

A generalized net model of an intuitionistic fuzzy expert system

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Abstract: A generalized net model of an intuitionistic fuzzy expert system is described. It is based on two other models of a standard expert system and on one of its extensions. The so constructed model corresponds to an expert system that can answer to different questions related to the time.

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1 Introduction

A lot of colleagues already assert that the Expert Systems (ESs) are dying. The author supports the idea that they will live their “Renaissance”, obtaining a special place in the instrumentation of Data Mining (DM). Preserving their basic purpose to generate new knowledge by answering hypotheses, we can essentially extend the area of their capabilities. When some unclear situation arises in a process controlled by DM-tools, and when some hypotheses for its future development are generated, then the new type of ESs can help.

In a series of papers collected in [3], the author described the basic steps of the process of the functioning and the results of work of ESs. There, nine Generalized Nets (GN, see [1, 5]) models of Expert Systems (ESs) are described. In Section 2, we discuss the GN-model that describes the

functioning and the results of the work of an arbitrary ES. It will be the basis of a new GN-models, describing in Sections 3 and 4.

We must mention also that in [10, 11], the functioning and the results of the work of relational databases, uncertain data and knowledge engineering processes are described by GNs.

The first of the models from [3] shows how a GN-model of a given ES can be constructed. This model contains information about the separate ES components (Data Base (DB), Knowledge Base (KB) hypotheses), which are represented by GN-transitions, places, tokens and token characteristics. It shows that we can construct one GN-model for *any* ES. In the second and third GN-models from [3], a part of the GN-components that correspond to the ES-components are changed only by token characteristics. In the second GN-model, the DB is represented by a characteristic of a specially constructed token, while in the third GN-model the same is done for the KB.

Therefore, the second and the third GN-models are already independent of the concrete DB and the concrete KB of a given ES, being modelled by the GN. Hence, the third GN-model is *universal* for the class of *all* standard ESs.

The fourth GN-model is an extension of the third one, but with the possibility to represent KB rules containing the operation “negation” in their antecedents. The remaining five GN-models, described in [3], are devoted to extensions of the concept of an ES. The new ESs contain new components, which can be represented by the GN-tools. Of course, these new components are not all of those necessary for modelling concrete expert processes, but they illustrate the possibility for extending the ES-structures, and in separate cases they can be useful.

The fifth GN-model represents the functioning and the results of the work of an ES, with priorities of the data within its DB. The data have now a specific priority. At the time of the ES functioning (within the context of a GN-model), new data (represented by GN-tokens) can enter the DB changing the existing information in it (if the new data are in contradiction with the data already existing, and if the priority of the new data is greater than the priority of the old), or confirm it. There are special tools, described by GN-subnets, which can check the correctness of the new data and this information will enter the DB only if it is not in contradiction with the existing rules of the ES’s KB.

The seventh GN-model is devoted to an analogous extension of the ES, but now related to its KB. The rules there have priorities and they can be changed or confirmed as in the previous case. Now, there are GN-subnets that check the correctness of the new rules and they will enter the DB only if they are not in contradiction with the existing rules of the ES’s KB and with the existing data in the ES’s DB.

The sixth GN describes an ES that contains “metafacts”. This new concept is similar to DB facts and simultaneously to the KB rules. In practice, the metafacts can be interpreted as facts about the DB-facts, as well as, as simple KB-rules.

The concept of an Intuitionistic Fuzzy ES (IFES) was introduced in [2] as an extension of fuzzy ESs. The estimations of the truth-values of the facts there have the form $\langle m, n \rangle$, where $m, n \in [0, 1]$ and $m + n \leq 1$. Numbers m and n correspond to the degree of validity and the degree of non-validity of the fact. In this case, there possibly exists a degree of uncertainty p , for which $p = 1 - m - n \geq 0$. The three components of the Intuitionistic Fuzzy Set (IFS) [4] and

its derivatives (intuitionistic fuzzy logics, intuitionistic fuzzy graphs, intuitionistic fuzzy abstract systems, intuitionistic fuzzy ESs, and so on), give greater possibilities for the real processes modelling than ordinary fuzzy objects. The eighth GN-model describes the functioning and the results of the work of an IFES.

On the basis of the modal types of operators defined over IFs, the eighth GN-model was constructed so that it represents the functioning and the results of an ES using modal logic operations.

Here, we must mention that in [2], the concept of an Intuitionistic Fuzzy ES (IFES) was introduced. It was essentially extended in [3, 10, 11]. In these ESs, each fact F has IF-estimations $\langle \mu(F), \nu(F) \rangle$, determining its degrees of validity and non-validity. So, the answer whether a given hypothesis is valid or not, obtains essentially more exact evaluation. In near future, we will introduce an extension of the IFES whose facts will have the IVIF-estimations $\langle M(F), N(F) \rangle$, where $M(F), N(F) \subseteq [0, 1]$ and $\sup M(x) + \sup N(x) \leq 1$. So, we will define Interval Valued IFES (IVIFES). A next step of the extensions will be introducing of facts that contain moments of time, when they started to be valid, and moments in which they ceased being valid (a sequence of time-moments t_1, t_2, \dots, t_n). Then (cf. [3]), on the one hand we can answer to questions related to the time (“at the moment”, “once”, “sometimes”, “for long/short time”, “often”, “rarely”, “for a short period”, “for a long period”, etc.). On the other hand, the IVIFES rules can have essentially more complex forms, containing different logical operations (conjunction, disjunction, implication, negation,...), quantifiers (“for existence” and “for all”) and modal operators in their antecedents (see [9]). In addition, the facts and the rules can have priorities that determine whether a given fact or a given rule can remain in the DB or must be changed with another one.

Finally, the ninth GN-model, described in [3], represents the functioning and the results of the work of an ES with rules, related to a fixed time-scale, which is given as characteristics of a special GN-token. In this way, we can construct a temporal ES using temporal logic operators.

2 A GN-model of a standard ES

As was mentioned above, the first two GN-models from [3] depend on the forms (DB and KB) of the described ESs, while the GN-model that will be given below (it is the third model from [3]), is the first one that is fully independent from the forms of the ESs whose functioning and results of work represents. It is shown in Figure 1.

For the sake of clarity, the places are marked by three different symbols: a , b and c , such that:

- the token α together with its descendants of all generations obtained after splitting will go to the a -places;
- the token β will go on b -places;
- the token γ will go on c -places.

Below, the tokens characteristics will be ordered tuples whose first component is in turn a vector with components natural numbers. At every step where tokens will split, they will be marked with the number of the current split, keeping the previous numeration, i.e., if the first component

of a token characteristic is $\langle s_1, s_2, \dots, s_{k-1} \rangle$ ($k \geq 0, s_1, s_2, \dots, s_{k-1}$ being natural numbers), then its next characteristic will be $\langle s_1, s_2, \dots, s_{k-1}, s_k \rangle$, where the natural number s_k will correspond to the number of the tokens's current splitting.

Let Δ be the DB of a given ES. Let a token α enter place a_1 of the GN with an initial characteristic $x_0^\alpha = \langle p, H \rangle$, where p is the current number of the α -token which enters place a_1 and H is a hypothesis.

Let a token β enter place b_1 with an initial characteristic $x_0^\beta = \Delta$.

Let the token γ enter the place c_1 with an initial characteristic $x_0^\gamma = R$, where $R = \{R_1, \dots, R_n\}$ is the list of the rules. Each rule R_i has the form ($1 \leq i \leq n$):

$$R_i = \langle C_i; A_{i,1}, \dots, A_{i,s_i} \rangle,$$

or

$$R_i = \langle A_{i,1}, \dots, A_{i,s_i} : \neg C_i \rangle,$$

where C_i is the consequent and $A_{i,1}, \dots, A_{i,s_i}$ are the elements of the conjunction which forms the antecedent, x_i denotes the i -th characteristic of the α -token with the highest priority in a given place.

The transitions of the GN are given in the following Figure 1. The first transition has the form:

$$Z_1 = \langle \{a_1, b_1, b_9, c_1, c_7\}, \{a_2, a_3, b_2, b_3, c_2, c_3\}, r_1, M_1, \square_1 \rangle,$$

where (for index matrices see [6]):

$r_1 =$	a_2	a_3	b_2	b_3	c_2	c_3
a_1	r_{a_1,a_2}	r_{a_1,a_3}	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
b_1	<i>false</i>	<i>false</i>	r_{b_1,b_2}	r_{b_1,b_3}	<i>false</i>	<i>false</i>
b_9	<i>false</i>	<i>false</i>	r_{b_9,b_2}	r_{b_9,b_3}	<i>false</i>	<i>false</i>
c_1	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	r_{c_1,c_2}	r_{c_1,c_3}
c_7	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	r_{c_7,c_2}	r_{c_7,c_3}

where

- $r_{a_1,a_2} = r_{b_1,b_2} = r_{b_9,b_2} = r_{c_1,c_2} = r_{c_7,c_2} = \text{"pr}_2 x_0^\alpha \notin x_0^\beta\text{"}$,
- $r_{a_1,a_3} = \neg r_{a_1,a_2}$,
- $r_{b_1,b_3} = r_{b_9,b_3} = r_{c_1,c_3} = r_{c_7,c_3} = \neg r_{a_1,a_2}$ & "there are no new α -tokens before place a_1 ", where $\neg P$ is the negation of predicate P ,

and

$M_1 =$	a_2	a_3	b_2	b_3	c_2	c_3
a_1	∞	∞	0	0	0	0
b_1	0	0	1	1	0	0
b_9	0	0	1	1	0	0
c_1	0	0	0	0	1	1
c_7	0	0	0	0	1	1

and

$$\square_1 = \wedge(a_1, \vee(b_1, b_9), \vee(c_1, c_7)).$$

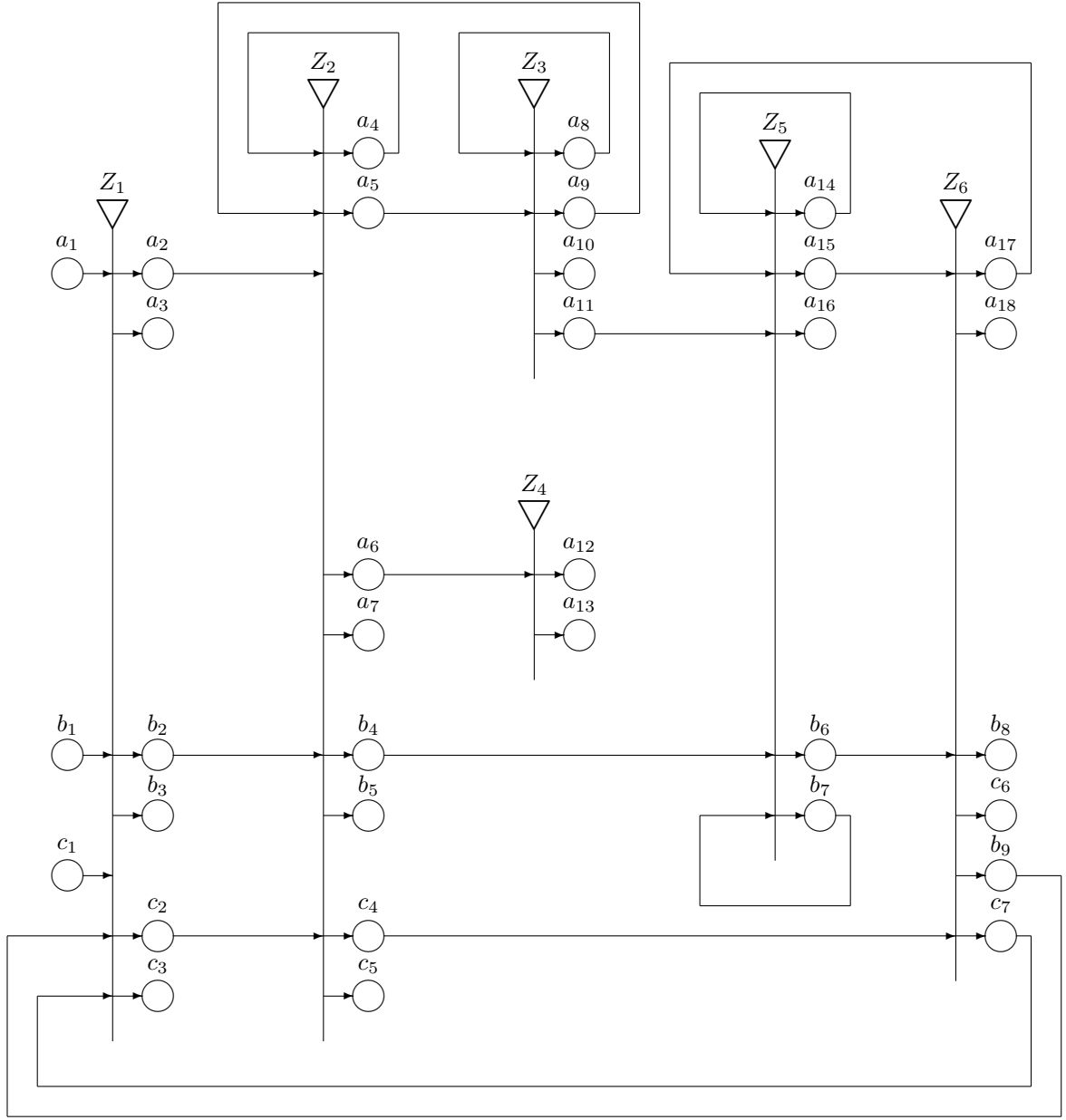


Figure 1. A GN-model of a standard ES

The tokens obtain the characteristic “ $\langle \text{pr}_1 x_0^\alpha, ! \text{pr}_2 x_0^\alpha \rangle$ ” in place a_3 , and they do not obtain any characteristic in the other output places.

The second transition has the form:

$$Z_2 = \langle \{a_2, a_4, a_9, b_2, c_2\}, \{a_4, a_5, a_6, a_7, b_4, b_5, c_4, c_5\}, r_2, M_2, \square_2 \rangle,$$

where

		a_4	a_5	a_6	a_7	b_4	b_5	c_4	c_5
$r_2 =$	a_2	r_{a_2,a_4}	r_{a_2,a_5}	r_{a_2,a_6}	$false$	$false$	$false$	$false$	$false$
	a_4	r_{a_4,a_4}	r_{a_4,a_5}	$false$	r_{a_4,a_7}	$false$	$false$	$false$	$false$
	a_9	r_{a_9,a_4}	r_{a_9,a_5}	r_{a_9,a_6}	r_{a_9,a_7}	$false$	$false$	$false$	$false$
	b_2	$false$	$false$	$false$	$false$	r_{b_2,b_4}	r_{b_2,b_5}	$false$	$false$
	c_2	$false$	$false$	$false$	$false$	$false$	$false$	r_{c_2,c_4}	r_{c_2,c_5}

where

- $r_{a_2,a_4} = r_{a_4,a_4} = r_{a_9,a_4} r_{a_2,a_4} = r_{a_4,a_4} = r_{a_9,a_4} = \text{"numb}(\text{pr}_1 x_0^\gamma, \text{pr}_2 x_{last}^\alpha) > \text{nc}(a_4, \alpha) + 1"$
 $\& \neg r_{a_4,a_7}$,
- $r_{a_2,a_5} = r_{a_4,a_5} = r_{a_9,a_5} = \text{"numb}(\text{pr}_1 x_0^\gamma, \text{pr}_2 x_{last}^\alpha) > \text{nc}(a_4, \alpha)" \& \neg r_{a_4,a_7}$,
- $r_{a_2,a_6} = r_{a_9,a_6} = \text{"numb}(\text{pr}_1 x_0^\gamma, \text{pr}_2 x_{last}^\alpha) = 0" \& \neg r_{a_4,a_7}$,
- $r_{a_4,a_7} = r_{a_9,a_7} = \text{"the current } \alpha\text{-token has } \text{pv}(\text{pv}(\text{pr}_1 x_{last}^\alpha))\text{-kin in at least one of the places } a_{11}, a_{14} \text{ or } a_{15}"$,

where the functions numb, nc and pv mean the following:

- numb(Y, y) is the number of the occurrences of the element y in the ordered set Y ,
- nc(l, α) is the number of cycles of the token α in place l ,
- pv($\langle s_1, s_2, \dots, s_{k-1}, s_k \rangle$) = $\langle s_1, s_2, \dots, s_{k-1} \rangle$;

and

$M_2 =$	a_4	a_5	a_6	a_7	b_4	b_5	c_4	c_5
a_2	∞	∞	∞	0	0	0	0	0
a_4	∞	∞	0	∞	0	0	0	0
a_9	∞	∞	∞	∞	0	0	0	0
b_2	0	0	0	0	1	1	0	0
c_2	0	0	0	0	0	0	1	1

and

$$\square_2 = \wedge(\vee(a_2, a_4, a_9), b_2, c_2).$$

The tokens obtain the characteristics

$$\begin{aligned} & \text{"} \langle \langle \text{pr}_1 x_{last}^\alpha; \text{nc}(l_4, \alpha) + 1 \rangle, \{A_1, \dots, A_i | \langle \text{pr}_2 x_{last}^\alpha, \{A_1, \dots, A_i\} \rangle \in x_0^\gamma \\ & \text{is appearing for a } \text{nc}(l_4, \alpha) + 1 \rangle - \text{step} \rangle - x_{last}^\beta \rangle \text{"} \end{aligned}$$

in place a_5 and " $\langle \text{pr}_1 x_{last}^\alpha, \neg! \text{pr}_2 x_{last}^\alpha \rangle$ " in place a_6 , and they do not obtain any characteristic in the other output places.

We must note that the output place priorities must satisfy the following inequality:

$$\pi_L(a_7) > \pi_L(a_6) > \pi_L(a_5) > \pi_L(a_4).$$

The third transition has the form:

$$Z_3 = \langle \{a_5, a_8\}, \{a_8, a_9, a_{10}, a_{11}\}, r_3, M_3, \vee(a_5, a_8) \rangle,$$

where

$$r_3 = \begin{array}{c|cccc} & a_8 & a_9 & a_{10} & a_{11} \\ \hline a_5 & r_{a_5,a_8} & r_{a_5,a_9} & r_{a_5,a_{10}} & r_{a_5,a_{11}} \\ a_8 & r_{a_8,a_8} & r_{a_8,a_9} & r_{a_8,a_{10}} & false \end{array}$$

where

- $r_{a_5,a_8} = r_{a_8,a_8} = \text{“}\underline{\text{card}}(\text{pr}_2 x_{last}^\alpha) > \underline{\text{nc}}(a_8, \alpha) + 1\text{”} \& \neg r(a_5, a_{10})$,
- $r_{a_5,a_9} = r_{a_8,a_9} = \text{“}\underline{\text{card}}(\text{pr}_2 x_{last}^\alpha) > \underline{\text{nc}}(a_8, \alpha)\text{”} \& \neg r_{a_5,a_{10}}$,
- $r_{a_5,a_{10}} = r_{a_8,a_{10}} = \text{“in the places } a_6, a_{12} \text{ or } a_{13} \text{ resides a token which is a last-kin of the token with the highest priority”}$,
- $r_{a_5,a_{11}} = r_{a_8,a_{11}} = \text{“}\underline{\text{card}}(x_{last}^\alpha) = 0\text{”} \& \neg r_{a_5,a_{10}}$,

and

$$M_3 = \begin{array}{c|cccc} & a_8 & a_9 & a_{10} & a_{11} \\ \hline a_5 & \infty & \infty & \infty & \infty \\ a_8 & \infty & \infty & \infty & 0 \end{array}.$$

The tokens do not obtain any characteristic in places a_8 and a_{10} and they obtain the characteristics

$$\text{“}\langle \text{pr}_1 x_{last}^\alpha; \underline{\text{nc}}(l_8, \alpha) + 1 \rangle, (\underline{\text{nc}}(l_8, \alpha) + 1)\text{-th element of the set } \text{pr}_2 x_{last}^\alpha \text{”}$$

in place a_9 and $\text{“}\langle \text{pr}_1 x_{last}^\alpha, ! \text{pr}_2 x_{last}^\alpha \rangle\text{”}$ in place a_{11} .

We must note that the output place priorities must satisfy the following inequality:

$$\pi_L(a_{11}) > \pi_L(a_{10}) > \pi_L(a_9) > \pi_L(a_8).$$

The fourth transition has the form:

$$Z_4 = \langle \{a_6\}, \{a_{12}, a_{13}\}, r_4, M_4, \vee(a_6) \rangle,$$

where

$$r_4 = \begin{array}{c|cc} & a_{12} & a_{13} \\ \hline a_6 & r_{a_6,a_{12}} & r_{a_6,a_{13}} \end{array},$$

where

- $r_{a_6,a_{12}} = \text{“there are tokens outside places } a_{14}, a_{15}, \dots, a_{18}\text{”}$,
- $r_{a_6,a_{13}} = \neg r_{a_6,a_{12}}$

and

$$M_4 = \begin{array}{c|cc} & a_{12} & a_{13} \\ \hline a_6 & \infty & \infty \end{array}.$$

The tokens do not obtain any characteristic in place a_{12} . They obtain the characteristic $\text{“}\langle \text{pr}_a x_{last}^\alpha, \neg ! \text{pr}_2 x_0^\alpha \rangle\text{”}$ in place a_{13} .

The fifth transition has the form:

$$Z_5 = \langle \{a_{11}, a_{14}, a_{17}, b_4, b_7\}, \{a_{14}, a_{15}, a_{16}, b_6, b_7\}, r_5, M_5, \square_5 \rangle,$$

where

$r_5 =$		a_{14}	a_{15}	a_{16}	b_6	b_7
a_{11}		$r_{a_{11},a_{14}}$	$r_{a_{11},a_{15}}$	$r_{a_{11},a_{16}}$	<i>false</i>	<i>false</i>
a_{14}		$r_{a_{14},a_{14}}$	$r_{a_{14},a_{15}}$	$r_{a_{14},a_{16}}$	<i>false</i>	<i>false</i>
a_{17}		$r_{a_{17},a_{14}}$	$r_{a_{17},a_{15}}$	$r_{a_{17},a_{16}}$	<i>false</i>	<i>false</i>
b_4		<i>false</i>	<i>false</i>	<i>false</i>	r_{b_4,b_6}	r_{b_4,b_7}
b_7		<i>false</i>	<i>false</i>	<i>false</i>	r_{b_7,b_6}	r_{b_7,b_7}

where

- $r_{a_{11},a_{14}} = r_{a_{14},a_{14}} = r_{a_{17},a_{14}} =$ “all last homogeneous kins of the token are in places $a_4, a_5, a_8, a_9, a_{11}$ or a_{14} and there are no last homogeneous kins in places a_6, a_{12} or a_{13} ”,
- $r_{a_{11},a_{15}} = r_{a_{14},a_{15}} = r_{a_{17},a_{15}} =$ “the token does not have last homogeneous kins”,
- $r_{a_{11},a_{16}} = r_{a_{14},a_{16}} = r_{a_{17},a_{16}} =$ “the token has last homogeneous kins in places a_6, a_{12} or a_{13} ”,
- $r_{b_4,b_6} = r_{b_7,b_6} =$ “all interior a -places, with the possible exception of places a_{15} and a_{17} are empty”,
- $r_{b_4,b_7} = r_{b_7,b_7} = \neg r_{b_4,b_6}$,

and

$M_5 =$		a_{14}	a_{15}	a_{16}	b_6	b_7
a_{11}		∞	∞	∞	0	0
a_{14}		∞	∞	∞	0	0
a_{17}		∞	∞	∞	0	0
b_4		0	0	0	1	1
b_7		0	0	0	1	1

and

$$\Box_5 = \wedge(\vee(a_{11}, a_{14}, a_{17}), \vee(b_4, b_7)).$$

All last kins merge in place a_{14} and the resulting token obtains no characteristic; the tokens obtain the characteristics “ $\langle \text{pv}(\text{pv}(\text{pr}_1 x_{last}^\alpha)), !\text{pr}_2 x_{last-2}^\alpha \rangle$ ” in place a_{15} and “ $x_{last}^\beta \cup \{\text{pr}_2 x_{last-2}^\alpha\}$ ” in place b_7 and they do not obtain any characteristic in places a_{16} and b_6 .

We must note that the β -token obtains the above mentioned characteristic in place b_7 which symbolises that the new (local) fact is added to the DB, only if this extension of the DB is possible. Otherwise, the β -token will not obtain any characteristic in place b_7 .

$$Z_6 = \langle \{a_{15}, b_6, c_4\}, \{a_{17}, a_{18}, b_8, b_9, c_6, c_7\}, r_6, M_6, \wedge(a_{15}, b_6, c_4) \rangle,$$

where

$r_6 =$		a_{17}	a_{18}	b_8	b_9	c_6	c_7
a_{15}		$r_{a_{15},a_{17}}$	$r_{a_{15},a_{18}}$	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
b_6		<i>false</i>	<i>false</i>	r_{b_6,b_8}	r_{b_6,b_9}	<i>false</i>	<i>false</i>
c_4		<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	r_{c_4,c_6}	r_{c_4,c_7}

where

- $r_{a_{15},a_{17}} = r_{b_6,b_9} = r_{c_4,c_7} = \neg r_{a_{15},a_{18}},$
- $r_{a_{15},a_{18}} = r_{b_6,b_8} = r_{c_4,c_6} = \text{“pr}_1 x_{last}^\alpha = \text{pr}_1 x_0^\alpha\text{”}$

and

$$M_6 = \begin{array}{c|cccccc} & a_{17} & a_{18} & b_8 & b_9 & c_6 & c_7 \\ \hline a_{15} & \infty & \infty & 0 & 0 & 0 & 0 \\ b_6 & 0 & 0 & \infty & \infty & 0 & 0 \\ c_4 & 0 & 0 & 0 & 0 & \infty & \infty \end{array}.$$

The tokens do not obtain any characteristic in places $l_{a_{17}}, b_8, b_9, c_6, c_7$ and they obtain the characteristic “ $\langle x_{last}^\alpha, !\text{pr}_2 x_0^\alpha \rangle$ ” in place $l_{a_{18}}$.

The GN described here has the following universal property: it does not depend on the particular modelled production system. The only constraint posed on it is the above-mentioned condition concerning the type of the rule – namely, that the members of the antecedents of the ES-rules must be conjunctions of positive variables.

3 A GN-model of an ES with priorities of its facts

The GN described below (in [3] it is called as Fifth GN-model) will be considered as an extension of the above net. All notations from there are kept here so that to allow the reader to trace the development of the idea of the GN-modelling of ESs.

On the other hand, we introduce a new idea related to the ESs, based on the GN-description capabilities. The main difference is that now we will couple each rule with a priority. Therefore, every fact will have the form $\langle \text{“fact”}, \text{“its priority”} \rangle$ or, more formally, $\langle A, p_A \rangle$, where $A \in \Delta$ is a fact and p_A is a natural number – its priority.

Now two facts can be compared by their priorities. This possibility is very useful in the particular cases when the two facts coincide or when they are controversial.

Let us assign to every fact A of the DB Δ a natural number p_A that corresponds to the priority of A . Let a new fact B with a priority p_B be generated in some way at a certain time-moment of the ES functioning. If both facts are not related, then the new fact can enter the DB. In the ordinary ESs, the new fact B substitutes the old fact A , when B coincides with, or contradicts to A . Now the ES will function in another way, based on the new component. When the facts A and B coincide, their representative (A or B) remains in the DB Δ , but with a new priority – the maximum of p_A and p_B .

On the other hand, when the facts A and B are in a contradiction, the fact with the maximum priority between p_A and p_B remains in the DB.

The GN constructed here (see Figure 2) represents the ESs with data priorities in the above sense.

In this form of the GN-model there is a danger of accepting facts which will violate the correct functioning of the ES’s process. For this reason, another modification of the above GN-model is discussed in [3] which avert conflicts on a logical level.

The Fifth GN-model from [3] contains as a subnet Third GN-model (described above).

Let token α enter place a_1 , token β enter place b_1 and token γ enter place c_1 of the GN with initial characteristics as above.

The difference between this and the previous net is in the new type of tokens which enter the net during its functioning.

Let these tokens be $\delta_1, \delta_2, \dots, \delta_s$ ($s \geq 0$) and let them enter place d_1 with initial characteristics

$$x_0^{\delta_{cu}} = \langle \text{"new fact"}, \text{"its priority"} \rangle,$$

where, here and below, "*cu*" symbolizes the current number of the δ -type token which enters the GN.

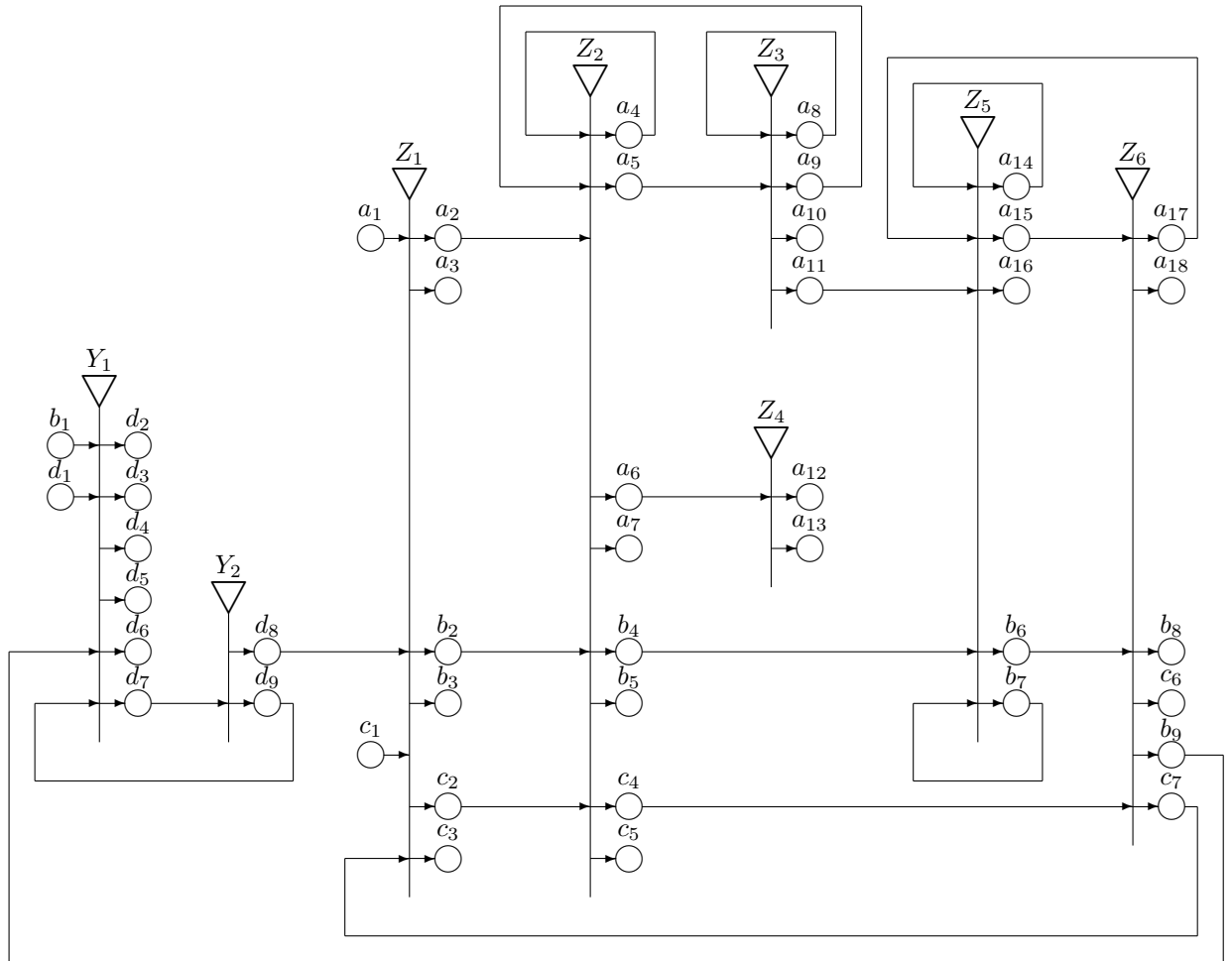


Figure 2. A GN-model of an ES with priorities of its facts

The essentially new transitions in the GN are described below. We will also describe these of the transitions from the previous GN which have some modifications, keeping, where appropriate, the notation from the Third GN-model.

The new transitions (those which do not have analogues in the previous nets) will be named Y_1 and Y_2 . For the needs of the subsequent GN-models later in this chapter, we must mention that these two transitions constitute a separate subnet of the present GN-model.

$$Y_1 = \langle \{b_1, b_9, d_1, d_9\}, \{d_2, d_3, d_4, d_5, d_6, d_7\}, r_{Y_1}, M_{Y_1}, \square_{Y_1} \rangle,$$

where

	d_2	d_3	d_4	d_5	d_6	d_7
$r_{Y_1} =$						
b_1	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
b_9	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
d_1	r_{d_1, d_2}	r_{d_1, d_3}	r_{d_1, d_4}	r_{d_1, d_5}	r_{d_1, d_6}	<i>false</i>
d_9	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>

where

- $r_{d_1, d_2} = \text{"pr}_1 x_0^{\delta_{cu}} \in x_{last}^\beta \text{"} \& \text{"pr}_2 x_0^{\delta_{cu}} \geq Q(\text{pr}_1 x_0^{\delta_{cu}}) \text{"}$, where $Q(A)$ is the priority of fact A of the point of view of the present status of the DB, i.e.,

$$Q(A) = \begin{cases} p_A, & \text{if } A \in x_{last}^\beta \\ 0, & \text{otherwise} \end{cases}$$

- $r_{d_1, d_3} = \text{"pr}_1 x_0^{\delta_{cu}} \in x_{last}^\beta \text{"} \& \text{"pr}_2 x_0^{\delta_{cu}} < Q(\text{pr}_1 x_0^{\delta_{cu}}) \text{"}$,
- $r_{d_1, d_4} = \text{"}\neg \text{pr}_1 x_0^{\delta_{cu}} \in x_{last}^\beta \text{"} \& \text{"pr}_2 x_0^{\delta_{cu}} \geq Q(\neg \text{pr}_1 x_0^{\delta_{cu}}) \text{"}$, where, as usual, $\neg P$ is the negation of the fact P ,
- $r_{d_1, d_5} = \text{"}\neg \text{pr}_1 x_0^{\delta_{cu}} \in x_{last}^\beta \text{"} \& \text{"pr}_2 x_0^{\delta_{cu}} < Q(\neg \text{pr}_1 x_0^{\delta_{cu}}) \text{"}$,
- $r_{d_1, d_6} = \text{"pr}_1 x_0^{\delta_{cu}} \notin x_{last}^\beta \text{"} \& \text{"}\neg \text{pr}_2 x_0^{\delta_{cu}} \notin x_{cu}^\beta \text{"}$,

and

	d_2	d_3	d_4	d_5	d_6	d_7
$M_{Y_1} =$						
b_1	0	0	0	0	0	1
b_9	0	0	0	0	0	1
d_1	1	1	1	1	1	0
d_9	0	0	0	0	0	1

and

$$\square_{Y_1} = \vee(b_1, b_9, \wedge(d_1, \vee(b_1, b_9, d_9))).$$

The tokens obtain the characteristic

$$\begin{cases} \text{"}(x_{last}^\beta - \{\langle \text{pr}_1 x_0^{\delta_{cu}}, Q(\text{pr}_1 x_0^{\delta_{cu}}) \rangle\}) \cup \{x_0^{\delta_{cu}}\}\text{"}, & \text{if the token } \delta_{cu} \text{ is in place } d_2 \\ *, & \text{if the token } \delta_{cu} \text{ is either in } d_3 \text{ or } d_5 \\ \text{"}(x_{last}^\beta - \{\langle \text{pr}_1 x_0^{\delta_{cu}}, Q(\neg \text{pr}_1 x_0^{\delta_{cu}}) \rangle\}) \cup \{x_0^{\delta_{cu}}\}\text{"}, & \text{if the token } \delta_{cu} \text{ is in place } d_4 \\ \text{"}x_{last}^\beta \cup \{x_0^{\delta_{cu}}\}\text{"}, & \text{if the token } \delta_{cu} \text{ is in place } d_6 \end{cases}$$

in place d_7 , and they do not obtain any characteristic in the other output places.

$$Y_2 = \langle \{d_7\}, \{d_8, d_9\}, r_{Y_2}, M_{Y_2}, \vee(d_7) \rangle,$$

where

	d_8	d_9
$r_{Y_2} =$		
d_7	$r_{46,47}$	$r_{46,48}$

where

- $r_{d_7, d_8} = \text{"}\bar{c}(d_1, TIME) = 0\text{"}$,
- $r_{d_7, d_9} = \text{"}\bar{c}(d_1, TIME) > 0\text{"}$,

where $\bar{c}(l, TIME)$ is the number of the tokens in place l at the current time-moment $TIME$. Moreover, $TIME$ can denote the current step of the functioning of the GN-models and

$$M_{Y_2} = \frac{\quad}{d_7} \left| \begin{array}{cc} d_8 & d_9 \\ 1 & 1 \end{array} \right.$$

The tokens do not obtain any characteristic in places d_8 and d_9 . The next transition has the form:

$$Z_1 = \langle \{a_1, c_1, c_7, d_8\}, \{a_2, a_3, b_2, b_3, c_2, c_3\}, r_1, M_1, \square_1 \rangle,$$

where

$r_1 =$		a_2	a_3	b_2	b_3	c_2	c_3
a_1		r_{a_1, a_2}	r_{a_1, a_3}	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
c_1		<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	r_{c_1, c_2}	r_{c_1, c_3}
c_7		<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	r_{c_7, c_2}	r_{c_7, c_3}
d_8		<i>false</i>	<i>false</i>	r_{d_8, b_2}	r_{d_8, b_3}	<i>false</i>	<i>false</i>

where

- $r_{a_1, a_2} = r_{c_1, c_2} = r_{c_7, c_2} = r_{d_8, b_2} = \text{"}\text{pr}_2 x_0^\alpha \notin x_0^\beta\text{"}$,
- $r_{a_1, a_3} = \neg r_{a_1, a_2}$,
- $r_{c_1, c_3} = r_{c_7, c_3} = r_{d_8, b_3} = \neg r_{a_1, a_2} \& \text{"there are no new } \alpha\text{-tokens before place } a_1\text{"}$,

and

$M_1 =$		a_3	b_2	b_3	c_2	c_3	f_1
a_1		1	0	0	0	0	1
c_1		0	0	0	1	1	0
c_7		0	0	0	1	1	0
d_8		0	1	1	0	0	0

and

$$\square_1 = \wedge(a_1, d_8, \vee(c_1, c_7)).$$

The tokens obtain the characteristic " $\text{pr}_1 x_0^\alpha, !\text{pr}_2 x_0^\alpha$ " in place a_3 and they do not obtain any characteristic in the other output places.

$$Z_2 = \langle \{a_2, a_9, b_2, c_2\}, \{a_5, b_4, b_5, c_4, c_5\}, r_2, M_2, \square_2 \rangle,$$

where

$r_2 =$		a_5	b_4	b_5	c_4	c_5
a_2		<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
a_9		<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
b_2		<i>false</i>	r_{b_2, b_4}	r_{b_2, b_5}	<i>false</i>	<i>false</i>
c_2		<i>false</i>	<i>false</i>	<i>false</i>	r_{c_2, c_4}	r_{c_2, c_5}

where

- $r_{b_2, b_4} = r_{c_2, c_4} = \text{“in places } a_8 \text{ or } a_9 \text{ of the subnet } E' \text{ there are } \alpha\text{-tokens”}$,
- $r_{b_2, b_5} = r_{c_2, c_5} = \neg r_{b_2, b_4}$,

and

$$M_2 = \begin{array}{c|ccccc} & a_5 & b_4 & b_5 & c_4 & c_5 \\ \hline a_2 & 1 & 0 & 0 & 0 & 0 \\ a_9 & 1 & 0 & 0 & 0 & 0 \\ b_2 & 0 & 1 & 1 & 0 & 0 \\ c_2 & 0 & 0 & 0 & 1 & 1 \end{array}$$

and

$$\square_2 = \wedge(\vee(a_2, a_9), b_2, c_2).$$

The tokens do not obtain any characteristic in those output places of transition Z_2 which are not E' -places.

The transitions Z_3 and Z_4 are components of the subnet E' while transitions Z_5 and Z_6 belong to both the set of the present GN's transitions and the set of the transitions of the subnet E' . They have the form of the corresponding transitions from Third GN-model and for this reason we do not discuss them.

The sixth GN-model of an ES, described in [3] is similar, but related to the rules with priorities. Its form is similar. Of course, we construct directly a new GN that is a union of both (fifth and sixth) GN-models.

In [3], a GN-model of an IFES is given, too.

4 A new (Tenth) GN-model of ESs

The GN-models, described in [3], are extended here to a new – Tenth, GN-model with means for taking into account temporal parameters of the facts.

Here, we shall construct a new GN-model that includes as a partial case the Fifth, Sixth, Eighth and Ninth GN-models from [3]. The most important characteristic of the new model is that the new facts enter the ES at the time of its functioning and the moments of their addition to the database are recorded. Thus, it will be possible to answer questions related to some temporal logical operators. While in the Ninth GN-model there are only two of the four basic temporal logic operators: “always” \mathcal{A} and “once” \mathcal{O} and two additional temporal operators – “sometimes” \mathcal{S} and “at the currently” \mathcal{C} , which do not have analogues in the temporal logic, here we shall increase these operators essentially.

Let T be a fixed set of real numbers which we shall call “time-scale” and it is strictly oriented by the relation “ $<$ ”.

Let p be a proposition and V be a truth-value function, which maps the ordered pair:

$$V(p, t) = \langle \mu(p, t), \nu(p, t) \rangle$$

to the proposition p and to the time-moment $t \in T$.

As it is mentioned in Section 1.1, we note that proposition p with intuitionistic fuzzy values $\langle a, b \rangle$ is called an “*Intuitionistic Fuzzy Tautology*” (IFT), if and only if $a \geq b$.

Let $x \in E$ be a fixed proposition and $A \subset E$, where here and below E is a set of propositions. Firstly, we shall introduce one new (for the IFS theory) operator as follows:

$$\tau(A(T), x) = \{t \mid \mu_A(x, t) > \nu_A(x, t) \ \& \ t \in T\}.$$

Obviously, for all $x \in E$:

$$\emptyset \subset \tau(A(T), x) \subset T.$$

For x we can assert that it is “*Intuitionistic Fuzzy Valid*” (IFV) in time-moment t , if and only if

$$\mu_A(x, t) \geq \nu_A(x, t). \quad (*)$$

Numbers $\mu_A(x, t)$ and $\nu_A(x, t)$ can be respectively interpreted as a “*degree of validity*” and a “*degree of non-validity*”.

Let us assume that in E for each element x there exists an element $\neg x$ and let for it be valid:

$$\tau(A(T), \neg x) = \{t \mid \nu_A(x, t) > \mu_A(x, t) \ \& \ t \in T\}.$$

Therefore, the predicate $\varphi(x) = “x \text{ has always been true}”$ will be IFV, if $(*)$ holds for all $t \in T$. Obviously, φ coincide with the above mentioned operator \mathcal{A} .

By similarity, we can define the following predicates, too:

$$\begin{aligned} \psi(x) &= “x \text{ has sometime been true, but not always}”, \\ \chi(x) &= “once x was true”, \\ \omega(x) &= “x \text{ has never been true}”. \end{aligned}$$

Obviously, χ coincide with the above mentioned operator \mathcal{O} . It can be easily seen that:

$$\begin{aligned} \varphi(x) &= 1, \text{ if and only if } \tau(A(T), x) = T, \\ \psi(x) &= 1, \text{ if and only if } \emptyset \neq \tau(A(T), x) \neq T, \text{ and} \\ &\quad (\exists t_1, t_2 \in \tau(A(T), x))(\exists t_3 \in T - \tau(A(T), x))(t_1 < t_3 < t_2), \\ \chi(x) &= 1, \text{ if and only if } \emptyset \neq \tau(A(T), x) \neq T, \text{ and} \\ &\quad (\forall t_1, t_2 \in \tau(A(T), x))(\neg \exists t_3 \in T - \tau(A(T), x))(t_1 < t_3 < t_2), \\ \omega(x) &= 1, \text{ if and only if } \tau(A(T), x) = \emptyset. \end{aligned}$$

All the above predicates $\varphi, \psi, \chi, \omega$ have values in set $\{0, 1\}$. Now, we can construct their IFVs.

Let below $\text{card}(X)$ be the cardinality of set X . Therefore, for the fixed elements $x \in X$ we can define the couple

$$\rho(x) = \left\langle \frac{\text{card}(\tau(A(X), x))}{\text{card}(T)}, \frac{\text{card}(\tau(A(X), \neg x))}{\text{card}(T)} \right\rangle.$$

It is an intuitionistic fuzzy couple, because

$$0 \leq \frac{\text{card}(\tau(A(X), x))}{\text{card}(T)} + \frac{\text{card}(\tau(A(X), \neg x))}{\text{card}(T)} \leq 1.$$

The second inequality will become an equality, if there was no time moment when for x : $\mu_A(x, t) = \nu_A(x, t)$. The set of all time-moments for which the later equality is not valid (let us denote it by Δ_x) determines the “*degree of uncertainty*” for x , and of course,

$$\frac{\text{card}(\tau(A(X), x))}{\text{card}(T)} + \frac{\text{card}(\tau(A(X), \neg x))}{\text{card}(T)} + \frac{\Delta_x}{\text{card}(T)} = 1.$$

We can define the following two new predicates

$$\begin{aligned} \xi(x) &= 1, \text{ if and only if } \rho(x) \text{ is an IFT,} \\ \sigma(x) &= 1, \text{ if and only if } \rho(\neg x) \text{ is an IFT.} \end{aligned}$$

These predicates can be interpreted as follows:

$$\begin{aligned} \xi(x) &= \text{“}x \text{ is often true”}, \\ \sigma(x) &= \text{“}x \text{ is rarely true”}. \end{aligned}$$

These two predicates can be generalized. For example, we can use the two real numbers $\lambda, \mu \in [0, 1]$ and we can define that $\langle \lambda, \mu \rangle$ is (λ, μ) -IFT if and only if $a \geq \lambda$ and $b \leq \mu$. Then

$$\begin{aligned} \xi^*(x) &= \text{“}x \text{ is } (\lambda, \mu)\text{-often true”}, \\ \sigma^*(x) &= \text{“}x \text{ is } (\lambda, \mu)\text{-rarely true”}. \end{aligned}$$

For them there will hold

$$\begin{aligned} \xi^*(x) \text{ is } (\lambda, \mu)\text{-often if and only if } \mu(\rho(x)) \geq \lambda \ \&\ \nu(\rho(x)) \leq \mu, \\ \sigma^*(x) \text{ is } (\lambda, \mu)\text{-rarely if and only if } \mu(\rho(\neg x)) \geq \lambda \ \&\ \nu(\rho(\neg x)) \leq \mu. \end{aligned}$$

Now, we shall return to the GN-model. We must note that the present model can be used as a basis for an extension of the Eighth GN-model from [3], where ESs are endowed with elements of intuitionistic fuzzy logic.

Our remarks concerning the way of modifying the DB of the ES modelled by the fifth GN, described in the previous section still hold here.

Let the new GN have a special transition Ω having only one (which is both an input and an output) place o where there is only one token ω . Let ω have an initial characteristic “ T ”, the time moment when the GN starts its functioning and let the characteristic function related to this place Φ_ω be defined in such a way that it gives the value: “ $x_{last}^\omega + t^o$ ”, where t^o is the elementary time step in the GN-model. In some cases it is convenient to assume that $t^o = 1$.

Let the token α enter place a_1 , the token β enter place b_1 and the token γ enter place c_1 of the GN with initial characteristics as in the Third GN-model mentioned above.

Unlike all models so far, the facts of DB Δ at the initial moment of the GN functioning will have the form:

$$\langle \text{"fact"}, \text{"its priority"}, \langle \langle T, * \rangle, * \rangle \rangle,$$

or more formally:

$$\langle A, p_A, \langle \langle T, * \rangle, * \rangle \rangle,$$

where $A \in \Delta$ is a fact and p_A is a natural number corresponding to its priority.

The last (third) component contains one or more ordered pairs. The first component of each pair is the time moment when the fact was stored in the DB (for this reason, the facts of the initial DB have a T component – such facts will be called “active”) and the second component of each pair is the time-moment of its deletion from the DB – now the fact is called “passive”). The second pair (if it exists) corresponds to the case when the fact enters the DB for a second time, etc. In this way we can trace the intervals of the facts’ existence in the DB.

Let tokens $\delta_1, \delta_2, \dots, \delta_s$ ($s \geq 0$) enter place d_1 with initial characteristics

$$\langle A_{cu}, p_{A,cu}, \theta_K(\delta_{cu}) \rangle,$$

where A is a fact, p_A is its priority (a natural number), “ cu ” symbolises the current number of the δ -type token which enters the GN and θ_K is the function which determines the time-moments at which the tokens enter the net.

In some cases it is appropriate to define the values of the function θ_K with respect to the internal GN-time.

With the aim of allowing the δ -tokens to obtain as a third component of their characteristic the time-moment of their entrance, we shall define that transition Ω has the highest priority among all GN-transitions. Thus, token ω will be the first one that will obtain a new characteristic among all other GN-tokens; moreover, all other tokens which must obtain the current time-moment as a current characteristic, will obtain it, using the newly obtained ω ’s characteristic.

The new GN (see Figure 3) will contain transitions Y_1 and Y_2 , coinciding with their counterparts from the Fifth GN-model of ESs, described above and representing here by subnet E_Y . The subnet E_Z corresponds to the set of transitions in the Sixth GN-model of ESs that, as was mentioned above, is similar to E_Y . The only differences will be in the form of the characteristic functions.

Here the tokens obtain the characteristics:

$$\left\{ \begin{array}{ll} \text{"}(x_{last}^\beta - \{\langle pr_1 x_0^{\delta_{cu}}, Q(pr_1 x_0^{\delta_{cu}}), S(pr_1 x_0^{\delta_{cu}}) \rangle\} \cup \{\langle pr_{1,2} x_0^{\delta_{cu}}, S(pr_1 x_0^{\delta_{cu}}) \rangle\})\text{"}, & \text{if the token } \delta_{cu} \text{ is in place } d_2 \\ * , & \text{if } \delta_{cu} \text{ is in either } d_3 \text{ or } d_5 \\ \text{"}(x_{last}^\beta - \{\langle \neg pr_1 x_0^{\delta_{cu}}, Q(\neg pr_1 x_0^{\delta_{cu}}), S(\neg pr_1 x_0^{\delta_{cu}}) \rangle\} \cup \{\langle pr_{1,2}, x_0^{\delta_{cu}}, \langle S(x_0^{\delta_{cu}}, \langle x_{last}^\omega, * \rangle), \langle \neg pr_1 x_0^{\delta_{cu}}, Q(\neg pr_1 x_0^{\delta_{cu}}), \langle LS(\neg pr_1 x_0^{\delta_{cu}}), \langle pr_1 RS(\neg pr_1 x_0^{\delta_{cu}}), x_{last}^\omega \rangle \rangle \rangle\})\text{"}, & \text{if } \delta_{cu} \text{ is in place } d_4 \\ \text{"}x_{last}^\beta \cup \{pr_{1,2} x_0^{\delta_{cu}}, \langle x_{last}^\omega, * \rangle\}\text{"}, & \text{if the token } \delta_{cu} \text{ is in place } d_6 \end{array} \right.$$

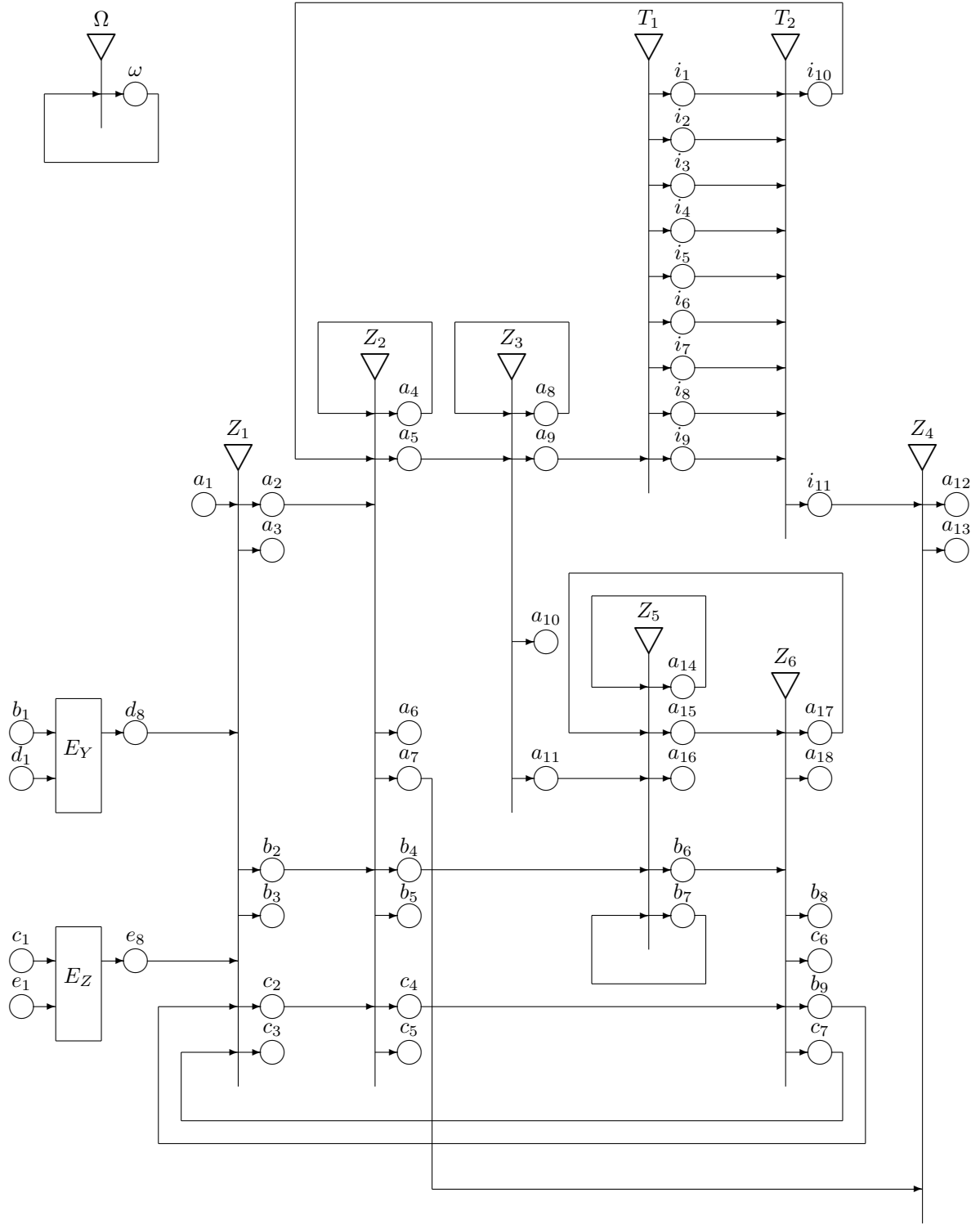


Figure 3. Tenth GN-model of ESs

in place d_7 , and they do not obtain any characteristic in the other output places. In these places (we assume that A is an ordered triple of the above form):

$$S(A) = \begin{cases} \text{pr}_3 A, & \text{if } A \in \text{pr}_1 x_{last}^\beta, \\ *, & \text{otherwise} \end{cases},$$

where $*$ stands for the empty value, and

$$\begin{aligned} LS(A) &= \text{pr}_{1,2,\dots,s-1} S(A), \\ RS(A) &= \text{pr}_s S(A), \end{aligned}$$

if $S(A)$ is an s -dimensional vector.

We must note that it is possible that some δ -tokens split and enter two output d -places. This situation corresponds to the case when fact $\text{pr}_1 x_0^{\delta_{cu}}$ (or its negation) is currently active in the DB and at the same time, its negation (or its positive form, respectively), is now passive but had been active in the DB at some previous moment.

Transitions Z_1 and Z_3 also coincide with their counterparts from the Third and the Fifth GN-models. However now there are minor changes in some predicates and tokens characteristics.

For transition Z_1 , the changes are in the forms of the following predicates:

$$r_{a_1,a_2} = r_{c_1,c_2} = r_{c_7,c_2} = r_{d_8,b_2} = "(\text{pr}_1 x_0^\alpha \notin \text{pr}_1 x_0^\beta)" \vee "(RS(\text{pr}_1 x_0^\alpha) \neq "*")"$$

and for transition Z_3 the change is in the characteristic that the tokens receive in place a_9 :

$$"(\langle \text{pr}_1 x_{last}^\alpha; \underline{\text{nc}}(a_8, \alpha) + 1 \rangle, \text{pr}_2 x_{last}^\alpha [\underline{\text{nc}}(a_8, \alpha) + 1], S(\text{pr}_2 x_{last}^\alpha [\underline{\text{nc}}(a_8, \alpha) + 1]))",$$

where $X[k]$ is the k -th element of the set X .

The transition Z_4 now has the form:

$$Z_4 = \langle \{a_7, i_{11}\}, \{a_{12}, a_{13}\}, r_4, M_4, \vee(a_7, i_{11}) \rangle,$$

where

$$r_4 = \begin{array}{c|cc} & a_{12} & a_{13} \\ \hline a_7 & r_{a_7,a_{12}} & r_{a_7,a_{13}} \\ i_{11} & r_{i_{11},a_{12}} & r_{i_{11},a_{13}} \end{array},$$

where

- $r_{a_7,a_{12}} = r_{i_{11},a_{12}} = \text{"there are tokens outside places } a_{14}, a_{15}, \dots, a_{18}"$,
- $r_{a_7,a_{13}} = r_{i_{11},a_{13}} = \neg r_{a_7,a_{12}}$

and

$$M_4 = \begin{array}{c|cc} & a_{12} & a_{13} \\ \hline a_7 & \infty & \infty \\ i_{11} & \infty & \infty \end{array}.$$

The tokens do not obtain any characteristic in place a_{12} and they obtain the characteristic

$$"(\text{pr}_1 x_0^\alpha, \neg! \text{pr}_2 x_0^\alpha)"$$

in place a_{13} .

Up to now we only considered simple variables in the hypotheses, i.e., not prefixed by signs. Now we assume that variables may be prefixed by one of the signs $\varphi, \psi, \chi, \omega, \xi, \sigma, \xi^*$ and σ^* .

We must note, that if we would like to use temporal operators ξ^* and σ^* , we must have their temporal parameters λ and μ . Let us assume that they are fixed for a concrete ES, or that they are given as a characteristic of α -token in place a_9 , as follows (cf. the above α -characteristic):

$$“\langle \langle \text{pr}_1 x_{last}^\alpha; \lambda, \mu; \underline{\text{nc}}(a_8, \alpha) + 1 \rangle, \text{pr}_2 x_{last}^\alpha[\underline{\text{nc}}(a_8, \alpha) + 1], S(\text{pr}_2 x_{last}^\alpha[\underline{\text{nc}}(a_8, \alpha) + 1]) \rangle”.$$

The new transitions are two described as follows.

$$T_1 = \langle \{a_9\}, \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9\}, r_{T_1}, M_{T_1}, \vee(a_9) \rangle,$$

where

$$r_{T_1} = \begin{array}{c|cccccccccc} & i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 & i_9 \\ \hline a_9 & r_{a_9, i_1} & r_{a_9, i_2} & r_{a_9, i_3} & r_{a_9, i_4} & r_{a_9, i_5} & r_{a_9, i_6} & r_{a_9, i_7} & r_{a_9, i_8} & r_{a_9, i_9} \end{array}$$

where

- r_{a_9, i_1} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the φ operator”,
- r_{a_9, i_2} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the ψ operator”,
- r_{a_9, i_3} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the χ operator”,
- r_{a_9, i_4} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the ω operator”,
- r_{a_9, i_5} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the ξ operator”,
- r_{a_9, i_6} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the σ operator”,
- r_{a_9, i_7} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the ξ^* operator”,
- r_{a_9, i_8} = “the variable $\text{pr}_2 x_{last}^\alpha$ is prefixed by the σ^* operator”,
- $r_{a_9, i_9} = \neg r_{a_9, i_1} \& \neg r_{a_9, i_2} \& \neg r_{a_9, i_3} \& \neg r_{a_9, i_4} \& \neg r_{a_9, i_5} \& \neg r_{a_9, i_6} \& \neg r_{a_9, i_7} \& \neg r_{a_9, i_8}$

and all elements of M_{T_1} are equal to ∞ .

The tokens obtain the characteristics, respectively:

- in place i_1 :

$$\begin{cases} “\langle ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{if } \text{pr}_1 RS(\text{pr}_1 x_{last}^\alpha) = x_0^\omega \text{ and } \text{pr}_2 RS(\text{pr}_1 x_{last}^\alpha) = * \\ “\langle \neg ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{otherwise} \end{cases}$$

- in place i_2 :

$$\begin{cases} “\langle ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{if } \text{pr}_1 x_{last}^\alpha \in \text{pr}_1 x_{last}^\beta \\ “\langle \neg ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{otherwise} \end{cases}$$

- in place i_3 :

$$\begin{cases} “\langle ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{if } (\exists s \geq 2)(\exists u_1, u_2, \dots, u_s \in N)(T \leq \text{pr}_1 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha)) \\ & \leq \text{pr}_2 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \leq \dots \leq \text{pr}_1 \text{pr}_{u_s} S(\text{pr}_1 x_{last}^\alpha) \\ “\langle \neg ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{otherwise} \end{cases}$$

- in place i_4 :

$$\begin{cases} “\langle ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{if } \text{pr}_2 RS(\text{pr}_1 x_{last}^\alpha) = * \\ “\langle \neg ! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle”, & \text{otherwise} \end{cases}$$

- in place i_5 :

$$\left\{ \begin{array}{l} \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{if } (\exists s \geq 2)(\exists u_1, u_2, \dots, u_s \in N)((T \leq \text{pr}_1 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \\ \leq \text{pr}_2 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \leq \dots \leq \text{pr}_1 \text{pr}_{u_s} S(\text{pr}_1 x_{last}^\alpha)) \\ \& (\sum_{i=1}^s (\text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_1 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha)) \\ \geq \sum_{i=1}^{s-1} (\text{pr}_1 \text{pr}_{u_{i+1}} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha))) \\ \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{otherwise} \end{array} \right.$$

- in place i_6 :

$$\left\{ \begin{array}{l} \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{if } (\exists s \geq 2)(\exists u_1, u_2, \dots, u_s \in N)((T \leq \text{pr}_1 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \\ \leq \text{pr}_2 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \leq \dots \leq \text{pr}_1 \text{pr}_{u_s} S(\text{pr}_1 x_{last}^\alpha)) \\ \& (\sum_{i=1}^s (\text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_1 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha)) \\ \leq \sum_{i=1}^{s-1} (\text{pr}_1 \text{pr}_{u_{i+1}} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha))) \\ \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{otherwise} \end{array} \right.$$

- in place i_7 :

$$\left\{ \begin{array}{l} \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{if } (\exists s \geq 2)(\exists u_1, u_2, \dots, u_s \in N)((T \leq \text{pr}_1 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \\ \leq \text{pr}_2 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \leq \dots \leq \text{pr}_1 \text{pr}_{u_s} S(\text{pr}_1 x_{last}^\alpha)) \\ \& (\sum_{i=1}^s (\text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_1 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha)) > \lambda \\ \& \sum_{i=1}^{s-1} (\text{pr}_1 \text{pr}_{u_{i+1}} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha) < \mu)) \\ \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{otherwise} \end{array} \right.$$

- in place i_8 :

$$\left\{ \begin{array}{l} \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{if } (\exists s \geq 2)(\exists u_1, u_2, \dots, u_s \in N)((T \leq \text{pr}_1 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \\ \leq \text{pr}_2 \text{pr}_{u_1} S(\text{pr}_1 x_{last}^\alpha) \leq \dots \leq \text{pr}_1 \text{pr}_{u_s} S(\text{pr}_1 x_{last}^\alpha)) \\ \& (\sum_{i=1}^s (\text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_1 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha)) < \lambda \\ \& \sum_{i=1}^{s-1} (\text{pr}_1 \text{pr}_{u_{i+1}} S(\text{pr}_1 x_{last}^\alpha) - \text{pr}_2 \text{pr}_{u_i} S(\text{pr}_1 x_{last}^\alpha) > \mu)) \\ \langle \neg! \text{pr}_1 x_{last}^\alpha, \text{pr}_{2,3} x_{last}^\alpha \rangle, \quad \text{otherwise} \end{array} \right.$$

- and in place i_9 they do not obtain any characteristic.

We must note that there are the following two possibilities for the characteristic of the token in place i_3 about $\text{pr}_2 \text{pr}_{u_s} S(\text{pr}_1 x_{last}^\alpha)$: (a) it is $= *$ or (b) it is $= \tau$, where $T \leq \tau \leq T + t^*$. Both cases can be interpreted as validity of the fact at the present time-moment. If there exists at least one previous time-moment in which the fact has been valid, the conditions of the operator ψ will be satisfied.

$$T_2 = \langle \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9\}, \{i_{10}, i_{11}\}, r_{T_2}, M_{T_2}, \vee(i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9) \rangle,$$

where

$$r_{T_2} = \begin{array}{c|cc} & i_{10} & i_{11} \\ \hline i_1 & r_{i_1, i_{10}} & r_{i_1, i_{11}} \\ i_2 & r_{i_2, i_{10}} & r_{i_2, i_{11}} \\ i_3 & r_{i_3, i_{10}} & r_{i_3, i_{11}} \\ i_4 & r_{i_4, i_{10}} & r_{i_4, i_{11}} \\ i_5 & r_{i_5, i_{10}} & r_{i_5, i_{11}} \\ i_6 & r_{i_6, i_{10}} & r_{i_6, i_{11}} \\ i_7 & r_{i_7, i_{10}} & r_{i_7, i_{11}} \\ i_8 & r_{i_8, i_{10}} & r_{i_8, i_{11}} \\ i_9 & r_{i_9, i_{10}} & r_{i_9, i_{11}} \\ i_{10} & true & false \end{array},$$

where

- $r_{i_1, i_{10}} = r_{i_2, i_{10}} = r_{i_3, i_{10}} = r_{i_4, i_{10}} = r_{i_5, i_{10}} = r_{i_6, i_{10}} = r_{i_7, i_{10}} = r_{i_8, i_{10}} = \text{“the procedure must continue”}$,
- $r_{i_1, i_{11}} = r_{i_2, i_{11}} = r_{i_3, i_{11}} = r_{i_4, i_{11}} = r_{i_5, i_{11}} = r_{i_6, i_{11}} = r_{i_7, i_{11}} = r_{i_8, i_{11}} = \neg r_{i_1, i_{10}}$

and

$$M_{T_2} = \begin{array}{c|cc} & i_{10} & i_{11} \\ \hline i_1 & \infty & \infty \\ i_2 & \infty & \infty \\ i_3 & \infty & \infty \\ i_4 & \infty & \infty \\ i_5 & \infty & \infty \\ i_6 & \infty & \infty \\ i_7 & \infty & \infty \\ i_8 & \infty & \infty \\ i_9 & \infty & 0 \end{array}.$$

Transitions Z_5 and Z_6 have the same forms as in the Third GN-model.

5 Conclusion

Briefly, generalized net models of expert systems:

- contain different logical operations (conjunction, disjunction, implication, negation, etc.), quantifiers (“for existence” and “for all”) and modal operators in the hypotheses and in the antecedents of the ES-rules,
- can be self-modifying at the time of their functioning,
- can have priorities of the facts and rules,
- can have metafacts and metarules,
- can have intuitionistic fuzzy values of the aspects of estimations of facts and rules (see [2,4]),

- can answer questions related to the time (“at the moment”, “once”, “sometimes”, “for a long/short time”, “often”, “rarely”, etc.),
- can be a combination of all of the above types.

Similar directions for extensions of DBs, data warehouses, OLAP-structures, etc. can be realized. The first steps for GN-modelling of these structures have been discussed in [10, 11].

We assume that solving each of the above problems or, of course, all of them, will promote not only the theory and application of GNs, but also the research in the area of DM, too (cf. [7, 8, 12]).

Bearing in mind all of the above, we think that it is clear that GNs can really make a claim for a place within DM. How central is this place? That will depend on how successfully the problems above can be solved.

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