

# Fuzzy coloring and total fuzzy coloring of various types of intuitionistic fuzzy graphs

R. Buvaeswari<sup>1</sup> and P. Revathy<sup>2</sup>

<sup>1</sup> Department of Mathematics, Sri Krishna Arts and Science College

Coimbatore - 8, Tamil Nadu, India

e-mail: buvaeswarir@skasc.ac.in

<sup>2</sup> Department of Science and Humanities, Sri Krishna College of Engineering and Technology

Coimbatore - 8, Tamil Nadu, India

e-mail: revaprabhu92@gmail.com

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**Abstract:** In this paper, fuzzy coloring and total fuzzy coloring of intuitionistic fuzzy graphs are introduced. The fuzzy chromatic number, fuzzy chromatic index, total fuzzy chromatic number and total fuzzy chromatic index of both vertices and edges in intuitionistic fuzzy graphs are defined and their properties are analysed with illustrations.

**Keywords:** Intuitionistic fuzzy graph, Fuzzy coloring, Total fuzzy coloring, Chromatic number, Fuzzy chromatic index, Total chromatic number.

**2020 Mathematics Subject Classification:** 05C72.

## 1 Introduction

The fuzzy coloring and total fuzzy coloring concepts is incited to intuitionistic fuzzy graphs, and then analyzed with its fuzzy vertex chromatic number, fuzzy edge chromatic number and the total fuzzy chromatic number with their indexes utilizing the concepts of fuzzy coloring and total fuzzy coloring.



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## 2 Preliminaries

**Definition 2.1.** An Intuitionistic fuzzy graph is of the form  $\hat{G} = (V, E)$ , where

(i)  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1 : V \rightarrow [0, 1]$  and  $\nu_1 : V \rightarrow [0, 1]$  notate degrees of membership and non-membership of the element  $v_i \in V$ , respectively, and  $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$ , for all  $v_i \in V, (i = 1, 2, \dots, n)$ .

(ii)  $E \subset V \times V$  where  $\mu_{ij} : V \times V \rightarrow [0, 1]$  and  $\nu_{ij} : V \times V \rightarrow [0, 1]$  such that

$$\begin{aligned}\mu_{ij} &\leq \min \{\mu_i, \mu_j\} \\ \nu_{ij} &\leq \max \{\nu_i, \nu_j\}\end{aligned}$$

Also  $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$  for all  $(v_i, v_j) \in E$  [6].

The triplets  $(v_i, \mu_{1i}, \nu_{1i})$  and  $(e_{ij}, \mu_{2ij}, \nu_{2ij})$  denotes the degrees of membership and non-membership of vertex  $v_i$  and edge relation  $e_{ij} = (v_i, v_j)$  on  $V \times V$ .

**Note 2.1.** [1] (i) When  $\mu_{2ij} = \nu_{2ij} = 0$  for some  $i$  and  $j$ , then there is no edge between  $v_i$  and  $v_j$ . Otherwise there exists an edge between  $v_i$  and  $v_j$ .

(ii) If one of the inequalities is not satisfied in (i) and (ii), then  $\hat{G}$  is not an IFG.

**Definition 2.2.** The family  $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k\}$  are fuzzy sets in  $V$  called  $k$ -fuzzy coloring of  $\hat{G} = (V, E)$  when

1.  $\bigvee \Gamma = 0$
2.  $\gamma_i \wedge \gamma_j = 0$
3.  $\forall xy$ , the strong edge of  $\hat{G}$ ,  $\wedge \{\gamma_i(x), \gamma_j(y)\} = 0, (1 \leq i \leq k)$ .

**Definition 2.3.** The family  $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k\}$  of intuitionistic fuzzy sets on a set  $V$  called  $k$ -vertex coloring [5] of  $\hat{G} = (V, E)$  when

1.  $\forall \gamma_i(x) = V \forall x \in V$
2.  $\gamma_i \wedge \gamma_j = 0$
3.  $\forall xy$ , strong edge of  $\hat{G}$ ,  $\min \{\gamma_i(\mu_1(x)), \gamma_i(\mu_1(y))\} = 0; \max \{\gamma_i(\nu_1(x)), \gamma_i(\nu_1(y))\} = 1, (1 \leq i \leq k)$ .

The Minimal utility of  $k$  of  $\hat{G}$  having  $k$ -vertex coloring is denoted by  $\chi(\hat{G})$ , implies vertex chromatic number of an intuitionistic fuzzy graph  $\hat{G}$ .

**Definition 2.4.** The family  $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k\}$  of intuitionistic fuzzy sets on  $E$ , the  $k$ -edge coloring of  $\hat{G} = (V, E)$  when

1.  $\forall \gamma_i(xy) = E \forall xy \in E$
2.  $\gamma_i \wedge \gamma_j = 0$
3.  $\forall xy$ , incident edges of  $E$ ,  $\min \{\gamma_i(\mu_2(xy))\} = 0; \max \{\gamma_i(\nu_2(xy))\} = 1, (1 \leq i \leq k)$ .

The minimal utility of  $k$  of  $\hat{G}$  having  $k$ -edge coloring denoted by  $\chi'(\hat{G})$ , implies edge chromatic number of an IFG  $\hat{G}$ .

**Definition 2.5.** The family  $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k\}$  of intuitionistic fuzzy sets on  $V$  and  $E$ ,  $k$ -total coloring of  $\hat{G} = (V, E)$  when

1.  $\forall \gamma_i(x) \vee \gamma_i(xy) = V \vee E \quad \forall x \in V, xy \in E$
2.  $\gamma_i \wedge \gamma_j = 0$
3.  $\forall xy$ , strong and incident vertices and edges of  $\hat{G}$ ,  $\min \{\gamma_i(\mu_1(x)), \gamma_i(\mu_1(y)), \gamma_i(\mu_2(xy))\} = 0$ ;  $\max \{\gamma_i(v_1(x)), \gamma_i(v_1(y)), \gamma_i(v_2(xy))\} = 1, (1 \leq i \leq k)$ .

In  $\hat{G}$ , the minimal utility of  $k$  having a  $k$ -total coloring denoted by  $\chi^T(\hat{G})$ , implies total chromatic number of an IFG  $\hat{G}$ .

### 3 Fuzzy coloring of an intuitionistic fuzzy graph

Generally, mixing any of two distinct colors, a new color is obtained. But, when white color is mixed with any one of the distinct color, the solidity of the distinct color gets reduced to some extent. Here the term solidity refers to the fuzzy term. Starting with a color  $C_k$  and  $\omega(\leq 1)$  units of color  $C_k$  is mixed with  $1 - \omega$  units of white, we obtain a standard mixture of  $C_k$ , and the resulting color is a fuzzy color of  $C_k$  with membership value  $\omega$ .

#### 3.1 Definition

Let  $\hat{C} = c_1, c_2, \dots, c_n, n \geq 1$  be as set of distinct colors. The fuzzy set  $(C, f)$  is called the set of fuzzy colors where  $f : C \rightarrow [0, 1]$ , with  $f(c_i)$ , the membership value of the color  $c_i$ , is the amount per unit of a standard mixture. The color  $\tilde{c}_i = (c_i, f(c_i))$  is called the fuzzy color corresponding to the basic color  $c_i$ . In contrast of fuzzy color, the membership value of a distinct color is 1. Thus a distinct color is a fuzzy color whose membership value is taken as 1.

Fuzzy coloring and total fuzzy coloring is performed with the identification of type of the edges of an intuitionistic fuzzy graphs (i) all strong edges, (ii) some strong edges (iii) all weak edges or no edges.

Case 1: Intuitionistic Fuzzy Graph with all strong edges. In fuzzy coloring and total coloring concept, a crisp graph, fuzzy graph, intuitionistic fuzzy graph containing only strong edges, the coloring is with the full solidity with any two distinct fuzzy colors.

Case 2: Intuitionistic Fuzzy Graph with some strong edges. Let  $V = \{V_i, i = 1, 2, \dots, n\}$  the neighborhood vertices set assuming that only  $(v, u_1), (v, u_2)$  have strong edges incident to  $v$  and all other edges  $(v, u_i), i = 3, 4, \dots, n$  are weak edges. The vertex  $v$  can be colored by the fuzzy color  $(C, 1)$  and  $u_1$  by another different fuzzy color  $(C_1, 1)$ . The same applies for  $(v, u_2)$  where  $v$  by the fuzzy color  $(C, 1)$  and  $u_2$  by another different fuzzy color  $(C_2, 1)$ . But at  $(v, u_3)$ , where it is a weak edge, the fuzzy color could be  $(C, 1)$  and  $(C, 0.9)$  at the other end of the vertex.

Case 3: Intuitionistic Fuzzy Graph with all weak edges or no edges. In an IFG, having all weak edges or no edges choose a vertex  $V$  and color with any of the fuzzy color say  $(C, 1)$ , then all other vertices can be colored with the same fuzzy color  $C_k$  by reducing its solidity. In the coloring of an IFG by fuzzy coloring, the membership value in the fuzzy color is the solidity of the color used.

### 3.2 Chromatic number in fuzzy coloring and total fuzzy coloring

The vertex chromatic number, edge chromatic number and total chromatic number is the minimal set of colors needed to properly color the vertex, edge and both vertex and edge of an IFG denoted by  $\chi(\hat{G})$ ,  $\chi'(\hat{G})$  and  $\chi^T(\hat{G})$ . Minimal value of  $k$ , in coloring vertices is vertex chromatic number of  $\hat{G} = (V, E)$ , has a  $k$ -vertex coloring denoted by  $\chi(\hat{G})$  and its fuzzy vertex chromatic number is denoted by  $\chi_F(\hat{G})$  with fuzzy vertex chromatic index,  $I[\chi_F(\hat{G})] = \{x, (C_k, 1)\}$  where  $x$  denotes the fuzzy vertex chromatic number. Minimal value of  $k$  in coloring edges is edge chromatic number of  $\hat{G} = (V, E)$ , has a  $k$ -edge coloring denoted by  $\chi'(\hat{G})$  and its fuzzy edge chromatic number is denoted by  $\chi'_F(\hat{G})$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{x', (C_K, 1)\}$  where  $x'$  denotes the fuzzy edge chromatic number. Minimal value of  $k$  in coloring both vertices and edges [3] is total chromatic number of  $\hat{G} = (V, E)$ , has a  $k$ -total coloring denoted by  $\chi^T(\hat{G})$  and its total fuzzy chromatic number is denoted by  $\chi_F^T(\hat{G})$ , with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{x^T, (C_k, 1)\}$  where  $x^T$  denotes the total fuzzy chromatic number.

## 4 Some various types of the intuitionistic fuzzy graph

In this paper, depending on the number of vertices and edges connectivity, some various types of intuitionistic fuzzy graphs [2], such as Null, Trivial, Directed, Undirected, Simple, Connected, Disconnected, Constant, Totally Constant, Complete, Cycle, Wheel, Cyclic, Acyclic and Star are discussed. For each type of an intuitionistic fuzzy graph here, Fuzzy coloring and Total Fuzzy coloring are introduced. With the Fuzzy coloring, the fuzzy vertex chromatic number and the fuzzy edge chromatic number with its indexes of an intuitionistic fuzzy graph are found and with the total fuzzy coloring, the total fuzzy chromatic number with its index is found. The fuzzy coloring to vertices in an intuitionistic fuzzy graph gives the fuzzy vertex chromatic number, the fuzzy coloring to edges in an intuitionistic fuzzy graph gives the fuzzy edge chromatic number and the total fuzzy coloring to both vertices and edges in an intuitionistic fuzzy graph gives the total fuzzy chromatic number. Depending upon the coloring to vertices and edges to an intuitionistic fuzzy graph, the values of the fuzzy vertex chromatic number  $\chi_F(\hat{G})$ , fuzzy edge chromatic number  $\chi'_F(\hat{G})$  and the total fuzzy chromatic number  $\chi_F^T(\hat{G})$  are determined.

The relationship between the chromatic number of an intuitionistic fuzzy graph and the fuzzy chromatic number of an intuitionistic fuzzy graph is analysed by the following theorem.

**Theorem 4.1.** *Let  $\hat{G}$  be an intuitionistic fuzzy graph. Then  $\chi(\hat{G}) \leq \chi_F(\hat{G})$ .*

*Proof.* Let  $\hat{G} = (V, \hat{E})$  be an intuitionistic fuzzy graph.

Let  $\hat{G}$  has  $n$  edges and then  $\chi(\hat{G}) = k$  with  $k \leq n$ . If all the edges of  $\hat{G}$  are strong, the chromatic number of  $\hat{G}$  is equal to the fuzzy chromatic number of an intuitionistic fuzzy graph  $\hat{G}$ . An additional value of fuzzy coloring is, it gives the name of the color and its solidity used with a notation. Also, two end vertices of an edge must be colored by two fuzzy colors obtained from the single color if the edge is weak or there is no edge. This also implies the chromatic number and the fuzzy chromatic number of an intuitionistic fuzzy graph are same, as the color used is same,

if the edges are weak and there is no edge, but the solidity of the colors at the vertices or edges differ, which is the novel concept of fuzzy coloring. Thus, if  $\hat{G}$  has  $n$  strong edges or weak edges or no edges, then  $\chi(\hat{G})$  may be less than  $\chi_F(\hat{G})$ . So,  $\chi(\hat{G}) \leq \chi_F(\hat{G})$ .  $\square$

**Note 4.1.** The number of weak edges of an intuitionistic fuzzy graph can be calculated if the chromatic number of an intuitionistic fuzzy graph and fuzzy chromatic number of the same intuitionistic fuzzy graph are well known.

**Note 4.2.** If all the edges of a complete intuitionistic fuzzy graph  $\hat{G} = (V, \hat{E})$  are strong, then  $\chi(\hat{G})$  and  $\chi_F(\hat{G})$  is the number of vertices of  $\hat{G}$ .

**Note 4.3.** Chromatic number of a crisp path is always 2. But, chromatic number of an intuitionistic fuzzy path is less than or equal to 2. If an intuitionistic fuzzy path has at least one strong edge, then the end vertices of that edge are colored by two distinct colors or two distinct fuzzy colors. So, the intuitionistic fuzzy path is 2-chromatic. If an intuitionistic fuzzy path has all weak edges or no edges, then it is 1-chromatic.

The various types of intuitionistic fuzzy graph are as follows.

## 4.1 Null intuitionistic fuzzy graph

A phrase “Null Intuitionistic Fuzzy Graph” refers to an IFG with no edges.

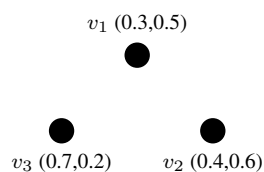


Figure 4.1

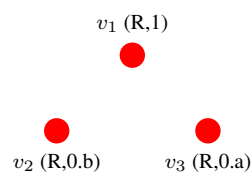


Figure 4.2

In Figure 4.1, there are three vertices  $v_1, v_2, v_3$  and there are no edges between them. For Figure 4.2, the vertex chromatic number of  $\hat{G}$  is  $\chi(\hat{G}) = 1$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 1$  with its fuzzy vertex chromatic index  $I[\chi(\hat{G})] = \{1, (R, 1)\}$  when fuzzy coloring is applied.

## 4.2 Trivial intuitionistic fuzzy graph

The term “Trivial Intuitionistic Fuzzy Graph” refers to an IFG with only precisely a single vertex. In Figure 4.3, there is only vertex  $v_1$  and there are no edges.

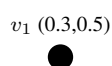


Figure 4.3



Figure 4.4

The vertex chromatic number of  $\hat{G}$  is  $\chi(\hat{G}) = 1$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 1$  with the fuzzy vertex chromatic index  $I[\chi(\hat{G})] = \{1, (R, 1)\}$  when fuzzy coloring is applied as shown in Figure 4.4.

### 4.3 Directed intuitionistic fuzzy graph

A “Directed Intuitionistic Fuzzy Graph” is an IFG in which each edge has a direction. It is to be noted that in a Directed Intuitionistic Fuzzy Graph in Figure 4.5,  $v_1v_2$  is different from  $v_2v_1$ .

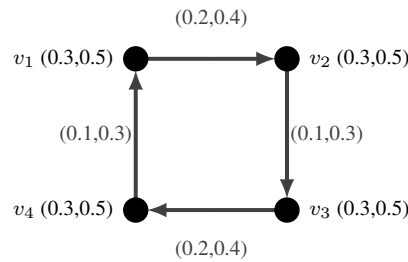


Figure 4.5

According to fuzzy coloring and total fuzzy coloring ideas, based on Figures 4.6 (a), (b), (c), the vertex chromatic number of  $\hat{G}$  is  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the edge chromatic number of  $\hat{G}$  is  $\chi'(\hat{G}) = 2$  and the fuzzy edge chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy edge chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ .

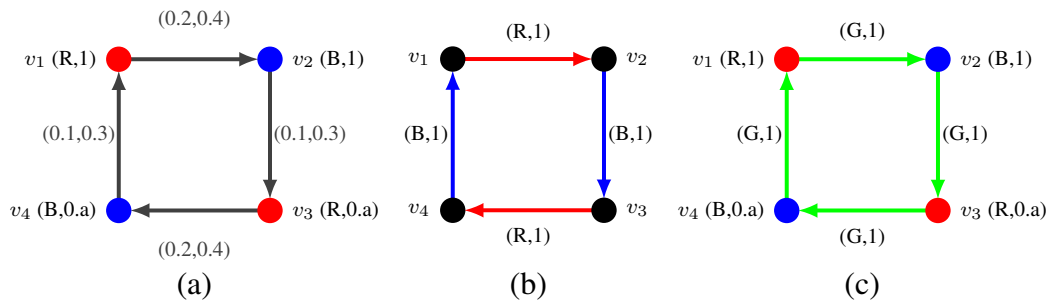


Figure 4.6

### 4.4 Undirected intuitionistic fuzzy graph

An IFG with no direction to each edge is an Undirected Intuitionistic Fuzzy Graph. It is to be noted that in an Undirected Intuitionistic Fuzzy Graph in Figure 4.7,  $v_1v_2$  is same as  $v_2v_1$ .

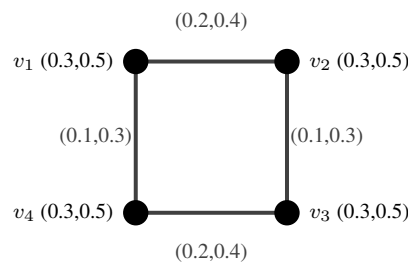


Figure 4.7

According to fuzzy coloring and total fuzzy coloring ideas, based on Figures 4.8 (a), (b), (c), the vertex chromatic number of  $\hat{G}$  is  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the edge chromatic number of  $\hat{G}$  is  $\chi'(\hat{G}) = 2$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 2$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ .

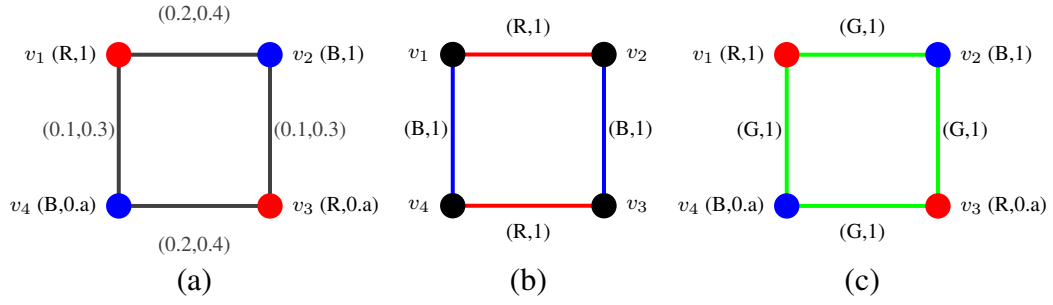


Figure 4.8

#### 4.5 Simple intuitionistic fuzzy graph

“Simple Intuitionistic Fuzzy Graph” is one that has no loops or parallel edges as in Figure 4.9.

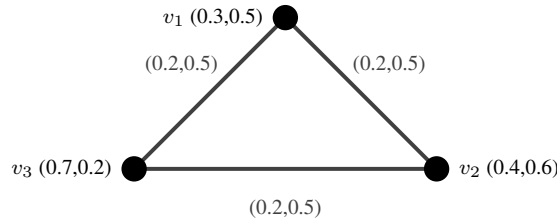


Figure 4.9

Figures 4.10 (a), (b), (c), illustrate the fuzzy coloring and total fuzzy coloring notions. According to these concepts, the graph's vertex chromatic number is  $\chi(\hat{G}) = 3$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 3$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the edge chromatic number is  $\chi'(\hat{G}) = 3$  and its fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 3$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the total chromatic number is  $\chi^T(\hat{G}) = 4$  and its total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 4$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ .

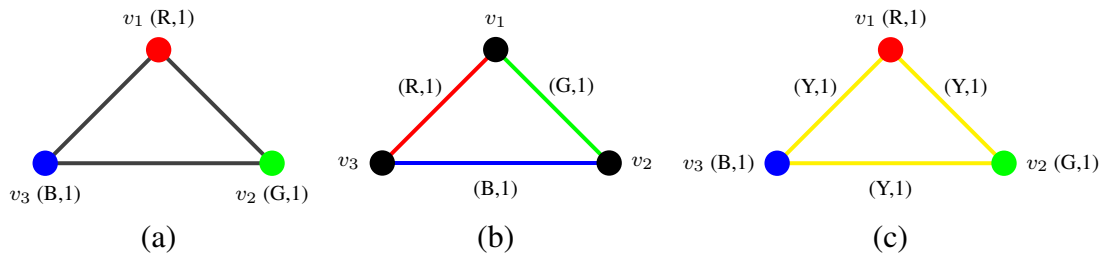


Figure 4.10

## 4.6 Connected intuitionistic fuzzy graph

An IFG is connected, if there exists a path between every pair of vertices as in Figure 4.11.

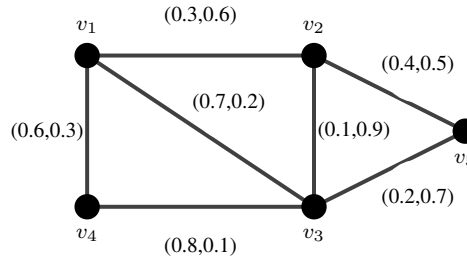


Figure 4.11

Figures 4.12 (a), (b), (c), show for  $\hat{G}$ , the vertex chromatic number is  $\chi(\hat{G}) = 3$  and fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 3$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the edge chromatic number is  $\chi_F(\hat{G}) = 4$  and the fuzzy edge chromatic number is  $\chi_F(\hat{G}) = 4$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ , the total chromatic number is  $\chi^T(\hat{G}) = 5$  and total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 5$  with the total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{5, (R, 1), (B, 1), (G, 1), (Y, 1), (BL, 1)\}$  all in accordance with the fuzzy coloring and total fuzzy coloring ideas.

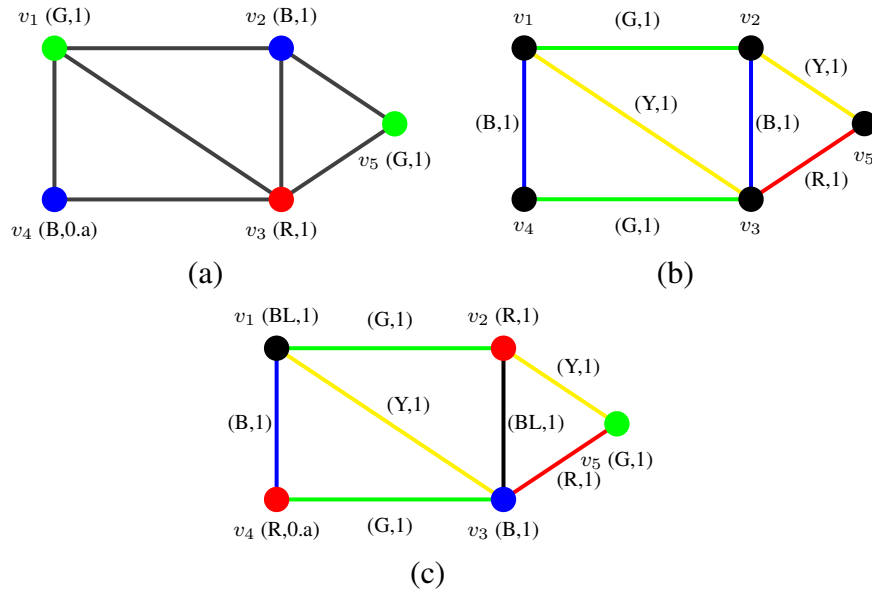


Figure 4.12

## 4.7 Disconnected intuitionistic fuzzy graph

When an intuitionistic fuzzy graph has at least two vertices that are not linked by an edge, we term it as a Disconnected intuitionistic fuzzy graph as in Figure 4.13.



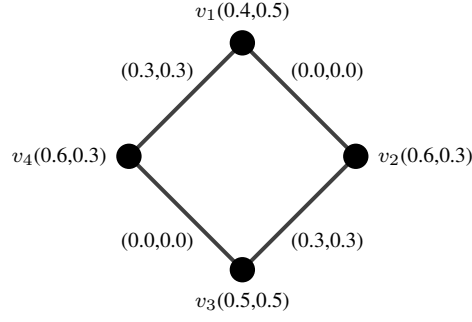


Figure 4.13

The vertex chromatic number is  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the edge chromatic number is  $\chi'(\hat{G}) = 1$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 1$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{1, (G, 1)\}$ , the total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$  from Figures 4.14 (a), (b), (c).

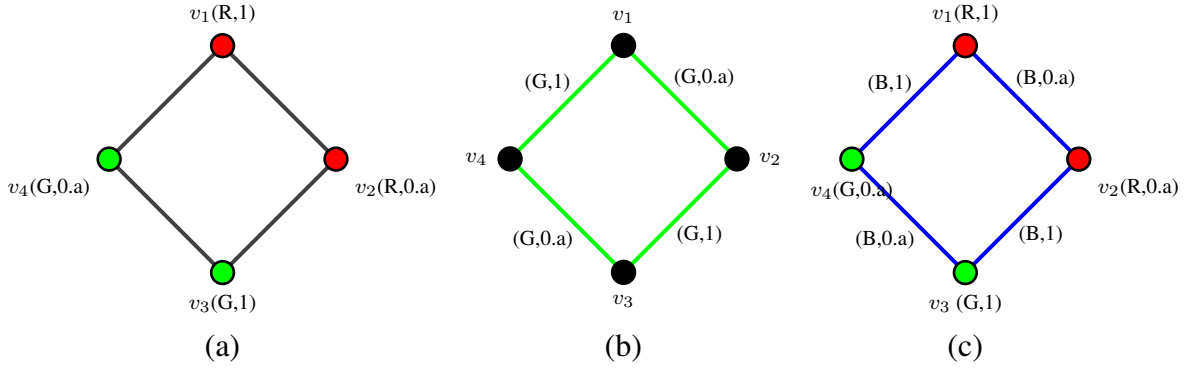


Figure 4.14

## 4.8 Constant Intuitionistic Fuzzy Graph

A constant IFG has the same degrees at every vertex. It charts the degree  $\Delta = k$ , where  $k = k_i + k_j$  in all vertices [1]. In Figure 4.15, the degree of  $v_1, v_2, v_3, v_4$  is  $(0.6, 0.7)$ .

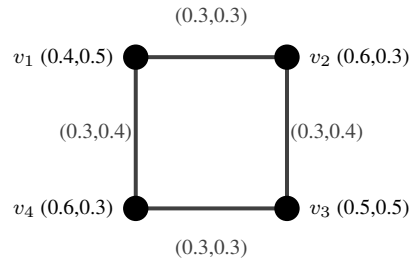


Figure 4.15

By the fuzzy coloring and total fuzzy coloring concepts, from Figures 4.16 (a), (b) and (c), the vertex chromatic number is  $\chi(\hat{G}) = 2$ , and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the edge chromatic number is  $\chi'(\hat{G}) = 2$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 2$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the total fuzzy chromatic number is  $\chi^T_F(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi^T_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$

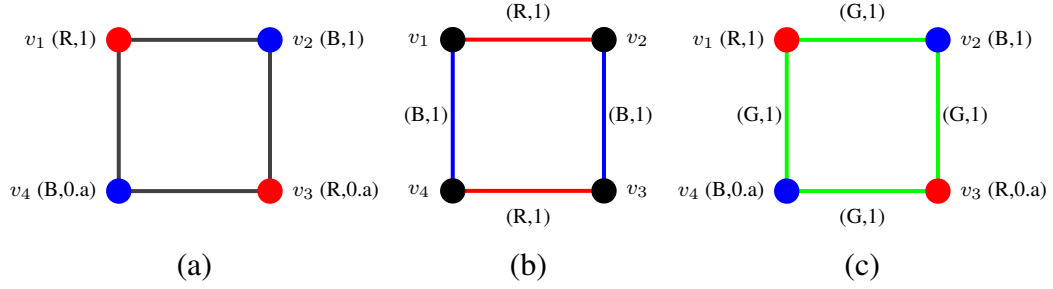


Figure 4.16

## 4.9 Totally constant intuitionistic fuzzy graph

An IFG is Totally Constant iff each of its vertices has the same degree. The Totally Constant Intuitionistic Fuzzy Graph in Figure 4.17 has the same degree in each of its vertices.

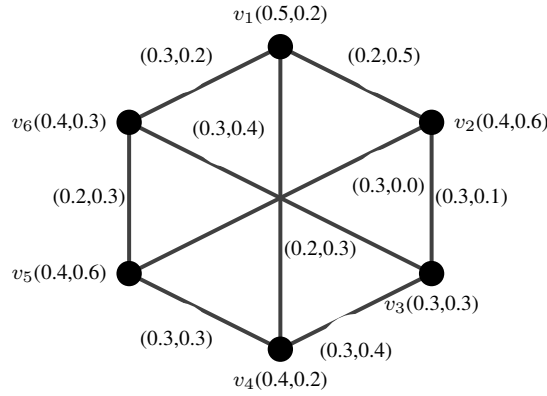


Figure 4.17

Figures 4.18 (a), (b), (c), imply that when using fuzzy coloring and total fuzzy coloring ideas, the vertex chromatic number is  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the edge chromatic number is  $\chi'(\hat{G}) = 5$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 5$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{5, (R, 1), (B, 1), (G, 1), (Y, 1), (BL, 1)\}$ , the total chromatic number is  $\chi^T(\hat{G}) = 5$  and the total fuzzy chromatic number of the graph is  $\chi^T_F(\hat{G}) = 5$  with the total fuzzy chromatic index  $I[\chi^T_F(\hat{G})] = \{5, (R, 1), (B, 1), (G, 1), (Y, 1), (BL, 1)\}$ .

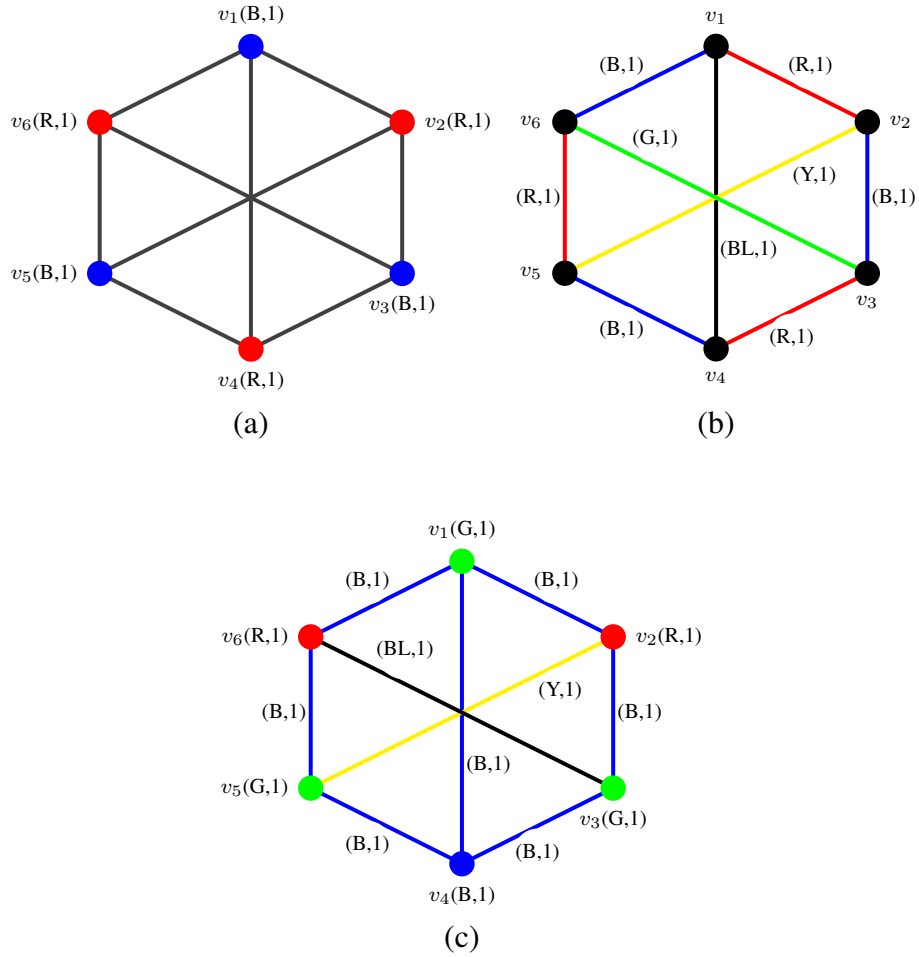


Figure 4.18

#### 4.10 Complete intuitionistic fuzzy graph

The term Comprehensive or Complete refers to an intuitionistic fuzzy graph connects to other vertices. In simple words, all the vertex are connected to any other vertices, then it is a Complete intuitionistic fuzzy graph as in Figure 4.19.

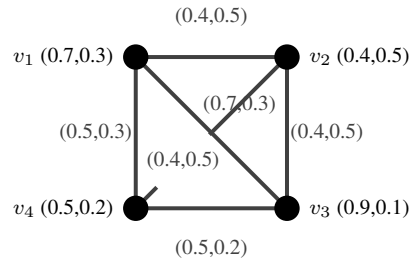


Figure 4.19

According to the fuzzy coloring and total fuzzy coloring ideas, Figures 4.20 (a), (b), (c) show that, the vertex chromatic number is  $\chi(\hat{G}) = 4$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ , the

edge chromatic number is  $\chi'(\hat{G}) = 4$  and the fuzzy edge chromatic index is  $\chi'_F(\hat{G}) = 4$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ , the total chromatic number is  $\chi^T(\hat{G}) = 4$  and the fuzzy total chromatic number is  $\chi_F^T(\hat{G}) = 4$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$

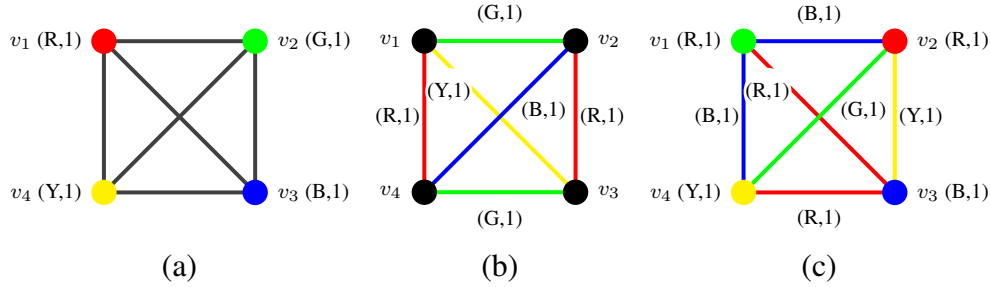


Figure 4.20

#### 4.11 Cycle intuitionistic fuzzy graph

An IFG which has vertices  $v_1, \dots, v_n$  and also  $n \geq 3$  i.e., there should be minimum three vertices and three edges to form a cycle. This shows that two edges will eventually create a cycle starting at a vertex which implies a Cycle Intuitionistic Fuzzy Graph. In Figure 4.21, there are five vertices and five edges which is developing a cycle  $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1$ .

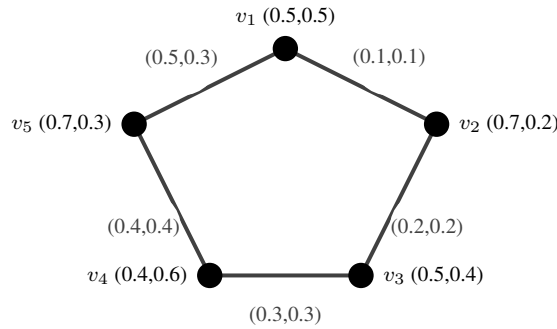


Figure 4.21

Figures 4.22 (a), (b), (c), show that the vertex chromatic number is  $\chi(\hat{G}) = 3$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 3$  with fuzzy vertex chromatic  $I[\chi_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the edge chromatic number is  $\chi'(\hat{G}) = 3$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 3$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the total chromatic number is  $\chi^T(\hat{G}) = 4$  and its total fuzzy chromatic number of the graph is  $\chi_F^T(\hat{G}) = 4$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$  all in accordance with the fuzzy coloring and total fuzzy coloring ideas.

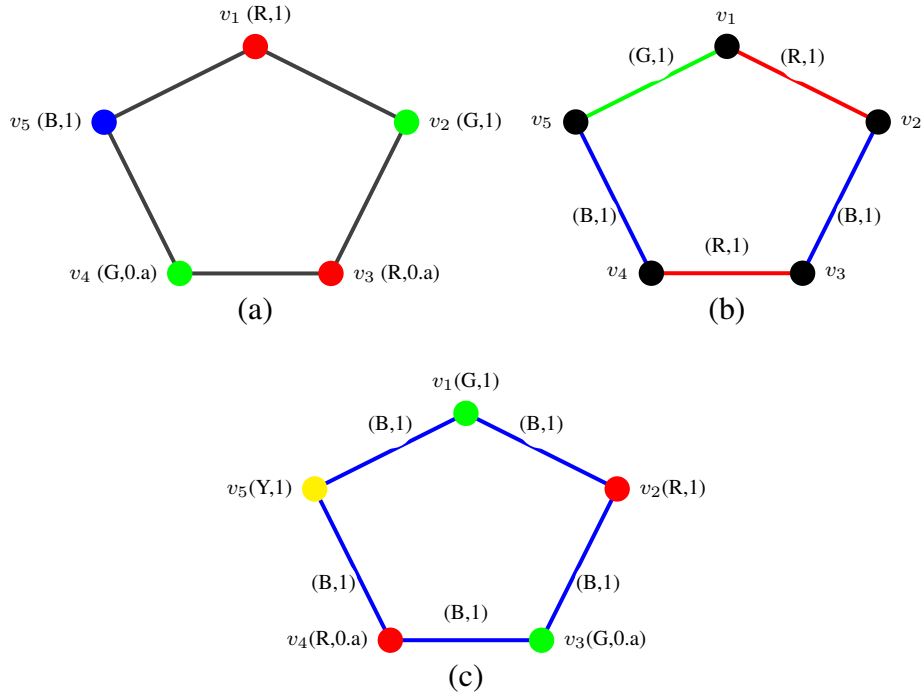


Figure 4.22

#### 4.12 Wheel intuitionistic fuzzy graph

Cycle intuitionistic fuzzy graphs are said to have the shape of a wheel when an additional vertex is added and it links each of the vertices through edges. In the wheel intuitionistic fuzzy graph in Figure 4.23, the vertex  $v_2$  is called as the hub of all the vertices.

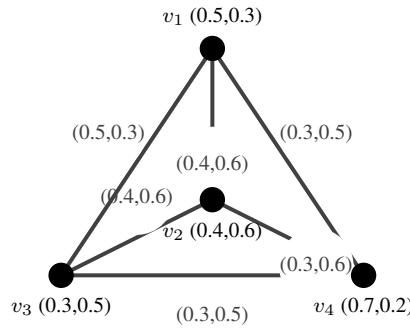


Figure 4.23

By fuzzy coloring and total fuzzy coloring concepts, from Figures 4.24 (a), (b), (c), the vertex chromatic number is  $\chi(\hat{G}) = 4$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 4$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ , the edge chromatic number of  $\hat{G}$  is  $\chi'(\hat{G}) = 3$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 3$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 4$  and the total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 4$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ .

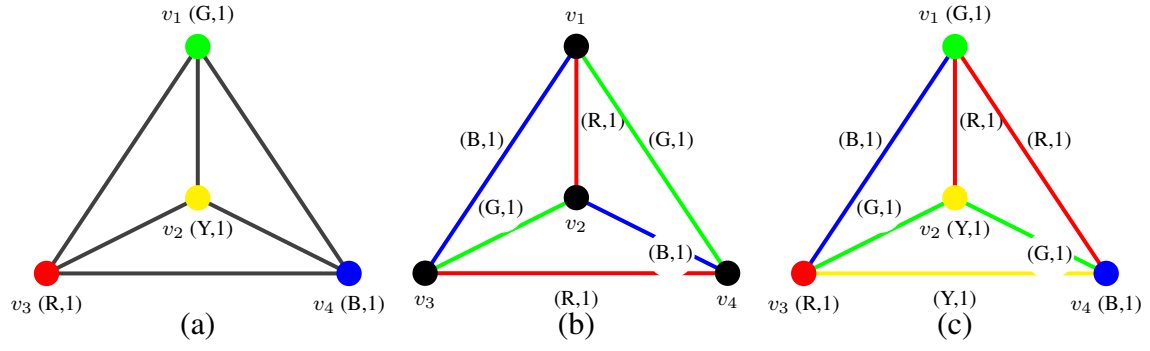


Figure 4.24

### 4.13 Cyclic intuitionistic fuzzy graph

For a Cyclic IFG, atleast one cycle is required it to be considered cyclic. In Figure 4.25, there are two Cycles  $v_1 - v_2 - v_3 - v_4 - v_1$  and  $v_3 - v_5 - v_6 - v_7 - v_3$ .

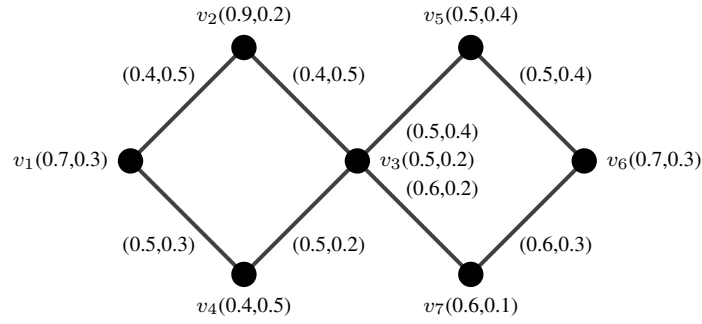
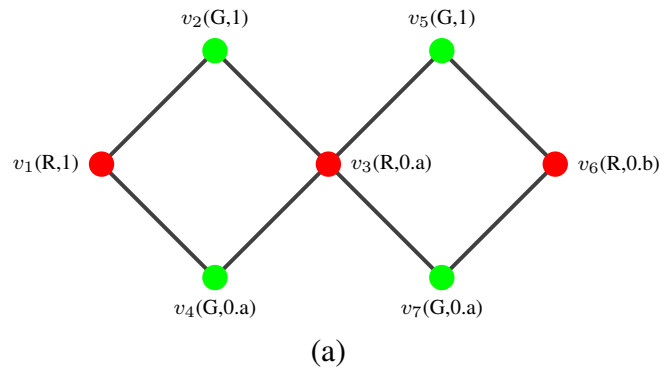


Figure 4.25

Figures 4.26 (a), (b), (c), demonstrate that the vertex chromatic number is  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (G, 1)\}$ , the edge chromatic number of  $\hat{G}$  is  $\chi'(\hat{G}) = 4$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 4$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{4, (R, 1), (B, 1), (G, 1), (Y, 1)\}$ , total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$  according to the fuzzy coloring and total fuzzy coloring ideas.



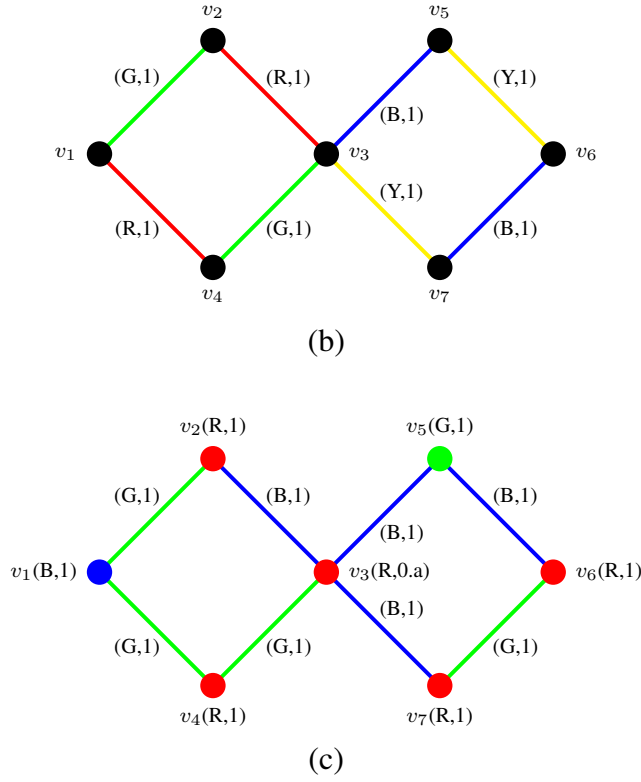


Figure 4.26

#### 4.14 Acyclic intuitionistic fuzzy graph

An IFG is acyclic iff it has no cycles in it. In Figure 4.27, the graph is not a proper cycle. Hence, it is acyclic.

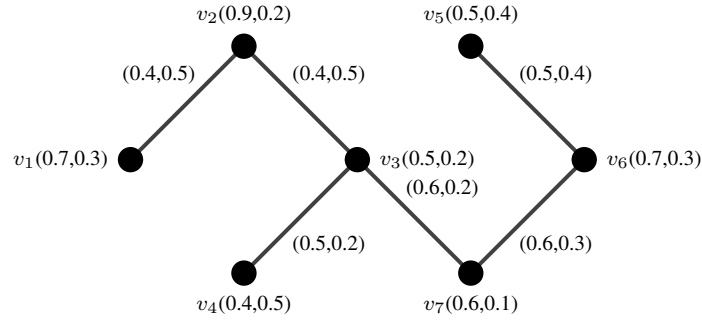


Figure 4.27

By the fuzzy coloring and total fuzzy coloring concepts, from Figures 4.28 (a),(b),(c), the vertex chromatic number is  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (G, 1)\}$ , the edge chromatic number of  $\hat{G}$  is  $\chi'(\hat{G}) = 3$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 3$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the fuzzy total chromatic number is  $\chi_F^T(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ .

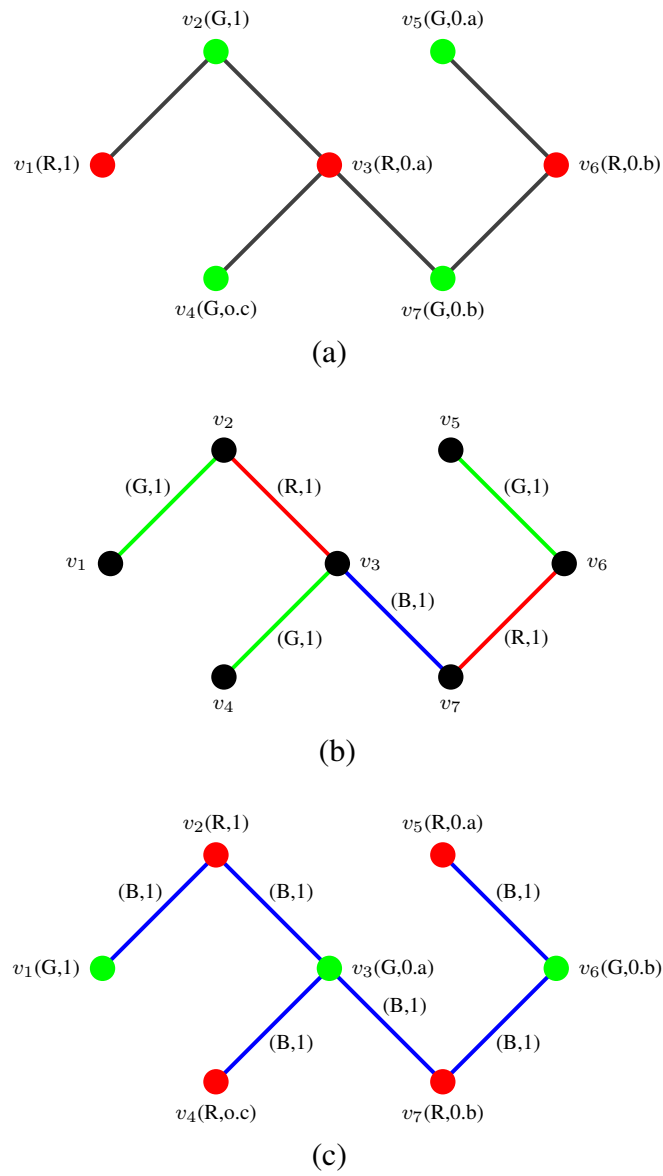


Figure 4.28

#### 4.15 Star intuitionistic fuzzy graph

Intuitionistic fuzzy graphs are said to be a star if every edge connects to the same single vertex [7]. In Figure 4.29, all the vertices  $v_2, v_3, v_4$  are connected to a single star vertex  $v_1$ .

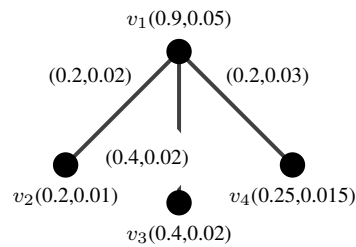


Figure 4.29



From Figures 4.30 (a), (b), (c), we can infer that the graph has a vertex chromatic number  $\chi(\hat{G}) = 2$  and the fuzzy vertex chromatic number is  $\chi_F(\hat{G}) = 2$  with fuzzy vertex chromatic index  $I[\chi_F(\hat{G})] = \{2, (R, 1), (B, 1)\}$ , the edge chromatic number of  $\hat{G}$  is  $\chi'(\hat{G}) = 3$  and the fuzzy edge chromatic number is  $\chi'_F(\hat{G}) = 3$  with fuzzy edge chromatic index  $I[\chi'_F(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$ , the total chromatic number of  $\hat{G}$  is  $\chi^T(\hat{G}) = 3$  and the total fuzzy chromatic number is  $\chi_F^T(\hat{G}) = 3$  with total fuzzy chromatic index  $I[\chi_F^T(\hat{G})] = \{3, (R, 1), (B, 1), (G, 1)\}$  using the fuzzy coloring and total fuzzy coloring ideas.

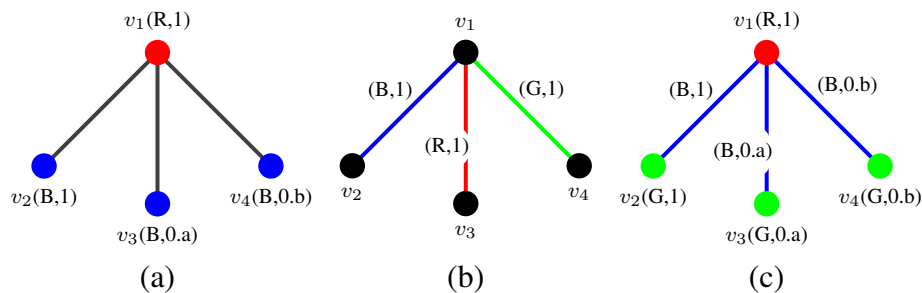


Figure 4.30

## 5 Conclusion

In this article, the chromatic number, chromatic index and total chromatic number of intuitionistic fuzzy graphs are found along with their vertex fuzzy chromatic number, edge fuzzy chromatic index and total fuzzy chromatic number and also their indexes by applying the methods of fuzzy coloring and total fuzzy coloring.

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