Notes on Intuitionistic Fuzzy Sets
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A note on intuitionistic fuzzy countable dense homogeneous spaces

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Abstract: The countable dense homogeneity (CDH) property which is an important tool in general topology, states that in a CDH space, any two countable dense sets are homeomorphic. In this paper, we have extended the CDH property to intuitionistic fuzzy topological spaces and found it as a proper extension.

Keywords: Countable dense homogeneity, Intuitionistic fuzzy homeomorphism, Intuitionistic fuzzy topology.
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1 Introduction

It was L. A. Zadeh [6] who introduced fuzzy sets in 1965, and the concept has widened into an active area of research that connects traditional mathematics with the study of uncertainty. Further, in 1983, Atanassov [2] proposed a generalization of the notion of fuzzy set called the Intuitionistic fuzzy set (IFS). Later, the mathematician Çoker [5] introduced the concept called Intuitionistic fuzzy topological space (IFTS). Many of the concepts in ordinary topological space like connectedness, compactness, continuity, separation axioms, etc., were extended to IFTS by various mathematicians. Among the different homogeneity concepts, countable dense homogeneity (CDH) [4] property is of great importance in general topology and a hot area of research. The property states that in a CDH space any two countable dense sets are homeomorphic. The fuzzy extension of CDH is initiated by Samer Al Ghour and Ali Fora [1] who laid the foundations for this paper. We found it necessary to extend CDH to include intuitionistic fuzzy topological spaces. One of the main goals of this paper is to show how the definition of countable dense homogeneous ordinary topological spaces can be amended to define a good extension of it in intuitionistic fuzzy topological spaces.

2 Preliminary concepts

Some basic notations and definitions needed for our study are given here [3]: Let Υ be a non-empty set. An *intuitionistic fuzzy set* (IFS in short) A is represented as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in \Upsilon\}$ where $\mu_A(x), \nu_A(x) : \Upsilon \to [0,1]$ denotes the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in \Upsilon$ to the set A, respectively, together with the constraint $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in \Upsilon$. The IFSs $\{\langle x, 0, 1\rangle \mid x \in \Upsilon\}$ and $\{\langle x, 1, 0\rangle \mid x \in \Upsilon\}$ are denoted by 0 ("Truth") and 1 ("Falsity"), respectively. If $\{A_i : i \in I\}$ is a collection of IFSs on Υ, then

- $A_i \subseteq A_j$ if and only if $\mu_{A_i}(x) \leq \mu_{A_j}(x)$ and $\nu_{A_i}(x) \geq \nu_{A_j}(x)$
- $A_i = A_j$ if and only if $A_i \subseteq A_j$ and $A_j \subseteq A_i$
- Complement of A_i is $A_i^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in \Upsilon\}$
- $\cup A_i = \{\langle x, \vee\mu_{A_i}, \wedge\nu_{A_i}\rangle \mid x \in \Upsilon\}$
- $\cap A_i = \{\langle x, \wedge\mu_{A_i}, \vee\nu_{A_i}\rangle \mid x \in \Upsilon\}$

Let $c \in \Upsilon$ and $0 < \alpha, \beta < 1$ with $\alpha + \beta \leq 1$, then an *intuitionistic fuzzy point* (IFP for short) $c(\alpha, \beta)$ is the IFS $\{\langle x, c_\alpha, 1 - c_{1-\beta}\rangle \mid x \in \Upsilon\}$, where c_α is a fuzzy point with support c and value α. An IFP $c(\alpha, \beta)$ is said to *belong to* an IFS $A = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in \Upsilon\}$, denoted as $c(\alpha, \beta) \in A$ if and only if $\alpha < \mu_A(x)$ and $\beta > \nu_A(x)$. A family $\mathcal{T}$ of IFSs is said to be an *intuitionistic fuzzy topology* abbreviated as IFT for Υ if it satisfies the three axioms:

- $0, 1 \in \mathcal{T}$;
- $\forall\, A_1, A_2 \in \mathcal{T} \implies A_1 \cap A_2 \in \mathcal{T}$;
- $\forall\, (A_i) \subseteq \mathcal{T} \implies \cup A_i \in \mathcal{T}$.

In this case $(\Upsilon, \mathcal{T})$ is called an intuitionistic fuzzy topological space (IFTS, for short) and any IFS in $\mathcal{T}$ is called an intuitionistic fuzzy open set (IFOS, for short). An IFS B is said to be closed with respect to $\mathcal{T}$ if $B = A^c$ for some $A \in \mathcal{T}$. The closure of an IFS A is B if any only if B is the smallest closed set containing A, expressed as $\mathrm{cl}(A) = B$.

Let $\Upsilon$and Y be two non-empty sets and $f : \Upsilon \to Y$ be a mapping. Then,

- If $B = \{\langle y, \mu_B, \nu_B\rangle \mid y \in Y\}$ is an IFS in Y, the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by: $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B), f^{-1}(\nu_B)\rangle \mid x \in \Upsilon\}$.
- If $A = \{\langle x, \mu_A, \nu_A\rangle \mid x \in \Upsilon\}$ is an IFS in Υ, the image of A under f, denoted by $f(A)$, is the IFS in Y defined by: $f(A) = \{\langle y, f(\mu_A), 1 - f(1 - \nu_A)\rangle \mid y \in Y\}$ where:

$$f(\mu_A)(y) = \begin{cases} \sup\{\mu_A(x) \mid x \in f^{-1}(y)\}; & f^{-1}(y) \neq \emptyset \\ 0; & f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \nu_A))(y) = \begin{cases} \inf\{\nu_A(x) \mid x \in f^{-1}(y)\}; & f^{-1}(y) \neq \emptyset \\ 1; & f^{-1}(y) = \emptyset \end{cases}$$

Thus, if f is a bijective function, then $f(A) = \{\langle y, \mu_A(f^{-1}(y)), \nu_A(f^{-1}(y))\rangle \mid y \in Y\}$. Let $(\Upsilon, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two IFTS and $h : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ be a mapping. Then h is said to be *intuitionistic fuzzy continuous* if $\forall B \in \mathcal{T}_2 \implies f^{-1}(B) \in \mathcal{T}_1$. Then, h is said to be an *intuitionistic fuzzy homeomorphism* if and only if h is bijective and both h, h^{-1} are continuous.

3 Intuitionistic fuzzy countable dense homogeneity

In the classical sense, a subset $A \subseteq X$ of a topological space X is said to be dense in X if and only if every open set of X intersects A; equivalently, the closure of A is X itself. Extending this notion to the intuitionistic fuzzy framework gives rise to two non-equivalent concepts, namely dense(I) and dense(II). Building upon these, we further generalize an important concept from ordinary topology—the Countable Dense Homogeneity (CDH) property. A space X is said to possess the CDH property if any two countable dense subsets of X are homeomorphic. This work extends the CDH concept to intuitionistic fuzzy topological spaces, introducing the notion of Intuitionistic Fuzzy Countable Dense Homogeneity.

Definition 1. *A collection D of IFPs in an IFTS $(\Upsilon, \mathcal{T})$ is said to be dense(I) if for every non-zero $O \in \mathcal{T}$ there exists $c(\alpha, \beta) \in D$ such that $c(\alpha, \beta) \subseteq O$.*

Example 1. *For a fixed $c \in X$, define $D_c = \{c(\alpha, \beta) \mid \alpha \in \mathbb{Q} \cap [0, 1]$ and $\alpha + \beta \leq 1\}$. Then D_c is dense(I), since for any $O \in \mathcal{T}$, there exist $\alpha < \mu_O(c)$ and $\beta = \nu_O(c) + \mu_O(c) - \alpha$ such that $c(\alpha, \beta) \subseteq O$.*

Definition 2. *An IFTS $(\Upsilon, \mathcal{T})$ is said to be separable(I) if it contains a dense(I) set of IFPs.*

Lemma 1. *If $f : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is an intuitionistic fuzzy homeomorphism and D is a dense(I) collection of IFPs in Y, then $f^{-1}(D)$ is dense(I) in X.*

Proof. Let $O = \{\langle x, \mu_O(x), \nu_O(x)\rangle \mid x \in X\} \in \mathcal{T}_1$. Then, f being a homeomorphism implies $f(O) = \{\langle y, \mu_O(f^{-1}(y)), \nu_O(f^{-1}(y))\rangle \mid y \in Y\} \in \mathcal{T}_2$. Since D is dense(I) in Y, there exist $c \in Y$ and $\alpha, \beta \in (0, 1)$ with $\alpha + \beta \leq 1$ such that $c(\alpha, \beta) \in D$ and $c(\alpha, \beta) \subseteq f(O)$. Then

$$\alpha \leq \mu_O(f^{-1}(c)) \text{ and } \beta \geq \nu_O(f^{-1}(c)) \tag{1}$$

Since $\{\langle y, c_\alpha, 1 - c_{1-\beta}\rangle\} = c(\alpha, \beta) \in D$,

$$\Longrightarrow \{\langle x, f^{-1}(c_\alpha)(x), f^{-1}(1 - c_{1-\beta})(x)\rangle \mid x \in X\} = f^{-1}[c(\alpha, \beta)] \in f^{-1}(D)$$
$$\Longrightarrow \{\langle x, c_\alpha(f(x)), 1 - c_{1-\beta}(f(x))\rangle \mid x \in X\} = f^{-1}[c(\alpha, \beta)] \in f^{-1}(D).$$

Thus $f^{-1}[c(\alpha, \beta)] \in O$ if and only if the membership value α and the non-membership value β that are obtained at $f(x) = c$ or $x = f^{-1}(c)$ satisfy $\alpha \leq \mu_O(f^{-1}(c))$ and $\beta \geq \nu_O(f^{-1}(c))$, which is true by (1). □

Theorem 1. *Separability(I) is a topological property.*

Proof. Let $(\Upsilon, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two IFTSs such that $(Y, \mathcal{T}_2)$ is separable(I). Then there exists a collection D of IFPs of Y, satisfying the dense(I) condition. Let $f : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ be a homeomorphism, then by Lemma 1, $f^{-1}(D)$ is a dense(I) collection of IFPs in X. Hence $(\Upsilon, \mathcal{T}_1)$ is separable(I). □

Definition 3. *A collection D of intuitionistic fuzzy points in an IFTS $(\Upsilon, \mathcal{T})$ is said to be dense(II) if*

$$\mathrm{cl}\Big(\bigcup_{c(\alpha,\beta)\in D} c(\alpha, \beta)\Big) = 1.$$

Example 2. *For any fixed $x \in X$, $\mathbb{Q}(D_x^\alpha) = \{x(\alpha, \beta) \mid \alpha \in \mathbb{Q} \cap [0, 1]$ and $\alpha + \beta \leq 1\}$ is dense(II). For, any closed set $A = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$, there exists $\alpha \in \mathbb{Q} \cap [0, 1]$ such that $\alpha \geq \nu_A(x)$ and $x(\alpha, \beta) \in \mathbb{Q}(D_x^\alpha)$.*

The same case pertains for $\mathbb{Q}'(D_x^\beta) = \{x(\alpha, \beta) \mid \beta \in \mathbb{Q} \cap [0, 1]$ and $\alpha + \beta \leq 1\}$, since there exists $\beta \in \mathbb{Q} \cap [0, 1]$ such that $\beta \leq \mu_A(x)$ and $x(\alpha, \beta) \in \mathbb{Q}'(D_x^\beta)$.

Definition 4. *An IFTS $(\Upsilon, \mathcal{T})$ is said to be separable(II) if it contains a dense(II) collection of IFPs.*

Lemma 2. *For a bijective function $f : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$, if D is a dense(II) set of IFPs in Υ, then $f(D)$ is dense(II) in Y.*

Proof. Let $D = \{c(\alpha, \beta) \mid 0 < \alpha, \beta < 1$ and $\alpha + \beta \leq 1\}$ be dense(II) in Υ. First we shall prove that $f(c(\alpha, \beta)) = f(c)(\alpha, \beta)$, thereby implying $f(D) = \{f(c)(\alpha, \beta) \mid c(\alpha, \beta) \in D\}$. Now,

$$\begin{aligned}
f(c(\alpha, \beta)) &= f(\{\langle x, c_\alpha(x), 1 - (c_{1-\beta})(x)\rangle \mid x \in \Upsilon\}) \\
&= \{\langle y, c_\alpha(f^{-1}(y)), 1 - c_{1-\beta}(f^{-1}(y))\rangle \mid y \in Y\} \\
&= f(c)(\alpha, \beta) \\
\therefore \bigcup_{f(c)(\alpha,\beta)\in f(D)} f(c)(\alpha, \beta) &= f\Big(\bigcup_{c(\alpha,\beta)\in D} c(\alpha, \beta)\Big) \\
\Longrightarrow \mathrm{cl}\Big(\bigcup_{f(c)(\alpha,\beta)\in f(D)} f(c)(\alpha, \beta)\Big) &= \mathrm{cl}\Big(f\Big(\bigcup_{c(\alpha,\beta)\in D} c(\alpha, \beta)\Big)\Big) \\
&= f\Big(\mathrm{cl}\Big(\bigcup_{c(\alpha,\beta)\in D} c(\alpha, \beta)\Big)\Big) \\
&= f(1), \text{ by the hypothesis} \\
&= 1.
\end{aligned}$$

□

Theorem 2. *Separability*(II) *is a topological property.*

Proof. Let $(\Upsilon, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two IFTSs such that $(\Upsilon, \mathcal{T}_1)$ is separable(II). Then there exists a collection D of IFPs of Υ, satisfying the dense(II) condition. Let $f : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ be an intuitionistic fuzzy homeomorphism, then by Lemma 2, $f(D)$ is dense(II) in Y. Hence $(Y, \mathcal{T}_2)$ is separable(II). □

Definition 5. *For a collection* $D = \{c(\alpha, \beta) \mid 0 < \alpha, \beta < 1$ *and* $\alpha + \beta \leq 1\}$ *of IFPs, we define the following:*

$$S(D) = \{c \in \Upsilon \mid c(\alpha, \beta) \in D\},$$
$$\mathbb{Q}(S(D)) = \{c(\alpha, \beta) \mid c \in S(D); \alpha, \beta \in \mathbb{Q} \cap (0, 1) \text{ and } \alpha + \beta \leq 1\}.$$

Remark 1. *Note that dense*(I) *and dense*(II) *are not equivalent concepts. That is,* D *is dense*(I) *need not imply* D *is dense*(II)*, and vice-versa. However, the following theorem shows that separability*(I) *and separability*(II) *are equivalent.*

Theorem 3. *For an IFTS* $(\Upsilon, \mathcal{T})$ *and a collection* D *of IFPs of* Υ*, the following are true:*

(i) If D is dense(I), then $\mathbb{Q}(S(D))$ is dense(II).
(ii) If D is dense(II), then $\mathbb{Q}(S(D))$ is dense(I).

Proof. (i) Let $D = \{c(\alpha, \beta) \mid 0 < \alpha, \beta < 1$ and $\alpha + \beta \leq 1\}$ be dense(I). Suppose $\mathbb{Q}(S(D))$ is not dense(II). Then, $\text{cl}(\bigcup_{c(\alpha,\beta)\in\mathbb{Q}(S(D))} c(\alpha, \beta)) \neq 1$. Clearly $\bigcup_{c(\alpha,\beta)\in\mathbb{Q}(S(D))} c(\alpha, \beta)$ will have membership value '1' and non-membership value '0' to all $c \in S(D)$ and membership value '0' and non-membership value '1' to all $c \notin S(D)$.
Let

$$\text{cl}(\bigcup_{c(\alpha,\beta)\in\mathbb{Q}(S(D))} c(\alpha, \beta)) = A \neq 1 \implies \bigcup_{c(\alpha,\beta)\in\mathbb{Q}(S(D))} c(\alpha, \beta) \subseteq A.$$

Thus A will also have membership value '1' and non-membership value '0' to all $c \in S(D)$, which implies A^c would have membership value '0' and non-membership value '1' for all $c \in S(D)$. But $A \neq 1$ being closed would imply A^c is a non-zero open set in Υ. And since D is dense(I), there exists $c(\alpha, \beta) \in D$ such that $c(\alpha, \beta) \in A^c$.

$$\implies 0 < \alpha < \mu_{A^c}(c) \text{ and } 1 > \beta \geq \nu_{A^c}(c) \text{ with } c \in S(D)$$
$$\implies \mu_{A^c}(c) > 0 \text{ and } \nu_{A^c}(c) < 1 \text{ for } c \in S(D),$$

which contradicts the above.

(ii) Let D be dense(II). Then the smallest closed set containing $\bigcup_{c(\alpha,\beta)\in D} c(\alpha, \beta)$ is 1. Suppose that $\mathbb{Q}(S(D))$ is not dense(I).

$$\implies \exists\, O \neq 0 \in \mathcal{T} \text{ such that } c(\alpha, \beta) \nsubseteq O, \forall c(\alpha, \beta) \in \mathbb{Q}(S(D))$$
$$\implies \mu_O(c) = 0 \text{ or } \nu_O(c) = 1, \forall c \in S(D)$$
$$\implies \mu_{O^c}(c) = 1 \text{ and } \nu_{O^c}(c) = 0, \forall c \in S(D)$$
$$\implies \bigcup_{c(\alpha,\beta)\in D} c(\alpha, \beta) \subseteq O^c \neq 1$$

which contradicts our hypothesis. □

Theorem 4. *An IFTS $(\Upsilon, \mathcal{T})$ is separable(I) if and only if it is separable(II).*

Proof. Let $(\Upsilon, \mathcal{T})$ be separable(I). Then, there exists a collection D of dense(I) IFPs and by Theorem 3, $Q(S(D))$ is dense(II). Therefore, $(\Upsilon, \mathcal{T})$ is separable(II).

The converse is similiar. □

Remark 2. *The above theorem encourages us to simply call an IFTS $(\Upsilon, \mathcal{T})$ separable if it is separable(I) or separable(II).*

Definition 6. *An IFTS $(\Upsilon, \mathcal{T})$ is said to be intuitionistic fuzzy countable dense homogeneous abbreviated as IFCDH if for every countable dense(I) sets C and D of intuitionistic fuzzy points, there exists an intuitionistic homeomorphism $h : \Upsilon \to \Upsilon$ such that $h(S(C)) = S(D)$.*

Lemma 3. *Let $h : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ be a bijective mapping and D be a collection of intuitionistic fuzzy points of the IFTS $(\Upsilon, \mathcal{T}_1)$. Then, $S(h(D)) = h(S(D))$.*

Proof. It is enough to prove when D is a singleton set. Let $D = \{c(\alpha, \beta)\}$. Then $S(D) = \{c\}$ $\implies h(S(D)) = \{h(c)\}$. Now, $h(D) = h(c(\alpha, \beta)) = h(c)(\alpha, \beta) \implies S(h(D)) = \{h(d)\}$. □

Theorem 5. *Intuitionistic fuzzy countable dense homogeneity is a topological property.*

Proof. Let $(\Upsilon, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two IFTSs and $(\Upsilon, \mathcal{T}_1)$ be IFCDH. Suppose $f : (\Upsilon, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is a homeomorphism and let C, D be two countable dense(I) sets of IFPs of Y. Then $f^{-1}(C), f^{-1}(D)$ are countable dense(I) sets of IFPs of Υ. Thus, there exists a homeomorphism $h : \Upsilon \to \Upsilon$ such that

$$\begin{aligned} & h(S(f^{-1}(C))) = S(f^{-1}(D)) \\ \implies & h(f^{-1}(S(C))) = f^{-1}(S(D)) \\ \implies & f(h(f^{-1}(S(C)))) = S(D). \end{aligned}$$

Thus, $f \circ h \circ f^{-1}$ is our required homemorphism. □

Definition 7. *Let $(\Upsilon, \mathcal{T})$ be any IFTS. Then Υ is said to be intuitionistic fuzzy semi-discrete (IFSD, in short) if for each $c \in \Upsilon$, there exist $0 < \alpha, \beta < 1$ with $\alpha + \beta \leq 1$ such that $c(\alpha, \beta) \in \mathcal{T}$.*

That is, $\mathcal{T}$ contains IFPs for each $c \in \mathcal{T}$.

Theorem 6. *If $(\Upsilon, \mathcal{T})$ is a countable separable space, then Υ is IFSD if and only if Υ is IFCDH.*

Proof. Let Υ be IFSD. Then corresponding to each $c \in \Upsilon$, there exist $0 < \alpha, \beta < 1$ with $\alpha + \beta \leq 1$ such that $c(\alpha, \beta) \in \mathcal{T}$. Let C and D be two countable dense collection of intuitionistic fuzzy points. Then for each $c(\alpha, \beta) \in \mathcal{T}$, there exist $0 < \alpha', \alpha'', \beta', \beta'' < 1$ with $\alpha' + \beta' \leq 1$ and $\alpha'' + \beta'' \leq 1$ such that $c(\alpha', \beta') \in C$, $c(\alpha'', \beta'') \in D$ and $c(\alpha', \beta'), c(\alpha'', \beta'') \subseteq c(\alpha, \beta)$. Thus, $S(C) = S(D) = \Upsilon$, making the identity function on Υ our required homeomorphism.

Conversely, let Υ be IFCDH. Then $\mathbb{Q}(\Upsilon)$ is a countable dense subset of IFPs. Suppose that there exists $c \in \Upsilon$ such that for any $0 < \alpha, \beta < 1$, with $\alpha + \beta \leq 1$, we have $c(\alpha, \beta) \notin \mathcal{T}$. Then $D = \mathbb{Q}(\Upsilon) \setminus \{c(\alpha, \beta) \mid \alpha, \beta \in \mathbb{Q} \cap [0, 1]$ and $\alpha + \beta \leq 1\}$ is a countable dense set of IFPs. Therefore, there exists a homeomorphism $h : \Upsilon \to \Upsilon$ such that $h(S(\mathbb{Q}(\Upsilon))) = S(D) \implies \Upsilon = \Upsilon \setminus \{c\}$, which is a contradiction. □

4 Conclusion

Different variations of the concept of homogeneity have been studied by several mathematicians. One such variation is countable dense homogeneity, an important notion in general topology. Building on the work of Samer Al Ghour and Ali Fora on the CDH property in fuzzy topological spaces, we have extended this property to intuitionistic fuzzy topological spaces and shown that it forms a proper extension. As a further variant of this study, the CDH property could also be investigated in the context of vanishing intuitionistic fuzzy points $c(\beta)$ in X with $\beta \in [0, 1)$, defined by $c(\beta) = \{\langle x, 0, 1 - c_{1-\beta}\rangle \mid x \in X\}$. Such an extension may provide deeper insights into the structural behavior of intuitionistic fuzzy topological spaces.

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