

**INEQUALITIES WITH INTUITIONISTIC FUZZY MODAL
AND TOPOLOGICAL OPERATORS**
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Abstract

In this article we are going to study the topological properties of the operators C_μ , C_v , I_μ , I_v and the modal operator $\square_{\alpha\beta\gamma\delta}$.

Keywords

Inequality, Intuitionistic fuzzy set, Modal operator, Topological operator.

The definition of Intuitionistic Fuzzy Set (IFS) can be found in [1]. For the needs of this research it will be used some properties of the following topological operators that are defined in [2]:

$$\begin{aligned} C_\mu(A) &= \left\{ \langle x, K, \min(1 - K, \nu_A(x)) \rangle \mid x \in E \right\} \\ C_v(A) &= \left\{ \langle x, \mu_A(x), L \rangle \mid x \in E \right\} \\ I_\mu(A) &= \left\{ \langle x, k, \nu_A(x) \rangle \mid x \in E \right\} \\ I_v(A) &= \left\{ \langle x, \min(1 - l), \mu_A(x) \rangle \mid x \in E \right\} \end{aligned}$$

where

$$K = \sup_{y \in E} \mu_A(y) \quad L = \min_{y \in E} \nu_A(y)$$

and

$$k = \inf_{y \in E} \mu_A(y) \quad l = \sup_{y \in E} \nu_A(y)$$

The following modal operator is defined in [3]:

$$\square_{\alpha\beta\gamma\delta}(A) = \left\{ \langle x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) + \delta \rangle \mid x \in E \right\}$$

The subject of this research is to proof the following:

Theorem: For IFS A and for every $\alpha, \beta, \gamma, \delta \in [0,1]$ such that $\max(\alpha, \beta) + \gamma + \delta \leq 1$.

1. $C_\mu \square_{\alpha\beta\gamma\delta}(A) \subseteq \square_{\alpha\beta\gamma\delta} C_\mu(A)$
2. $C_v \square_{\alpha\beta\gamma\delta}(A) = \square_{\alpha\beta\gamma\delta} C_v(A)$
3. $I_\mu \square_{\alpha\beta\gamma\delta}(A) = \square_{\alpha\beta\gamma\delta} I_\mu(A)$
4. $I_v \square_{\alpha\beta\gamma\delta}(A) \supseteq \square_{\alpha\beta\gamma\delta} I_v(A)$

Proof 1:

$$\begin{aligned}
C_\mu \square_{\alpha\beta\gamma\delta}(A) &= C_\mu \left(\left\{ \langle x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) + \delta \rangle \mid x \in E \right\} \right) \\
&= \left\{ \left\langle x, \sup_{y \in E} (\alpha\mu_A(y) + \gamma), \min(1 - \sup_{y \in E} (\alpha\mu_A(y)\gamma), \beta\nu_A(x) + \delta) \right\rangle \mid x \in E \right\} \\
&= \left\{ \langle x, \alpha K + \gamma, \min(1 - \alpha K - \gamma, \beta\nu_A(x) + \delta) \rangle \mid x \in E \right\} \\
\square_{\alpha\beta\gamma\delta} C_\mu(A) &= \square_{\alpha\beta\gamma\delta} \left(\left\{ \langle x, K, \min(1 - K, \nu_A(x)) \rangle \mid x \in E \right\} \right) \\
&= \left\{ \langle x, \alpha K + \gamma, \beta \min(1 - K, \nu_A(x)) + \delta \rangle \mid x \in E \right\}
\end{aligned}$$

The first components of both sets coincide.

Let

$$\begin{aligned}
x &\equiv \min(1 - \alpha K - \gamma, \beta\nu_A(x) + \delta) - \beta \min(1 - K, \nu_A(x)) - \delta \\
&= \min(1 - \alpha K - \gamma, \beta\nu_A(x) + \delta) - \min(\beta - \beta K + \delta, \beta\nu_A(x) + \delta).
\end{aligned}$$

Because:

$$\begin{aligned}
1 - \alpha K - \gamma - \beta + \beta K - \delta &= 1 - \gamma - \delta - K(\alpha - \beta) \\
&\geq \max(\alpha, \beta) - K(\alpha - \beta) \geq \max(\alpha, \beta) - (\alpha - \beta) \geq 0,
\end{aligned}$$

then $x \geq 0$

Therefore it is true that

$$C_\mu \square_{\alpha\beta\gamma\delta}(A) \subseteq \square_{\alpha\beta\gamma\delta} C_\mu(A)$$

Proof 2:

$$\begin{aligned}
C_\nu \square_{\alpha\beta\gamma\delta}(A) &= C_\nu(\alpha\mu_A(x) + \gamma, \beta\nu_A(x) + \delta) \\
&= \alpha\mu_A(x) + \gamma, \beta L + \delta = \square_{\alpha\beta\gamma\delta}(\mu_A(x), L) = \square_{\alpha\beta\gamma\delta} C_\nu(A)
\end{aligned}$$

The proof of the other inequalities is analogous.

References

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