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Intuitionistic fuzzy contra semi-generalized continuous mappings

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Abstract: The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy contra semi-generalized continuous mappings in intuitionistic fuzzy topological space. We investigate some of its fundamental properties and its characterizations.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy semi-generalized closed set, Intuitionistic fuzzy semi-generalized open set, Intuitionistic fuzzy contra semi-generalized continuous mapping, Intuitionistic fuzzy semi-$T_{1/2}$ space.

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1 Introduction

Fuzzy set (FS), proposed by Zadeh [15] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. By adding the degree of non-membership to FS, Atanassov proposed intuitionistic fuzzy set (IFS) in 1983 [1], which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations.

Later on, fuzzy topology was introduced by Chang in 1967. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In last few years, various concepts in fuzzy were extended to intuitionistic fuzzy sets. In 1997, Çoker introduced the concept of intuitionistic fuzzy topological space. After this many concepts in fuzzy topological spaces were extended to intuitionistic fuzzy topological spaces.

We introduce the concepts of intuitionistic fuzzy contra semi-generalized continuous mappings as an extension of work done in the papers [10, 11]. We have studied some of the basic properties regarding it. We also obtained some characterizations and preservation theorems with the help of intuitionistic fuzzy semi-$T_{1/2}$ space.

2 Preliminaries

Definition 2.1 [1] An *intuitionistic fuzzy set* (IFS, for short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,/\, x \in X\}$$

where the functions μ_A: X → [0,1] and ν_A: X → [0,1] denote the degree of the membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x∈X to the set A respectively, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X.

Definition 2.2 [1] Let A and B be IFS's of the forms
$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,/\, x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \,/\, x \in X\}$
Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ∈ X,
(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle \,/\, x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$
(f) $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$
(g) $\overline{\overline{A}} = A$, $\overline{1_\sim} = 0_\sim$, $\overline{0_\sim} = 1_\sim$.

Definition 2.3 [1] Let α,β ∈ [0,1] with α + β ≤ 1. An *intuitionistic fuzzy point* (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta), & \text{if } x = p \\ (0,1), & \text{otherwise} \end{cases}$$

Definition 2.4 [4] An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFS's in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $\overline{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.5 [4] Let X and Y be two non-empty sets and f: X→ Y be a function. If

$$B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \,/\, y \in Y\}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle \,/\, x \in X\}$$

Definition 2.6 [4] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,/\, x \in X\}$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* of A are defined by

$$\text{int}(A) = \cup \{G \mid G \text{ is an IFOS in X and } G \subseteq A\},$$

$$cl(A) = \cap \{K \mid K \text{ is an IFCS in X and } A \subseteq K\}.$$

Note that, for any IFS A in (X, τ), we have

$$cl(\bar{A}) = \overline{int(A)} \text{ and } int(\bar{A}) = \overline{cl(A)}$$

Definition 2.7 An IFS A = {< x, μ_A (x), ν_A (x) > / x ∈ X} in an IFTS (X, τ) is called an

(i) *intuitionistic fuzzy semiopen set* (IFSOS) if A ⊆ cl(int(A)) [6].

(ii) *intuitionistic fuzzy α-open set* (IFαOS) if A ⊆ int(cl(int(A))) [6].

(iii) *intuitionistic fuzzy preopen set* (IFPOS) if A ⊆ int(cl(A)) [6].

(iv) *intuitionistic fuzzy regular open set* (IFROS) if int(cl(A))=A [6].

An IFS A is called an intuitionistic fuzzy semiclosed set, intuitionistic fuzzy α-closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi-preclosed set, respectively (IFSCS, IFαCS, IFPCS and IFRCS resp.), if the complement Ā is an IFSOS, IFαOS, IFPOS and IFROS respectively.

The family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy α-open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semi-preopen) sets of an IFTS (X,τ) is denoted by IFSO(X) (resp IF_α(X), IFPO(X), IFRO(X) and IFSPO(X)).

Definition 2.8 [10] An IFS A of an IFTS (X,τ) is called an *intuitionistic fuzzy semi-generalized closed (intuitionistic fuzzy sg-closed) set* (IFSGCS) if scl (A) ⊆ U, whenever A ⊆ U and U is an IFSOS.

The complement Ā of an intuitionistic fuzzy semi-generalized closed set A is called an *intuitionistic fuzzy semi-generalized open (intuitionistic fuzzy sg-open) set* (IFSGOS).

Definition 2.9 [10] An IFTS (X,τ) is said to be an *intuitionistic fuzzy semi-$T_{1/2}$ space*, if every intuitionistic fuzzy sg-closed set in X is an intuitionistic fuzzy semiclosed in X.

Definition 2.10 [8] Let $p_{(\alpha,\beta)}$ be an IFP of an IFTS (X, τ). An IFS A of X is called an *intuitionistic fuzzy neighbourhood* (IFN) of $p_{(\alpha,\beta)}$, if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition 2.11 [8] Let (X,τ) be an IFTS and A = ⟨x, μ_A, ν_A⟩ be an IFS in X. Then the intuitionistic fuzzy semi-interior and intuitionistic fuzzy semi-closure of A are defined by

$$sint(A) = \cup \{G \mid G \text{ is an IFSOS in X and } G \subseteq A\},$$
$$scl(A) = \cap \{K \mid K \text{ is an IFSCS in X and } A \subseteq K\}.$$

Definition 2.12 [11] Let (X,τ) be an IFTS and A = ⟨ x, μ_A, ν_A⟩ be an IFS in X. Then the intuitionistic fuzzy semi-interior and intuitionistic fuzzy semi-closure of A are defined by

$$sgint(A) = \cup \{G \mid G \text{ is an IFSGOS in X and } G \subseteq A\},$$
$$sgcl(A) = \cap \{K \mid K \text{ is an IFSGCS in X and } A \subseteq K\}.$$

Definition 2.13 [10] An IFTS (X,τ) is said to be an *intuitionistic fuzzy semi-$T_{1/2}$ space*, if every intuitionistic fuzzy sg-closed set in X is an intuitionistic fuzzy semiclosed in X.

Definition 2.14 [14] Two IFSs are said to be *q-coincident* (A q B in short) if and only if there exists an element x ∈ X such that $\mu_A(x) > \nu_A(x)$ or $\nu_A(x) < \mu_A(x)$.

Definition 2.15 Let f: X→Y be a mapping from an IFTS X into an IFTS Y. The mapping f is called an

(i) *intuitionistic fuzzy continuous*, if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y [6].

(ii) *intuitionistic fuzzy contra continuous*, if $f^{-1}(B)$ is an IFCS in X, for each IFOS B in Y [2].

(iii) *intuitionistic fuzzy contra semi-continuous*, if $f^{-1}(B)$ is an IFSCS in X, for each IFOS B in Y [2].

(iv) *intuitionistic fuzzy contra α-continuous*, if $f^{-1}(B)$ is an IFαCS in X, for each IFOS B in Y [2].

(v) *intuitionistic fuzzy sg-continuous*, if $f^{-1}(B)$ is an IFSGOS in X, for each IFOS B in Y [11].

Definition 2.16 [11] A mapping f: (X,τ) → (Y,κ) from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy semi-generalized irresolute (intuitionistic fuzzy sg-irresolute) mapping if $f^{-1}(A)$ is an IFSGCS in X, for every IFSGCS A in Y.

Definition 2.17 [2] Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle / i \in I\}$ of IFOSs (IFROSs) in X satisifies the condition $1_{\sim} = \cup \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle / i \in I\}$ is called a fuzzy open (fuzzy regular open) cover of X.

Definition 2.18 [2] A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle / i \in I\}$ of X which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle / i \in I\}$.

Definition 2.19 [2] An IFTS X is called fuzzy S-closed if each fuzzy regular closed cover of X has a finite subcover for X.

Definition 2.20 [2] An IFTS X is called fuzzy S-Lindelof if each fuzzy regular closed cover of X has a countable subcover for X.

Definition 2.21 [2] An IFTS X is called fuzzy countable S-closed if each countable fuzzy regular closed cover of X has a finite subcover for X.

Definition 2.22 [2] An IFTS X is called fuzzy strongly S-closed if each fuzzy closed cover of X has a finite subcover for X.

Definition 2.23 [2] An IFTS X is called fuzzy countable strongly S-closed if each countable fuzzy closed cover of X has a finite subcover for X.

Definition 2.24 [2] An IFTS X is called fuzzy almost Lindelof if each fuzzy open cover of X has a countable subcover the closure of whose members cover X.

Definition 2.25 [2] An IFTS X is called fuzzy countable almost compact if each countable fuzzy open cover of X has a finite subcover the closure of whose members cover X.

Definition 2.26 [2] An IFTS X is called fuzzy almost compact if each fuzzy open cover of X has a finite subcover the closure of whose members cover X.

3 Intuitionistic fuzzy contra semi-generalized mappings

In this section we introduce intuitionistic fuzzy contra semi-generalized continuous mapping and investigate some of its basic properties and given their characterizations.

Definition 3.1 A mapping f: X→Y from an IFTS X into an IFTS Y is called an intuitionistic fuzzy contra semi-generalized continuous (intuitionistic fuzzy contra sg-continuous) mapping if $f^{-1}(B)$ is an IFSGCS in X for each IFOS B in Y.

Example 3.2 Let X= {a, b}, Y= {u, v}.

$$\text{Let } A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$$
$$B = \langle y, \left(\frac{u}{0.4}, \frac{v}{0.3}\right), \left(\frac{u}{0.6}, \frac{v}{0.7}\right)\rangle$$

Then τ = {$0_\sim$, $1_\sim$, A} and κ = {$0_\sim$, $1_\sim$,B} are IFTSs on X and Y, respectively. Define a mapping f: (X, τ) → (Y, κ) by f(a) = u, f(b) = v.
Clearly $0_\sim, 1_\sim$are IFOS in Y, and then $f^{-1}(0_\sim), f^{-1}(1_\sim)$are IFSGCS in X.
Now B is an IFOS in Y, then

$$f^{-1}(B) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{06}, \frac{b}{0.7}\right)\rangle \text{ and } scl(f^{-1}(B)) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}\right)\rangle.$$

Then $f^{-1}(B)$ is an IFSGCS in X, since $scl(f^{-1}(B)) \subseteq U$, whenever $f^{-1}(B) \subseteq U$, where U is an IFSOS. Therefore, f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.3 Every intuitionistic fuzzy contra continuous mapping is an intuitionistic fuzzy contra sg-continuous mapping.

Proof: Let f: X→Y be an intuitionistic fuzzy contra continuous mapping. Let B be an IFOS in Y. Since f is an intuitionistic fuzzy contra continuous mapping, $f^{-1}(B)$ is an IFCS in X. In [10], it has been proved that every IFCS is an IFSGCS. Therefore $f^{-1}(B)$ is an IFSGCS in X for every IFOS B in Y. Hence f is an intuitionistic fuzzy contra sg-continuous mapping.

Example 3.4 Let X= {a, b}, Y= {u, v}.

$$\text{Let } A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$$
$$B = \langle y, \left(\frac{u}{0.4}, \frac{v}{0.3}\right), \left(\frac{u}{0.6}, \frac{v}{0.7}\right)\rangle$$

Then τ = {$0_\sim$, $1_\sim$, A} and κ = {$0_\sim$, $1_\sim$,B} are IFTSs on X and Y, respectively. Define a mapping g: (X, τ) → (Y, κ) by g(a) = u, g(b) = v.
Clearly $0_\sim, 1_\sim$are IFOS in Y, and then $f^{-1}(0_\sim), f^{-1}(1_\sim)$are IFSGCS in X.
Now B is an IFOS in Y, then

$$f^{-1}(B) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle \text{ and } scl(f^{-1}(B)) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle.$$

Then $f^{-1}(B)$ is an IFSGCS in X, since $scl(f^{-1}(B)) \subseteq U$, whenever $f^{-1}(B) \subseteq U$, where U is an IFSOS. Therefore, f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.5 Every intuitionistic fuzzy contra semi-continuous mapping is an intuitionistic fuzzy contra sg-continuous mapping.

Proof: Let f: X→Y be an intuitionistic fuzzy contra semi-continuous mapping. Let B be an IFOS in Y. Since f is an intuitionistic fuzzy contra continuous mapping, $f^{-1}(B)$ is an IFSCS in

X. In [10], it has been proved that every IFSCS is an IFSGCS. Therefore $f^{-1}(B)$ is an IFSGCS in X for every IFOS B in Y. Hence f is an intuitionistic fuzzy contra sg-continuous mapping.

Example 3.6 Let X = {a, b}, Y = {u, v}.

$$\text{Let } A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$$

$$B = \langle y, \left(\frac{u}{0.4}, \frac{v}{0.3}\right), \left(\frac{u}{0.6}, \frac{v}{0.7}\right)\rangle$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTSs on X and Y respectively. Define a mapping g: $(X, \tau) \rightarrow (Y, \kappa)$ by g(a) = u, g(b) = v.

Clearly $0_\sim, 1_\sim$ are IFOS in Y, and then $f^{-1}(0_\sim), f^{-1}(1_\sim)$ are IFSGCS in X.

Now B is an IFOS in Y, then

$$f^{-1}(B) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle \text{ and } scl(f^{-1}(B)) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle.$$

Then $f^{-1}(B)$ is an IFSGCS in X, since $scl(f^{-1}(B)) \subseteq U$, whenever $f^{-1}(B) \subseteq U$, where U is an IFSOS. Therefore, f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.7 Every intuitionistic fuzzy contra continuous mapping is an intuitionistic fuzzy contra sg-continuous mapping.

Proof: Let f: X→Y be an intuitionistic fuzzy contra continuous mapping. Let B be an IFOS in Y. Since f is an intuitionistic fuzzy contra continuous mapping, $f^{-1}(B)$ is an IFαCS in X. In [10], it has been proved that every IFαCS is an IFSGCS. Therefore, $f^{-1}(B)$ is an IFSGCS in X for every IFSOS B in Y. Hence, f is an intuitionistic fuzzy contra sg-continuous mapping.

Example 3.8 Let X= {a, b}, Y= {u, v}.

$$\text{Let } A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$$

$$B = \langle y, \left(\frac{u}{0.4}, \frac{v}{0.3}\right), \left(\frac{u}{0.6}, \frac{v}{0.7}\right)\rangle$$

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTSs on X and Y respectively. Define a mapping g: $(X, \tau) \rightarrow (Y, \kappa)$ by g(a) = u, g(b) = v.

Clearly $0_\sim, 1_\sim$ are IFOS in Y, and then $f^{-1}(0_\sim), f^{-1}(1_\sim)$ are IFSGCS in X.

Now B is an IFOS in Y, then

$$f^{-1}(B) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle \text{ and } scl(f^{-1}(B)) = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle.$$

Then $f^{-1}(B)$ is an IFSGCS in X, since $scl(f^{-1}(B)) \subseteq U$, whenever $f^{-1}(B) \subseteq U$, where U is an IFSOS. Therefore, f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.9 Let f: X→Y be a mapping. Then the following are equivalent:

(i) f is an intuitionistic fuzzy contra sg-continuous mapping;

(ii) $f^{-1}(B)$ is an IFSGOS in X, for every IFCS B in Y.

Proof: (i) ⇒ (ii): Let B be an IFCS in Y. Then $\overline{B}$ is an IFOS in Y. By hypothesis $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is an IFSGCS in X. Hence $f^{-1}(B)$ is an IFSGOS in X.

(ii) ⇒ (i): Let B be an IFOS in Y. Then $\overline{B}$ is an IFCS in Y. By (ii), $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is an IFSGOS in X. Hence $f^{-1}(B)$ is an IFSGCS in X. Therefore f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.10 Let f: X→Y be a mapping. Suppose that one of the following properties hold:

(i) $f(scl(A)) \subseteq int(f(A))$ for each IFS A in X;

(ii) $scl(f^{-1}(B)) \subseteq f^{-1}(int(B))$, for each IFS B in Y;

(iii) $f^{-1}(cl(B)) \subseteq sint(f^{-1}(B))$, for each IFS B in Y.

Then f is an intuitionistic fuzzy contra sg-continuous mapping.

Proof: (i) ⇒ (ii): Let B be any IFS in Y. From the assumption we have, $f\left(scl(f^{-1}(B))\right) \subseteq int\left(f(f^{-1}(B))\right) \subseteq int(B)$. Now $scl(f^{-1}(B)) \subseteq f^{-1}(int(B))$.

(ii) ⇒ (iii): By taking complement, we get the result.

Suppose that (iii) holds. Let B be any IFCS in Y. Then cl(B) =B. By our assumption, we have $f^{-1}(B) \subseteq f^{-1}(cl(B)) \subseteq sint(f^{-1}(B))$. But $sint(f^{-1}(B)) \subseteq f^{-1}(B)$ always. Hence, $sint(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSOS in X and hence $f^{-1}(B)$ is an IFSGOS in X. Thus f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.11 Let f: X→Y be a bijective mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy contra sg-continuous mapping if $cl(f(A)) \subseteq f(sint(A))$ for every IFS A in X.

Proof: Let A be an IFCS in Y. Then cl(A) = A and $f^{-1}(A)$ is an IFS in X. By hypothesis, $cl\left(f(f^{-1}(A))\right) \subseteq f\left(sint(f^{-1}(A))\right)$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl\left(f(f^{-1}(A))\right) \subseteq f\left(sint(f^{-1}(A))\right)$. Now $f^{-1}(A) \subseteq f^{-1}\left(f\left(sint(f^{-1}(A))\right)\right) = sint(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFSOS in X and $f^{-1}(A)$ is an IFSGOS in X. Thus f is an intuitionistic fuzzy contra semi-generalized continuous mapping.

Theorem 3.12 If f: X→Y is an intuitionistic fuzzy contra sg-continuous mapping, where X is an intuitionistic fuzzy semi-$T_{1/2}$ space, then the following conditions hold:

(i) $scl(f^{-1}(B)) \subseteq f^{-1}\left(int(scl(B))\right)$, for every IFOS B in Y;

(ii) $f^{-1}\left(cl(sint(B))\right) \subseteq sint(f^{-1}(B))$, for every IFCS B in Y.

Proof: (i) Let B be an IFOS in Y. By hypothesis $f^{-1}(B)$ is an IFSGCS in X. Since X is an intuitionistic fuzzy semi-$T_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. This implies $scl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(int(B)) \subseteq f^{-1}\left(int(scl(B))\right)$.

(ii) By taking the complement of (i) we get the result.

Theorem 3.13 Let f: X→Y be a mapping, where X is an intuitionistic fuzzy semi-$T_{1/2}$ space. Then, the following are equivalent:

(i) f is an intuitionistic fuzzy contra sg-continuous mapping;

(ii) for each $p_{(\alpha,\beta)}$ in X and IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFSOS A in X containing $p_{(\alpha,\beta)}$ such that $A \subseteq f^{-1}(B)$;

(iii) for each $p_{(\alpha,\beta)}$ in X and IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFSOS A in X containing $p_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

Proof: (i) ⇒ (ii) Let f be an intuitionistic fuzzy contra sg-continuous mapping and let B be an IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFSGOS in X. Since X is an intuitionistic fuzzy semi-$T_{1/2}$ space, $f^{-1}(B)$ is an IFSOS in X. Now $A = sint(f^{-1}(B)) \subseteq f^{-1}(B)$.

(ii) ⇒ (iii): The result follows from the relations $f(A) \subseteq f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i): Let B be an IFCS in Y and let $p_{(\alpha,\beta)}$ be an IFP in X, such that $f(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFSOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. This implies $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. That is $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since A is an IFSOS, $A = sin(A) \subseteq sint(f^{-1}(B))$. Therefore $p_{(\alpha,\beta)} \in sint(f^{-1}(B))$. But

$$f^{-1}(B) = \cup_{p_{(\alpha,\beta)} \in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq sint(f^{-1}(B)) \subseteq f^{-1}(B).$$

Hence, $f^{-1}(B)$ is an IFSOS in X and hence $f^{-1}(B)$ is an IFSGOS in X. Hence f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.14 Let f: X→Y and g: Y→Z be any two mappings. Then the following statements hold:

(i) If f is an intuitionistic fuzzy contra sg-continuous mapping and g is an intuitionistic fuzzy continuous mapping, then $g \circ f$ is an intuitionistic fuzzy contra sg-continuous mapping.

(ii) If f is an intuitionistic fuzzy contra sg-continuous mapping and g is an intuitionistic fuzzy contra continuous mapping, then $g \circ f$ is an intuitionistic fuzzy sg-continuous mapping.

(iii) If f is an intuitionistic fuzzy sg-irresolute mapping and g is an intuitionistic fuzzy contra sg-continuous mapping, then $g \circ f$ is an intuitionistic fuzzy contra sg-continuous mapping.

Proof: (i) Let B be an IFOS in Z. Since g is an intuitionistic fuzzy continuous mapping, $g^{-1}(B)$ is an IFOS in Y. Also since f is an intuitionistic fuzzy contra sg-continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFSGCS in X. Therefore $g \circ f$ is an intuitionistic fuzzy contra sg-continuous mapping.

(ii) Let B be an IFOS in Z. Since g is an intuitionistic fuzzy contra continuous mapping, $g^{-1}(B)$ is an IFCS in Y. Also since f is an intuitionistic fuzzy contra sg-continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFSGOS in X. Therefore $g \circ f$ is an intuitionistic fuzzy sg-continuous mapping.

(iii) Let B be an IFOS in Z. Since g is an intuitionistic fuzzy contra sg-continuous mapping, $g^{-1}(B)$ is an IFSGCS in Y. Also since f is an intuitionistic fuzzy sg-irresolute mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFSGCS in X. Therefore $g \circ f$ is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.15 For a mapping f: X→Y, the following are equivalent, where X is an intuitionistic fuzzy semi-$T_{1/2}$ space.

(i) f is an intuitionistic fuzzy contra sg-continuous mapping;

(ii) for every IFCS A in Y, $f^{-1}(A)$ is an IFSGOS in X;

(iii) for every IFCS B in Y, $f^{-1}(B)$ is an IFSGCS in X;

(iv) for any IFCS A in Y and for any IFP $p_{(\alpha,\beta)}$ in X, if $f(p_{(\alpha,\beta)})\, q\, A$, then $p_{(\alpha,\beta)}\, q\, sint(f^{-1}(A))$;

(v) for any IFCS A in Y and for any IFP $p_{(\alpha,\beta)}$ in X, if $f(p_{(\alpha,\beta)})$ q A, then there exists an IFSGOS B such that $p_{(\alpha,\beta)}$ q B and f(B) ⊆ A.

Proof: (i) ⇔ (ii) and (ii) ⇔(iii) are obvious.

(ii) ⇒ (iv): Let A be an IFCS in Y and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})$ q A. Then, $p_{(\alpha,\beta)}$ q $f^{-1}(A)$. By hypothesis, $f^{-1}(A)$ is an IFSGOS in X. Since X is an intuitionistic fuzzy semi-$T_{1/2}$ space, $f^{-1}(A)$ is an IFSOS in X. This implies $\mathrm{sint}(f^{-1}(A)) = f^{-1}(A)$. Hence, $p_{(\alpha,\beta)}$ q $\mathrm{sint}(f^{-1}(A))$.

(iv) ⇒ (ii): Let A be an IFCS in Y and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})$ q A. Then, $p_{(\alpha,\beta)}$ q $f^{-1}(A)$. By hypothesis, $p_{(\alpha,\beta)}$ q $\mathrm{sint}(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \mathrm{sint}(f^{-1}(A))$. Therefore, $f^{-1}(A) = \mathrm{sint}(f^{-1}(A))$. Thus $f^{-1}(A)$ is an IFSOS and hence $f^{-1}(A)$ is an IFSGOS in X.

(iv) ⇒ (v): Let A be an IFCS in Y and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})$ q A. Then, $p_{(\alpha,\beta)}$ q $f^{-1}(A)$. By hypothesis $p_{(\alpha,\beta)}$ q $\mathrm{sint}(f^{-1}(A))$. Thus $f^{-1}(A)$ is an IFSOS in X and hence $f^{-1}(A)$ is an IFSGOS in X. Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)}$q B and $f(B) = f(f^{-1}(A)) \subseteq A$.

(v) ⇒ (iv): Let A ⊆ Y be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})$ q A. Then, $p_{(\alpha,\beta)}$ q $f^{-1}(A)$. By hypothesis there exists an IFSGOS B in X such that $p_{(\alpha,\beta)}$qB and f(B) ⊆ A. Let $B = f^{-1}(A)$. Since X is an intuitionistic fuzzy semi-$T_{1/2}$ space, $f^{-1}(A)$ is an IFSOS in X. Therefore $p_{(\alpha,\beta)}$ q $\mathrm{sint}(f^{-1}(A))$.

Theorem 3.16 A mapping f: X→Y is an intuitionistic fuzzy contra sg-continuous mapping if $f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{int}(f^{-1}(B))$ for every IFS B in Y.

Proof: Let B be an IFCS in Y. Then cl(B) =B. Since every IFCS is an IFSCS. scl(B) = B. Now by hypothesis $f^{-1}(B) = f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{int}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFOS in X. Therefore f is an intuitionistic fuzzy contra continuous mapping. Then by Theorem.3.3, f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.17 A mapping f: X→Y is an intuitionistic fuzzy contra sg-continuous mapping, where X is an intuitionistic fuzzy semi-$T_{1/2}$ space if and only if

$$f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{sint}\left(f^{-1}(\mathrm{cl}(B))\right)$$

for every IFS B in Y.

Proof: Necessity: Let B be an IFS in Y. Then cl(B) is an IFCS in Y. By hypothesis $f^{-1}(\mathrm{cl}(B))$ is an IFSGOS in X. Since X is an intuitionistic fuzzy semi $T_{1/2}$ space, $f^{-1}(\mathrm{cl}(B))$ is an IFSOS in X. Therefore $f^{-1}(\mathrm{scl}(B)) \subseteq f^{-1}(\mathrm{cl}(B)) \subseteq \mathrm{sint}\left(f^{-1}(\mathrm{cl}(B))\right)$.

Sufficiency: Let B be an IFCS in Y. Then cl(B) = B. By hypothesis,

$$f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{sint}\left(f^{-1}(\mathrm{cl}(B))\right) = \mathrm{sint}(f^{-1}(B)).$$

But scl(B) = B. Therefore $f^{-1}(B) = f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{sint}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSGOS in X and hence $f^{-1}(B)$ is an IFSGOS in X. Hence f is an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 3.18 An intuitionistic fuzzy continuous mapping f: X→Y is an intuitionistic fuzzy contra sg-continuous mapping if IFSGO(X) = IFSGC(X).

Proof: Let A be an IFOS in Y. By hypothesis, $f^{-1}(A)$ is an IFOS in X and hence $f^{-1}(A)$ is an IFSGOS in X. Thus, $f^{-1}(A)$ is an IFSGCS in X. Therefore, f is an intuitionistic fuzzy contra sg-continuous mapping.

4 Applications to fuzzy compact spaces

In this section we define the concepts of fuzzy sg-compact, fuzzy sg-Lindelof, fuzzy countable sg-compact using the intuitionistic fuzzy semi-generalized closed set. Using the above concepts, we give some characterizations of intuitionistic fuzzy contra sg-continuous mappings

Definition 4.1 Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, / \, i \in I\}$ of IFSGOSs in X satisfies the condition $1_\sim = \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, / \, i \in I\}$ is called a fuzzy sg-open cover of X.

Definition 4.2 A finite subfamily of a fuzzy sg-open cover $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, / \, i \in I\}$ of X which is also a fuzzy sg-open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, / \, i \in I\}$.

Definition 4.3 An IFTS X is called fuzzy sg-compact if every fuzzy sg-open cover of X has a finite subcover.

Definition 4.4 An IFTS X is called fuzzy sg-Lindelof if each fuzzy sg-open cover of X has a countable subcover for X.

Definition 4.5 An IFTS X is called fuzzy countable sg-compact if each countable fuzzy sg-open cover of X has a finite subcover for X.

Theorem 4.6 Let f: X→Y be an intuitionistic fuzzy contra sg-continuous mapping from an IFTS X onto an IFTS Y. If X is fuzzy sg-compact, then Y is fuzzy strongly S-closed (fuzzy sg-Lindelof, fuzzy countable S-closed).

Proof: Let $\{G_i \, / \, i \in I\}$ be any fuzzy closed cover of Y. Then $1_\sim = \bigcup_{i\in I} G_i$. From the relation $1_\sim = f^{-1}(\bigcup_{i\in I} G_i)$ follows that $1_\sim = \bigcup_{i\in I} f^{-1}(G_i)$, so $\{f^{-1}(G_i) \, / i \in I\}$ is a fuzzy sg-open cover of X. Since X is fuzzy sg-compact, there exists a finite subcover $\{f^{-1}(G_i) / \, i = 1,2,3, \dots, n\}$. Therefore $1_\sim = \bigcup_{i=1}^{n} f^{-1}(G_i)$. Hence

$$1_\sim = f\left(\bigcup_{i=1}^{n} f^{-1}(G_i)\right) = \bigcup_{i=1}^{n} f(f^{-1}(G_i)) = \bigcup_{i=1}^{n} G_i.$$

Hence Y is fuzzy strongly S-closed.

Corollary 4.7 Let f: X→Y be an intuitionistic fuzzy contra sg-continuous mapping. If X is fuzzy sg-Lindelof (fuzzy countable sg-compact), then Y is fuzzy strongly S-Lindelof (fuzzy countable strongly S-closed).

Theorem 4.8 Let f: X→Y be an intuitionistic fuzzy contra sg-continuous mapping. If X is fuzzy sg-compact (fuzzy sg-Lindelof, fuzzy countable sg-compact), then Y is fuzzy S-closed (fuzzy S-Lindelof, fuzzy countable S-closed).

Proof: Let $\{G_i \, / \, i \in I\}$ be any fuzzy regular closed cover of Y. Then, $\{G_i \, / \, i \in I\}$ be a fuzzy closed cover of Y. Then, $1_\sim = \bigcup_{i\in I} G_i$. From the relation $1_\sim = f^{-1}(\bigcup_{i\in I} G_i)$ follows that

$1_\sim = \bigcup_{i\in I} f^{-1}(G_i)$, so $\{f^{-1}(G_i) / i \in I\}$ is a fuzzy sg-open cover of X. Since X is fuzzy sg-compact, there exists a finite subcover $\{f^{-1}(G_i) / i = 1,2,3, \dots, n\}$. Therefore, $1_\sim = \bigcup_{i=1}^{n} f^{-1}(G_i)$. Hence, $1_\sim = f(\bigcup_{i=1}^{n} f^{-1}(G_i)) = \bigcup_{i=1}^{n} f(f^{-1}(G_i)) = \bigcup_{i=1}^{n} G_i$. Hence, Y is fuzzy strongly S-closed.

Theorem 4.9 Let f: X→Y be an intuitionistic fuzzy contra sg-continuous mapping. If X is fuzzy sg-compact(fuzzy sg-Lindelof, fuzzy countable sg-compact), then Y is fuzzy S-closed (fuzzy S-Lindelof, fuzzy countable S-closed).

Proof: Let $\{G_i / i \in I\}$ be any fuzzy open cover of Y. Then, $\{G_i / i \in I\}$ be a fuzzy closed cover of Y. Then $1_\sim = \bigcup_{i\in I} G_i$. It follows that $1_\sim = \bigcup_{i\in I} cl(G_i)$. From the relation $1_\sim = f^{-1}(\bigcup_{i\in I} cl(G_i))$ follows that $1_\sim = \bigcup_{i\in I} f^{-1}(cl(G_i))$, so $\{f^{-1}(cl(G_i)) / i \in I\}$ is a fuzzy sg-open cover of X. Since X is fuzzy sg-compact, there exists a finite subcover $\{f^{-1}(cl(G_i)) / i = 1,2,3, \dots, n\}$. Therefore, $1_\sim = \bigcup_{i=1}^{n} f^{-1}(cl(G_i))$. Hence $1_\sim = f(\bigcup_{i=1}^{n} f^{-1}(cl(G_i))) = \bigcup_{i=1}^{n} f(f^{-1}(cl(G_i))) = \bigcup_{i=1}^{n} cl(G_i)$. Hence, Y is fuzzy almost compact.

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