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Performance measurement using intuitionistic fuzzy sets: A case study from debt collection centers

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Abstract: Performance measurement (PM) is a vital aspect of management, conducted periodically to assess the success or quality of a specific process or an organization. Performance encompasses both goals and relational models crucial for achieving those goals within a given time frame. From another perspective, performance measurement is about determining related performance indicators and providing a numerical summarized result which can be used for comparisons. Converting various indicators to an overall performance score can be modeled as a multi-criteria decision making problem. In this study, we focus on performance measurement of debt collection centers which are organizations specialized in collecting money from debtors.

To this end, a decision model is developed and intuitionistic fuzzy Analytic Hierarchy Process is used to obtain the criteria weights. The proposed framework is shown in a real-world case study.
Keywords: Performance measurement, Debt collection, Intuitionistic fuzzy sets, Analytic hierarchy process.
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1 Introduction

Performance measurement (PM) is a vital aspect of management, conducted periodically to assess the success or quality of a specific process. Lebas [9] suggests that performance encompasses both goals and relational models crucial for achieving those goals within the stipulated time frame. However, since the definition of company performance can vary depending on various factors, it is inherently subjective. Meyer [10] emphasizes that performance should be defined in relation to actions taken and their consequences, which ought to be benchmarked to gauge the degree of achievement. Folan et al. [5] further elaborate on three key terms related to performance measurement: relation, goal, and characteristics. Relation underscores the connection between individuals, teams, or companies and their context; goal delineates the expected level of performance; and characteristics stipulate that measurement should encompass relevant numerical aspects such as cost, quality, and flexibility. PM is utilized across different management levels.

Debt collection stands out as a significant challenge for all organizations, exerting considerable influence on their financial well-being and competitive viability. It entails a series of purposeful efforts aimed at recuperating overdue receivables from debtors in cases of unpaid invoices or credit. With the ongoing global economic downturn and the erosion of individuals' financial stability, instances of unpaid debts are on the rise, significantly impacting cash flows, turnovers, credit standings, and even organizational credibility (Çevik Onar et al. [1, 2]). Consequently, identifying the most pivotal factors for successful debt collection is paramount when dealing with outstanding debts. Taneta-Skwiercz [13] characterizes the debt collection process as a multidimensional phenomenon, encompassing legal, economic, and psychosocial perspectives, with particular emphasis on requisite legal reforms. In practice, debts are typically pursued either by the creditor themselves, utilizing their name and personnel, or through a third-party debt collection agency acting on behalf of the creditor firm. In numerous jurisdictions, collection fees are appended to the original debt as part of the debt recovery costs, which the debtor is obligated to settle.

The assessment of debt collection centers involves a comprehensive process that considers various indicators and aggregation of these indicators involve uncertainty and imprecision. Traditional performance evaluation methods often fail to account for the inherent subjectivity in this process, particularly in determining indicator weights. To address this, fuzzy sets have been widely utilized in literature to offer more realistic and precise outcomes for decision-making (Öztayşi et al. [11]). However, conventional fuzzy sets have limitations, leading to the proposal of various extensions such as type-2 fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, and fuzzy multi-sets (Kahraman et al. [6]; Esterella et al. [4]; Kahraman et al. [7]; Kahraman et al. [8]).

This paper is motivated by the need for an effective analytical tool that can efficiently incorporate human perceptions, given the inherently uncertain and imprecise nature of debt collection center performance evaluation. To address this, we propose a model incorporating Interval-Valued Intuitionistic Fuzzy Analytic Hierarchy Process. These sets consider both membership and non-membership values, providing a more nuanced representation of human reasoning in assessing debt collection center performance. The rest of the paper is as follows: in Section 2, Interval-Valued Intuitionistic Fuzzy Analytic Hierarchy Process is presented. Section 3 introduces performance measurement model and its application steps including the indicators used, performance measurement methodology and sample application. Finally, suggestions for further studies are given in the Conclusion section.

2 Interval intuitionistic fuzzy analytic hierarchy process

In this paper, we use an interval-valued intuitionistic AHP method proposed by (Çevik Onar et al. [3], Öztayşi et al. [12]) for performance measurement. In the following, we present the steps of our proposed method.

Step 1. Linguistic pairwise comparison matrices are formed according to the decision model and decision makers fill the matrices using linguistic scale given in Table 1.

Table 1. Linguistic scale and its corresponding IVIFS

Linguistic Terms	Membership & Non-membership values
Absolutely Low (AL)	([0.10, 0.25], [0.65, 0.75])
Very Low (VL)	([0.15, 0.30], [0.60, 0.70])
Low (L)	([0.20, 0.35], [0.55, 0.65])
Medium Low (ML)	([0.25, 0.4]), [0.50, 0.60])
Approximately Equal (AE)	([0.45, 0.55], [0.30, 0.45])
Medium High (MH)	([0.50, 0.60], [0.25, 0.40])
High (H)	([0.55,0.65], [0.20, 0.35])
Very High (VH)	([0.60,0.70], [0.15,0.30])
Absolutely High (AH)	([0.65,0.75], [0.10,0.25])
Exactly Equal (EE)	([0.5, 0.5], [0.5, 0.5]).

Step 2. The linguistic pairwise matrices are converted to their corresponding interval-valued intuitionistic fuzzy sets using the scale given in Table 1 in order to obtain intuitionistic pairwise comparison matrices and aggregated pairwise comparison matrix ($\tilde{R}_g$).

$$\tilde{R}_g = \begin{bmatrix} \left(\left[\mu^-_{g_{11}}, \mu^+_{g_{11}}\right], \left[\nu^-_{g_{11}}, \nu^+_{g_{11}}\right]\right) & \cdots & \left(\left[\mu^-_{g_{11}}, \mu^+_{g_{11}}\right], \left[\nu^-_{g_{11}}, \nu^+_{g_{11}}\right]\right) \\ \vdots & \ddots & \vdots \\ \left(\left[\mu^-_{g_{n1}}, \mu^+_{g_{n1}}\right], \left[\nu^-_{g_{n1}}, \nu^+_{g_{n1}}\right]\right) & \cdots & \left(\left[\mu^-_{g_{nn}}, \mu^+_{g_{nn}}\right], \left[\nu^-_{g_{nn}}, \nu^+_{g_{nn}}\right]\right) \end{bmatrix} \quad (1)$$

Interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) proposed by (Xu and Cai, [14])

$$IVIFWA_w(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left\langle\left[1-\prod_{i=1}^{n}(1-\mu_i^-)^{w_i},1-\prod_{i=1}^{n}(1-\mu_i^+)^{w_i}\right],\left[\prod_{i=1}^{n}(\nu_i^-)^{w_i},\prod_{i=1}^{n}(\nu_i^+)^{w_i}\right]\right\rangle,$$

where $\tilde{\alpha}_1=\left[\mu_i^-,\mu_i^+\right],\left[\nu_i^-,\nu_i^+\right]$ (i = 1, 2, …, n) is a collection of IVIFNs, and w_i is the weight of the i-th expert.

Step 3. Score judgement matrices ($\tilde{S}$) are formed using the scoring function given in Eq. 8. (2)?

$$\tilde{S}=\begin{bmatrix}\left[\mu_{g_{11}}^- - \nu_{g_{11}}^+,\mu_{g_{11}}^+ - \nu_{g_{11}}^-\right] & \cdots & \left[\mu_{g_{1n}}^- - \nu_{g_{1n}}^+,\mu_{g_{1n}}^+ - \nu_{g_{1n}}^-\right]\\ \vdots & \ddots & \vdots\\ \left[\mu_{g_{n1}}^- - \nu_{g_{n1}}^+,\mu_{g_{n1}}^+ - \nu_{g_{n1}}^-\right] & \cdots & \left[\mu_{g_{nn}}^- - \nu_{g_{nn}}^+,\mu_{g_{nn}}^+ - \nu_{g_{nn}}^-\right]\end{bmatrix}. \tag{2}$$

Step 4. Interval exponential matrices ($\tilde{A}$) are calculated as given in Eq. (3).

$$\tilde{A}=\begin{bmatrix}\left[e^{(\mu_{g_{11}}^- - \nu_{g_{11}}^+)},e^{(\mu_{g_{11}}^- - \nu_{g_{11}}^+)}\right] & \cdots & \left[e^{(\mu_{g_{1n}}^- - \nu_{g_{1n}}^+)},e^{(\mu_{g_{1n}}^- - \nu_{g_{1n}}^+)}\right]\\ \vdots & \ddots & \vdots\\ \left[e^{(\mu_{g_{n1}}^- - \nu_{g_{n1}}^+)},e^{(\mu_{g_{n1}}^- - \nu_{g_{n1}}^+)}\right] & \cdots & \left[e^{(\mu_{g_{nn}}^- - \nu_{g_{nn}}^+)},e^{(\mu_{g_{nn}}^- - \nu_{g_{nn}}^+)}\right]\end{bmatrix}$$
$$=\begin{bmatrix}\left[\tilde{a}_{g_{11}}^-,\tilde{a}_{g_{11}}^+\right] & \cdots & \left[\tilde{a}_{g_{1n}}^-,\tilde{a}_{g_{1n}}^+\right]\\ \vdots & \ddots & \vdots\\ \left[\tilde{a}_{g_{n1}}^-,\tilde{a}_{g_{n1}}^+\right] & \cdots & \left[\tilde{a}_{g_{nn}}^-,\tilde{a}_{g_{nn}}^+\right]\end{bmatrix}. \tag{3}$$

Step 5. Priority vectors of the interval exponential matrices are calculated using Eq. (4).

$$\tilde{w}_i=\left[\frac{\sum_{j=1}^{n}\tilde{a}_{ij}^-}{\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{a}_{ij}^+},\frac{\sum_{j=1}^{n}\tilde{a}_{ij}^+}{\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{a}_{ij}^-}\right]=[w_i^-,w_i^+],\quad i=1,...,n. \tag{4}$$

Step 6. Possibility degree matrices are obtained using Eq. (5) and (6).

$$P(\tilde{w}_i>\tilde{w}_j)=p_{ij}=\frac{\max(0,w_i^+-w_j^-)-\max(0,w_i^--w_j^+)}{(w_i^+-w_j^-)+(w_i^+-w_j^-)} \tag{5}$$

$$P(\tilde{w}_j>\tilde{w}_i)=p_{ij}=\frac{\max(0,w_j^+-w_i^-)-\max(0,w_j^--w_i^+)}{(w_i^+-w_i^-)+(w_j^+-w_j^-)} \tag{6}$$

Step 7. Possibility degrees are prioritized using Eq. (7).

$$w_i = \frac{\sum_{j=1}^{n} p_{ij} - 1}{n} + 0.5 . \tag{7}$$

Step 8. The weights are normalized as given in Eq. (8).

$$w_i^T = \frac{w_i}{\sum_{i=1}^{n} w_i} . \tag{8}$$

Step 9. The steps are repeated for each criterion with respect to the goal. Finally, the global weight of each criterion is calculated.

3 Proposed performance measurement model for debt collection centers

Performance measurement aims to obtain an overall numerical representation for showing the success or quality of an action. One of the important aspects of performance is to give feedback and make comparisons between organizations or between different time intervals. This requires an objective evaluation system. In order to maintain such an evaluation system, business environment and related performance indicators are investigated. After a brief literature review and interviews with domain experts, a decision model is developed. The performance indicators can be explained as in the following.

Payment Performance (C1): It is an indicator based on the amount of collection made. Essentially, it is included in the performance management system with two indicators. The criterion involves two sub-criteria:

- *Payment Rate (C11):* The ratio of total amount of payment collected to total monetary value of all files.
- *Collection Rate (C12):* The ratio between the number of cases which a collection has been started to the total number of files.

Promise to Pay Acquisition Rate (C2): Institutions managing receivables can make agreements regarding receivables for future periods. This indicator is calculated based on the ratio of the agreed amount to the total receivables. The criterion involves two sub-criteria:

- *Promise to Pay Rate (C21):* Ratio of the amount promised to be paid to the total receivable amount.
- *Promise to Collect Rate (C22):* Ratio of the number of files for which promises to pay are obtained to the total number of files.

Operational Performance (C3): It is a performance evaluation indicator created based on all activities carried out in files. Inputs to be considered within this scope. The criterion involves four sub-criteria:

- *Finalized Follow-up Rate (C31):* It is the confirmation of follow-up after the first action is taken once the file is assigned to the relevant institution. This indicator shows the proportion of files in which follow-up is finalized among all files.

- *Identified Asset Rate (C32):* After the finalization of follow-up, another significant step is determined as identifying the debtor's asset status. This indicator shows the proportion of files in which the asset status has been identified among all files.
- *Collections Initiated Rate (C33):* One of the most important steps on the path to collection is determined to be initiating collection by reaching an agreement with the debtor. This indicator shows the proportion of files in which collection has been initiated among all files.
- *Success Closure Rate (C34):* Files that have been closed are defined as those where the collection has been completed and there are no outstanding receivables. This indicator shows the proportion of closed files among all files.

In the weight determination phase, three domain experts evaluate the pairwise comparison matrices formed according to the above-mentioned performance measurement model. The evaluations of the experts are given in Tables 1–4.

Table 1. Expert evaluations of the main criteria

Criteria	**C1**			**C2**			**C3**		
	E1	**E2**	**E3**	**E1**	**E2**	**E3**	**E1**	**E2**	**E3**
C1	EE	EE	EE	H	H	EE	MH	VH	MH
C2	L	L	EE	EE	EE	EE	MH	H	MH
C3	ML	VL	ML	ML	L	ML	EE	EE	EE

Table 2. Expert evaluations of the criteria with respect to payment performance

C1 Subcriteria	**C11**			**C12**		
	E1	**E2**	**E3**	**E1**	**E2**	**E3**
C11	EE	EE	EE	ML	EE	ML
C12	MH	EE	MH	EE	EE	EE

Table 3. Expert evaluations of the criteria with respect to promise-to-pay performance

C2 Subcriteria	**C21**			**C22**		
	E1	E2	E3	E1	E2	E3
C21	EE	EE	EE	H	MH	MH
C22	L	ML	ML	EE	EE	EE

Table 3. Expert evaluations of the criteria with respect to operational performance

C3 Subcriteria	**C31**			**C32**			**C33**			**C34**		
	E1	**E2**	**E3**	**E1**	**E2**	**E3**	**E1**	**E2**	**E3**	**E1**	**E2**	**E3**
C31	EE	EE	EE	L	L	VL	VL	VL	VL	AL	VL	AL
C32	H	H	VH	EE	EE	EE	L	L	VL	VL	L	AL
C33	VH	VH	VH	H	H	VH	EE	EE	EE	VL	L	VL
C34	AH	VH	AH	VH	H	AH	VH	H	VH	EE	EE	EE

Based on these evaluations the indicator weights are calculated. Due to page constraints only, the calculations regarding to the first pairwise comparison matrix are given here. According to the steps of the methodology, first the linguistic evaluations are transformed into interval intuitionistic fuzzy sets. Then the aggregated decision matrix is formed (Table 4). The next step is to calculate the interval exponential matrix by using Eq. (3) (Table 5). Next, by using these values, the priority vectors are identified and then the possibility degree matrix is form by using Eqs. (5)–(6) (Table 7).

Table 4. Aggregated decision matrix

	C1	**C2**	**C3**
C1	[0.5,0.5],[0.5,0.5]	[0.542,0.748],[0,0.252]	[0.578,0.792],[0,0.208]
C2	[0.072,0.273],[0.524,0.727]	[0.5,0.5],[0.5,0.5]	[0.536,0.738],[0,0.262]
C3	[0.068,0.239],[0.559,0.761]	[0.068,0.268],[0.531,0.732]	[0.5,0.5],[0.5,0.5]

Table 5. Interval exponential matrix

	C1	**C2**	**C3**
C1	[1,1]	[1.95,5.6]	[2.35,6.19]
C2	[0.22,0.56]	[1,1]	[1.88,5.47]
C3	[0.2,0.48]	[0.22,0.55]	[1,1]

Table 6. Possibility degree matrix, priority degrees and normalized weights

	C1	**C2**	**C3**	**Priority Degrees**	**Normalized Weights**
C1	0.5000	0.7102	1.0000	0.8026	0.428
C2	0.2898	0.5000	0.9102	0.6750	0.360
C3	0.0000	0.0898	0.5000	0.3975	0.212

The final step is to calculate the priority degrees and normalized weights by using Eq. (7) and Eq. 8. The priority degrees and normalized weights are given in Table 6. When the steps are applied to other decision matrices, the local and global weights are calculated as in Table 7.

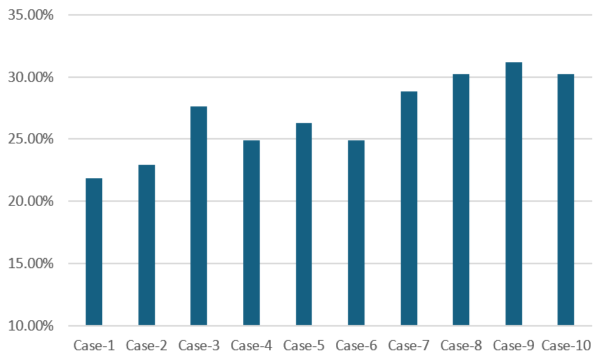
Table 7. Local weights and global weights of the criteria

	Local Weights	**Global Weights**		**Local Weights**	**Global Weights**
C1		0.428	**C22**	0.483	0.1739
C2		0.360	**C31**	0.177	0.0375
C3		0.212	**C32**	0.230	0.0488
C11	0.489	0.209	**C33**	0.271	0.0575
C12	0.511	0.218	**C34**	0.322	0.0683
C21	0.517	0.186			

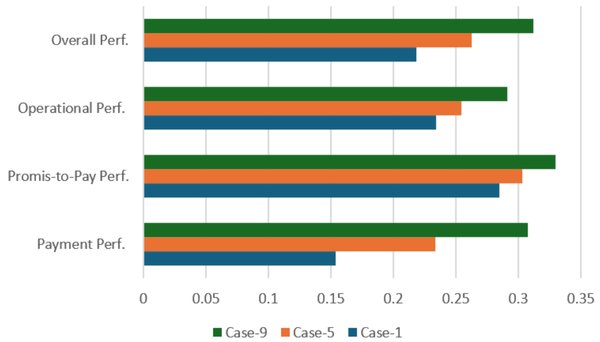
Table 8. Indicator values and overall performance of sample cases

	Payment Performance		Promise to Pay Acquisition Performance		Operational Performance				Overall Perf
	C11	C12	C21	C22	C31	C32	C33	C34	.
Case-1	18.10%	12.80%	29.10%	27.90%	33.60%	25.10%	27.90%	12.80%	21.82%
Case-2	21.60%	14.90%	31.30%	25.40%	35.00%	22.40%	25.40%	14.90%	22.91%
Case-3	30.40%	20.50%	35.40%	26.50%	38.10%	25.40%	26.50%	20.50%	27.64%
Case-4	24.10%	18.10%	31.10%	27.90%	36.30%	21.20%	27.90%	18.10%	24.88%
Case-5	27.60%	19.30%	34.10%	26.30%	38.60%	22.70%	26.30%	19.30%	26.30%
Case-6	21.00%	18.50%	33.90%	27.00%	38.80%	24.00%	27.00%	18.50%	24.89%
Case-7	31.90%	20.00%	36.30%	30.20%	38.90%	24.30%	30.20%	20.00%	28.80%
Case-8	35.60%	22.00%	34.80%	32.10%	39.40%	22.10%	32.10%	22.00%	30.22%
Case-9	36.80%	25.00%	34.90%	30.90%	38.80%	25.30%	30.90%	25.00%	31.21%
Case-10	34.50%	25.10%	33.90%	28.30%	41.50%	27.90%	28.30%	25.10%	30.20%

As it can be observed from Table 8, using the indicator weights, obtained in the previous step, the overall performance of each case can be calculated. This score enables effective comparison of cases (Figure 1a.). By using the decision model, the operational performance, promise-to-pay performance and payment performances can also be calculated and compared (Figure 1b).



(a)



(b)

Figure 1. Comparison of cases according to performance scores

4 Conclusion

In this paper, we focus on performance measurement problem of debt collection centers. The problem is vital for the economy since debt collection centers play a vital role for companies. The aim of the proposed quality of the processes performed. To this end, a decision model with three main and eight sub-criteria are developed. Interval-valued intuitionistic fuzzy AHP methodology is used to find the weights of the criteria and sub-criteria. The results show that the most important main criteria are payment performance and the most important sub-criteria are collection-rate.

These indicator weights are later used to find overall performance and criteria-based performance values. Interviews with the experts show that the proposed model satisfies the managerial expectations by providing agreeable performance values. In the following studies, the decision model can be improved by other related indicators and other multi-criteria decision-making models can be used to find the indicator weights.

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