Notes on Intuitionistic Fuzzy Sets
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A short note on a new ranking for intuitionistic fuzzy alternatives

Kristina Zhivkova [1], **Peter Vassilev** [2] **and Lyudmila Todorova** [3]

Department of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
Acad. G. Bonchev Str. bl. 105, Sofia-1113, Bulgaria
[1] e-mail: kristina@biomed.bas.bg
[2] e-mail: peter.vassilev@gmail.com
[3] e-mail: lpt@biomed.bas.bg

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Abstract: In the present work, we consider a new ranking for intuitionistic fuzzy alternatives, that is, alternatives represented by intuitionistic fuzzy pairs. This ranking is inspired by the "necessity" and "possibility" operators corresponding to such alternative. Some properties of the new ranking method are studied and a comparison with other ranking methods is provided.
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1 Introduction

Intuitionistic fuzzy sets were introduced in 1983 in [1] as an extension of L. Zadeh's fuzzy sets. The modal operators "necessity" ($\Box$) and "possibility" ($\Diamond$), which have no analogues in the fuzzy case, were defined there for the first time. In various decision making problems set in

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intuitionistic fuzzy setting, experts are tasked with evaluating alternatives with scores in the form of intuitionistic fuzzy pair – which may be viewed as an intuitionistic fuzzy set with a single element x. We will briefly recall some of the notions which will be used further

Definition 1 (cf. [3]). *An intuitionistic fuzzy pair is an ordered couple of non-negative numbers in the unit interval $x = \langle a, b\rangle$, such that:*

$$a + b \leq 1. \tag{1}$$

The hesitancy margin of the IFP x is the quantity:

$$h(x) = 1 - a - b$$

The geometrical representation of an IFP in the interpretational triangle is given in Figure 1.

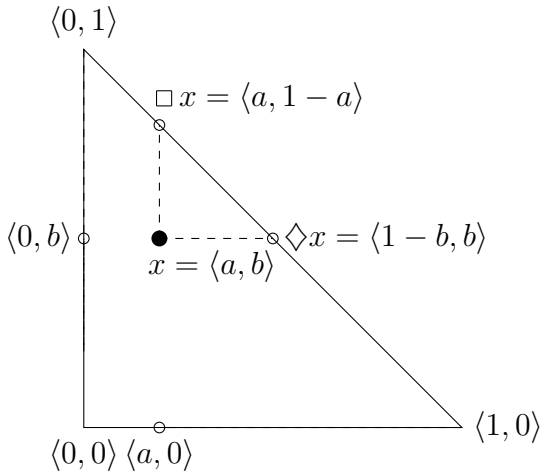


Figure 1. A geometrical interpretation of an IFP $x = \langle a, b\rangle$ and the result of the "necessity" and "posibility" operators

Different orderings have been defined over IFPs. The most recognizable are due to Atanassov [2] and Bustince and Burillo [4] (the latter one has been extensively studied by E. Marinov [5]). Both of these, of course, are partial orderings. Therefore, when two alternatives are incomparable under the chosen ordering the decision maker is faced with a difficult choice. There are several approaches employed most often in practice. One is to introduce another ordering that preserves the existing order of the other alternatives while allowing the considered alternatives to be compared. Another is to use a ranking function that assigns a real-valued number to each alternative, permitting the natural ordering of real numbers to be used to select the more beneficial one. A further approach is to incorporate a distance-based measure reflecting how far the considered alternative x is from the ideal alternative $\langle 1, 0\rangle$, which also results in a real-valued ranking.

The classical partial ordering introduced by Atanassov (see [2]) is given by

Definition 2 (cf. [2, 3]). *For two IFPs: $x = \langle x_1, x_2\rangle$ and $y = \langle y_1, y_2\rangle$, we say that x is less or equal to y, and we write:*

$$x \leq y,$$

if and only if

$$\begin{cases} x_1 \leq y_1 \\ x_2 \geq y_2. \end{cases} \tag{2}$$

On the other hand, the ordering proposed by Bustince and Burillo is defined by the hesitancy margin of the IFPs.

Definition 3 (cf. [4,5]). *For two IFPs: $x = \langle x_1, x_2 \rangle$ and $y = \langle y_1, y_2 \rangle$, we say that x precedes y (or equivalently, x has greater or equal hesitancy margin than y), and we write:*

$$x \preceq y,$$

if and only if

$$\begin{cases} x_1 \leq y_1 \\ x_2 \leq y_2. \end{cases} \tag{3}$$

Its is quite clear that in some sense the two partial orderings (barring the very particular case when two IFPs are equal) are incompatible. If two IFPs are comparable under the first ordering, they are incomparable under the second and vice versa.

In a previous research involving the second and third authors of the present study, a family of orderings depending on the power mean M_p were investigated. We refer the interested reader to [7] for further details. Here we only provide the definition of the power mean based orderings and we mention some of their properties:

Definition 4 (see [6]). *Given two IFPs $x = \langle x_1, x_2 \rangle$ and $y = \langle y_1, y_2 \rangle$, we say that x is first component biased power mean-based with a value of p less or equal to y and write $x \preceq_{\mu;M_p} y$ if*

$$\begin{cases} 1 - x_1 \geq 1 - y_1 \\ M_p(1 - x_1, x_2) \geq M_p(1 - y_1, y_2). \end{cases} \tag{4}$$

where the power mean of two non-negative numbers x, y is given by

$$M_p(s,t) = \left(\frac{s^p + t^p}{2} \right)^{\frac{1}{p}}.$$

There are special cases (obtained as a limit) of the power mean for particular values of p, which we remind here: $M_{-\infty}(s,t) = \min(s,t)$, $M_0(s,t) = \sqrt{st}$, $M_\infty(s,t) = \max(s,t)$.

These family of orderings allow us to compare the element of any set of IFPs which does not contain the $\langle 0, 0 \rangle$ alternative, for a suitable choice of p. They also preserve the orderings for all lower values of p. In particular the classical ordering (2) is obtained as (4) for $p = -\infty$.

We also remind the following definition.

Definition 5 (cf. [8]). *For two u, v IFPs, the order $\preceq_P$ is defined as follows:*

$$u \preceq_P v \Leftrightarrow P(u) \leq P(v) \tag{5}$$

where $P(u) = 1 - M_2(1 - u_1, u_2)$, with M_2 is the power mean of order 2.

Here we will investigate an alternative approach for ranking alternatives in the form of IFPs based on distance measure, and the triangle with vertices $\Box x$, $\Diamond x$, and x.

2 The proposed ranking function

Let two IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$ be given. Let the triangle with vertices $\Box x$, $\Diamond x$, and x, be denoted by A. Let the triangle with vertices $\Box y$, $\Diamond y$, and y, be denoted by B.

We define the following functions

$$\begin{cases} T(x,y) = \left(\sqrt{2} - \sqrt{(1-a)^2 + b^2}\right)(1 - 2SA) \\ T(y,x) = \left(\sqrt{2} - \sqrt{(1-c)^2 + d^2}\right)(1 - 2SB), \end{cases} \tag{6}$$

where SA is the area of the triangle A

$$SA = \frac{1}{2}(1-a-b)^2$$

SB is the area of the triangle B

$$SB = \frac{1}{2}(1-c-d)^2.$$

Definition 6. *We will say that the alternative x is better than the alternative y under the ranking T and write $x \succ_T y$, if $T(x,y) > T(y,x)$. If $T(x,y) = T(y,x)$, we say that the alternatives are equivalent with respect to T-ranking.*

Proposition 1. *The proposed ranking T is transitive. That is if $x \succ_T y$, and $y \succ_T z$, we have $x \succ_T z$.*

Proof. Let $x = \langle a, b\rangle$, $y = \langle c, d\rangle$, $z = \langle e, f\rangle$ In order to simplify the notation let us denote $\theta_x = \left(\sqrt{2} - \sqrt{(1-a)^2 + b^2}\right)$, $\theta_y = \left(\sqrt{2} - \sqrt{(1-c)^2 + d^2}\right)$, $\theta_z = \left(\sqrt{2} - \sqrt{(1-e)^2 + f^2}\right)$. Then, let $x \succ_T y$, and $y \succ_T z$. This means that

$$\begin{cases} \theta_x(1 - 2SA) > \theta_y(1 - 2SB) \\ \theta_y(1 - 2SB) > \theta_z(1 - 2SC) \end{cases}$$

Hence,

$$\theta_x(1 - 2SA) > \theta_z(1 - 2SC)$$

which is exactly $x \succ_T z$. □

The original idea of the authors involved a different version of the T-ranking, which also included the area of the intersection of the two triangles A and B

$$S\Delta_{xy} = S(A \cap B),$$

where if the intersection is a single point or an empty set, the area is taken as 0. In that interpretation the analogue of (6) would look like:

$$\begin{cases} T^*(x,y) = \left(\sqrt{2} - \sqrt{(1-a)^2 + b^2}\right)(1 - 2SA + 2S\Delta_{xy}) \\ T^*(y,x) = \left(\sqrt{2} - \sqrt{(1-c)^2 + d^2}\right)(1 - 2SB + 2S\Delta_{xy}), \end{cases} \tag{7}$$

This variation, however, ruins the transitivity in general.

Remark 1. *The proposed ranking T is not always compatible with the classical order in* (2)*. If $x \geq y$, and $h(x) \leq h(y)$ they are compatible. Indeed,* (2) *implies $\theta_x \geq \theta_y$, and since $1 - 2SA = 1 - h^2(x)$, $1 - 2SB = 1 - h^2(y)$, hence in this case $x \succ_T y$.*

Remark 2. *The ranking T is mostly compatible with the ranking proposed in* [8] *(see Definition 5 above). They coincide for all IFPs x, y for which $h(x) = h(y)$. This is true since, $\theta_x = \sqrt{2}M_2(1 - a, b)$, hence for such pairs the two rankings coincide up to a scaling factor.*

3 Software implementation

To facilitate the geometric interpretation and experimental investigation of the proposed ranking method, an interactive web-based software tool, *IFS Ranking Geometry Explorer*, was developed. The application provides a graphical representation of intuitionistic fuzzy pairs (IFPs) within the intuitionistic fuzzy interpretational triangle and allows users to investigate the relationships between their associated triangular regions. Its current version 0.2.0 is available from `https://biomed.bas.bg/bg/projects/kp-06-n72-8/software/`.

The software supports the interactive placement and modification of multiple IFPs, either by directly entering their membership and non-membership degrees or by moving the corresponding points within the interpretational triangle. For each selected pair of alternatives, the application visualizes the associated triangular regions and their intersection, while automatically calculating the corresponding areas, hesitation margins, and distance measures.

Particular attention is given to the comparison between the proposed transitive ranking function T and its original pair-dependent formulation T^*, which incorporates the area of intersection. An adjustable parameter λ allows users to investigate the transition between these two approaches. The application also implements the generalized family of ranking functions T_p, based on power means, for positive values of p.

In addition to numerical comparisons, the software provides graphical representations of preference regions, pairwise comparison matrices, and automatic detection of preference cycles among multiple alternatives. These features make it possible to explore the conditions under which different ranking methods produce consistent or conflicting preferences and to identify potential counterexamples to conjectured mathematical properties.

The application is implemented as a standalone HTML-based tool and can be executed locally in a web browser without additional software installation. It also supports the export of graphical results and numerical data, facilitating their use in further analysis and scientific presentations.

Thus, IFS Ranking Geometry Explorer serves not only as a visualization aid but also as an exploratory computational environment for investigating the geometric and ordering properties of intuitionistic fuzzy alternatives.

4 Conclusion

We proposed a new ranking method inspired by the necessity and possibility enveloping the IFP. We showed how it is related to some other ranking methods. In the future, we will try to

investigate other possible extensions. One possible venue, in view of Remark 2 is to consider a family of rankings depending on the power mean of order p, T_p, defined as:

$$\begin{cases} T_p(x,y) = 2^{\frac{1}{p}} \left(1 - M_p(1-a,b)\right)\left(1 - h^p(x)\right) \\ T_p(y,x) = 2^{\frac{1}{p}} \left(1 - M_p(1-c,d)\right)\left(1 - h^p(y)\right). \end{cases} \tag{8}$$

and by analogy, $x \succ_{T_p} y$ would be true when $T_p(x,y) > T_p(y,x)$.

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