokens have the lowest priority among all tokens of the net.

$$\theta_K^* = \theta_K \cup \theta_{\{l_Z | Z \in A\}},$$

where $\theta_{\{l_Z|Z\in A\}}$ determines that each α_Z-token stays in the initial time-moment T in the l_Z place.

$$X^* = X \cup \{x_0^{\alpha_Z} | Z \in A\},$$

where $x_0^{\alpha_Z}$ is the initial characteristic of token α_Z and it is a list of all output places for the transition Z:

$$\text{“}\left\langle l_1'', l_2'', \cdots l_n'' \right\rangle\text{”}$$

$$\Phi^* = \Phi \cup \Psi^*_{\{l_Z | Z \in A\}},$$

where the function $\Psi^*_{\{l_Z|Z\in A\}}$ determines the characteristics of the α_Z-tokens in the form

$$\Psi^*_{\{l_Z|Z\in A\}}(\alpha_Z) = \text{“}\{\langle l_j'', \Psi(l_j'')\rangle | l_j'' \in L''\}\text{”},$$

Here $\Psi(l_j'')$ is the characteristic of the place l_j'' in the sense of IFGNCP1 and the degrees of validity and non-validity of the predicates are included in this characteristic.

$$b^* = b \cup b_{\{\alpha_Z | Z \in A\}}$$

where the function $b_{\{\alpha_Z|Z\in A\}}(\alpha) = \infty$ determines the number of characteristics the α_Z tokens can keep.

We shall prove that both nets E and G function equally. We shall compare the functioning of one arbitrary transition Z of IFGNCP1 E and its corresponding transition Z^* of G and use the theorem for the completeness of the GN transitions. Obviously, these two transitions become active simultaneously and have equal duration of functioning and priorities. Let $\alpha \in K$ and $\beta \in K^*$ be two tokens of the same type with equal characteristics which are at two corresponding places of the transitions at some moment of time. Apparently neither of them is α_Z token. Depending on the execution of the operator for permission or prohibition of tokens' splitting the token α will be transferred either to all permitted output places or to the place with the highest priority among all output places. The transfer of α is determined by one of the conditions $C1, C2, \cdots, C7$. Let the conditions allow the transfer to output place l''_j (the case where splitting of tokens is allowed is analogous). At the same time β in place l'^*_i will be transferred to the output place l''^*_j (here l'^*_i is the corresponding input place to l'_i and l''^*_j the corresponding to l''_j output place in Z^*) because the function $f^*(r_{i,j}) = 1$ if the corresponding condition for the transfer from l'_i to l''_j is satisfied. Upon entering l''^*_j token β obtains the same characteristic as the token α in l''_j because the characteristic functions Φ and Φ^* assign the same characteristics in the corresponding places (since neither of them is l_Z place). In E the function Ψ assigns characteristic to the output place l''_j . At the end of the current time step the same characteristic is assigned to the α_Z token in place l_Z. Since the tokens were arbitrarily chosen we conclude that both transitions function equally. From the theorem for the completeness of the GN transitions it follows that the GN G represents the functioning and results of work of E. □

3 Intuitionistic fuzzy generalized nets with characteristics of the places of Type 3

The Intuitionistic fuzzy generalized nets of type 3 (IFGN3) are extension of IFGN1 (see [4]). An IFGN3 has the same components as IFGN1. The only difference is that in $IFGN3$ the characteristic function Φ assigns to every token α characteristics in the form:

$$x^{\alpha}_{cu} = \langle \overline{x}^{\alpha}_{cu}, \mu(r_{i,j}), \nu(r_{i,j}), \mu(x^{\alpha}_{cu}), \nu(x^{\alpha}_{cu}) \rangle$$

where $\overline{x}^{\alpha}_{cu}$ is the standard characteristic of the token in the sense of GNs, $\mu(r_{i,j})$ and $\nu(r_{i,j})$ are the degrees of truth and falsity of the corresponding predicate and $\mu(x^{\alpha}_{cu})$ and $\nu(x^{\alpha}_{cu})$ are estimations in intuitionistic fuzzy sense of the characteristics of the tokens. $\mu(x^{\alpha}_{cu})$ and $\nu(x^{\alpha}_{cu})$ are real numbers from $[0, 1]$ for which $\mu(x^{\alpha}_{cu}) + \nu(x^{\alpha}_{cu}) \leq 1$. The pair $\langle \mu(x^{\alpha}_{cu}), \nu(x^{\alpha}_{cu}) \rangle$ represents the validity and non-validity of the characteristics and through them the model determines its status.

Analogously to the previous section, we will combine the features of GNCP and IFGN3. The idea of intuitionistic fuzzy estimation of the token's characteristics can be applied to the places in the sense of GNCP. We will call the new type of net Intuitionistic fuzzy generalized net with characteristics of the places of type 3(IFGNCP3). An IFGNCP3 has the form:

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, \Psi, b \rangle \rangle$$

Every transition of an IFGNCP3 has the same components as in the standard GNs and they are explained in the previous section. All other components except the characteristic functions Φ and Ψ are the same as in IFGNCP1. The function Φ assigns characteristics to the tokens in the sense of IFGN3:

$$x^{\alpha}_{cu} = \langle \overline{x}^{\alpha}_{cu}, \mu(r_{i,j}), \nu(r_{i,j}), \mu(x^{\alpha}_{cu}), \nu(x^{\alpha}_{cu})\rangle$$

where $\mu(x^{\alpha}_{cu})$, $\nu(x^{\alpha}_{cu}) \in [0,1]$ and $\mu(x^{\alpha}_{cu}) + \nu(x^{\alpha}_{cu}) \leq 1$. $\overline{x}^{\alpha}_{cu}$ is the standard GN characteristic of the token. The tokens obtain characteristics only when the estimations of the characteristics satisfy one of the following conditions:

C1 $\mu(x^{\alpha}_{cu}) = 1, \nu(x^{\alpha}_{cu}) = 0$ (the case of ordinary GN)

C2 $\mu(x^{\alpha}_{cu}) > \frac{1}{2}$ $(> \nu(x^{\alpha}_{cu}))$

C3 $\mu(x^{\alpha}_{cu}) \geq \frac{1}{2}$ $(\geq \nu(x^{\alpha}_{cu}))$

C4 $\mu(x^{\alpha}_{cu}) > \nu(x^{\alpha}_{cu})$

C5 $\mu(x^{\alpha}_{cu}) \geq \nu(x^{\alpha}_{cu})$

C6 $\mu(x^{\alpha}_{cu}) > 0$

C7 $\nu(x^{\alpha}_{cu}) < 1$

The function Ψ assigns to the places characteristics in the form:

$$\psi^{\ l}_{cu} = \langle \overline{\psi}^{l}_{cu}, \mu(r_{i,j}), \nu(r_{i,j}), \mu(\psi^{l}_{cu}), \nu(\psi^{l}_{cu})\rangle$$

where $\mu(\psi^{l}_{cu})$, $\nu(\psi^{l}_{cu}) \in [0,1]$ and $\mu(\psi^{l}_{cu}) + \nu(\psi^{l}_{cu}) \leq 1$. $\overline{\psi}^{l}_{cu}$ is the standard characteristic of the place l in the sense of GNCP. The places obtain characteristics only when the estimations of the characteristics satisfy one of the following conditions:

C1 $\mu(\psi^{l}_{cu}) = 1, \nu(\psi^{l}_{cu}) = 0$ (the case of ordinary GN)

C2 $\mu(\psi^{l}_{cu}) > \frac{1}{2}$ $(> \nu(\psi^{l}_{cu}))$

C3 $\mu(\psi^{l}_{cu}) \geq \frac{1}{2}$ $(\geq \nu(\psi^{l}_{cu}))$

C4 $\mu(\psi^{l}_{cu}) > \nu(\psi^{l}_{cu})$

C5 $\mu(\psi^{l}_{cu}) \geq \nu(\psi^{l}_{cu})$

C6 $\mu(\psi^{l}_{cu}) > 0$

C7 $\nu(\psi^{l}_{cu}) < 1$

The algorithm for transition functioning in IFGNCP3 is the same as in IFGNCP1.

The following theorem can be proven in the same way as Theorem 1.

Theorem 2 *The functioning and the results of the work of every IFGNCP3 can be described by an ordinary GN.*

4 Conclusion

The new types of GNs defined in this paper are closely related to GNCP on one hand and to IFGN1 and IFGN3 on the other. The two theorems show how $\Sigma_{IFGNCP1}$ and $\Sigma_{IFGNCP3}$ are related to the class Σ. In [1] the connection between GNCP and IFGN1 and IFGN2 is studied in details. It is shown how we can represent the functioning and result of the work of a GNCP in terms of IFGN1 and IFGN2 and vice versa. To continue the comparative study of the different types of IFGN, GNCP, IFGNCP1 and IFGNCP3 in future we will investigate how the newly defined types are related to the other with regard to the functioning and the results of their work.

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