Notes on Intuitionistic Fuzzy Sets
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Solving second order intuitionistic fuzzy differential equations by intuitionistic fuzzy Sumudu transform method

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Abstract: In this work, the main finding associated with the intuitionistic fuzzy Sumudu transforms is proved in detail. Furthermore, the resolution of second-order intuitionistic fuzzy differential equations is investigated by means of the intuitionistic fuzzy Sumudu transforms method. Finally, a numerical example is provided to demonstrate the accuracy of the suggested approach.

Keywords: Intuitionistic fuzzy solution, Intuitionistic fuzzy number, Intuitionistic fuzzy Sumudu transform.
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1 Introduction

The theory of intuitionistic fuzzy sets was originally suggested by Atanassov [1,2] as an extension of fuzzy sets [16].

This theory has many applications in various disciplines including programming, medical diagnosis, medicine, and decision-making problems. The application of intuitionistic fuzzy sets in the development of the field of intuitionistic fuzzy differential equations (IFDEs) is an extension of both classical and fuzzy context of differential equations for handling the imprecision, vagueness, and uncertainty in real-life systems. In order to improve this field, integral and differential calculus notions for intuitionistic fuzzy valued functions (IFVF) have been constructed using the Hukuhara difference in the intuitionistic fuzzy framework [5]. For example, S. Melliani *et al.* [11, 12] investigated differential and partial differential equations under intuitionistic fuzzy theory respectively. R. Ettoussi, *et al.* [5] discussed the existence and uniqueness of the solution of (IFDEs) by means of successive approximations method, while in [3] they applied the theorem of the fixed point in the space $(\mathrm{IF}_1, \mathcal{D}_\infty)$ and presented the explicit solution through the method of γ-cuts. Furthermore, Intuitionistic fuzzy Laplace and Sumudu transforms were applied to solve differential equations of second and first orders in intuitionistic fuzzy environments respectively in [4, 6].

On the other hand, the Sumudu transform (ST), which was proposed by Watugala in 1993, has been extensively used to solve differential equations, especially those concerning engineering control systems [14, 15]. Recently, in responding to the new challenges in giving an appropriate solution covering uncertainty and imprecision, the framework of intuitionistic fuzzy sets has been introduced. The Intuitionistic Fuzzy Sumudu Transform (IFST) combines the classical Sumudu transform with the theory of intuitionistic fuzzy sets which is suitable for systems having dual uncertainty (i.e., membership and non-membership functions). The generalization has been useful in dealing with solving intuitionistic fuzzy differential equations and integral equations, providing a more robust mathematical tool for modeling real-world phenomena with inherent vagueness [7, 8].

Motivated by the previous works, the goal of this study is to extend the Sumudu transform method in an intuitionistic fuzzy environment as a new transform to solve differential equations of second order, which represents the first work to improve the intuitionistic fuzzy differential equations field and handle uncertainty in control systems and dynamic models.

The paper is structured as follows: In Section 2, we present preliminaries and establish key properties related to the intuitionistic fuzzy Sumudu transform, which is crucial to our study. In Section 3, we propose an approach for solving the initial value problem of the second order in an intuitionistic fuzzy framework through the method of IFST. Finally, in Section 4, we proved an application for the validity of this research paper.

2 Preliminaries

To achieve this work, several definitions and results are presented, where $\mathbb{I} = [b, c] \subseteq \mathbb{R}$.

Definition 2.1. [10] *We define the space of any intuitionistic fuzzy number as*

$$IF_1 = \mathbb{IF}(\mathbb{R}) = \left\{ \mathcal{E}_{\langle \Phi,\Psi \rangle} \ : \ \mathbb{R} \to [0,1]^2 \,, | \, \textit{for all } z \in \mathbb{R} | 0 \leq \Phi(z) + \Psi(z) \leq 1 \right\}.$$

An element $\mathcal{E}_{\langle \Phi,\Psi \rangle}$ of IF_1 is called an intuitionistic fuzzy number if the following assertions hold:

1. $\mathcal{E}_{\langle \Phi,\Psi \rangle}$ *is normal, i.e., there exist $z_0, z_1 \in \mathbb{R}$ such that $\Phi(z_0) = 1$ and $\Psi(z_1) = 1$,*
2. Φ *is fuzzy convex and* Ψ *is fuzzy concave,*
3. Φ *is upper semi-continuous and* Ψ *is lower semi-continuous,*
4. $\operatorname{supp}(\mathcal{E}_{\langle \Phi,\Psi \rangle}) = \operatorname{cl}\{z \in \mathbb{R} \ : | \ \Psi(z) < 1\}$ *is bounded.*

Definition 2.2. [9] *The parametric form (PF) of intuitionistic fuzzy number $\mathcal{E}_{\langle \Phi,\Psi \rangle}$ denoted as $\mathcal{E}_{\langle \Phi,\Psi \rangle}(\gamma) = \Big((\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,l}(\gamma), \mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,r}(\gamma)), (\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,l}(\gamma), \mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,r}(\gamma)) \Big)$ where the functions $\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,l}(\gamma)$, $\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,r}(\gamma)$, $\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,l}(\gamma)$ and $\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,r}(\gamma)$, verifies the following conditions:*

i) $\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,l}(\gamma)$ *is monotonically increasing bounded continuous,*

ii) $\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,r}(\gamma)$ *is monotonically decreasing bounded continuous,*

iii) $\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,l}(\gamma)$ *is monotonically increasing bounded continuous,*

iv) $\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,r}(\gamma)$ *is monotonically decreasing bounded continuous,*

v) $\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,l}(\gamma) \leq \mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,r}(\gamma)$ *and* $\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,l}(\gamma) \leq \mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,r}(\gamma)$*, for every* $0 \leq \gamma \leq 1$.

Example 2.1. *A Triangular Intuitionistic Fuzzy Number (TIFN) $\mathcal{E}_{\langle \Phi,\Psi \rangle}$ is an intuitionistic fuzzy set in $\mathbb{R}$ with their membership function Φ and non-membership function Ψ, respectively given by:*

$$\Phi(z) = \begin{cases} \dfrac{z - b_1}{b_2 - b_1} & \textit{if } b_1 \leq z \leq b_2, \\ \dfrac{b_3 - z}{b_3 - b_2} & \textit{if } b_2 \leq z \leq b_3, \\ 0. & \textit{otherwise}, \end{cases}$$

$$\Psi(z) = \begin{cases} \dfrac{b_2 - z}{b_2 - b_1'} & \textit{if } b_1' \leq z \leq b_2, \\ \dfrac{z - b_2}{b_3' - b_2} & \textit{if } b_2 \leq z \leq b_3', \\ 1. & \textit{otherwise}, \end{cases}$$

where $b_1' \leq b_1 \leq b_2 \leq b_3 \leq b_3'$, so this TIFN is expressed as $\mathcal{E}_{\langle \Phi,\Psi \rangle} = \left\langle b_1, b_2, b_3; b_1', b_2, b_3' \right\rangle$ and its parametric form given by

$$\mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,l}(\gamma) = b_1 + \gamma(b_2 - a_1), \ \mathcal{E}^{+}_{\langle \Phi,\Psi \rangle,r}(\gamma) = b_3 - \gamma(b_3 - b_2),$$
$$\mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,l}(\gamma) = b_1' + \gamma(b_2 - b_1'), \ \mathcal{E}^{-}_{\langle \Phi,\Psi \rangle,r}(\gamma) = b_3' - \gamma(b_3' - b_2).$$

Let $\gamma \in [0,1]$ and $\mathcal{E}_{\langle \Phi,\Psi \rangle} \in IF_1$, the lower and upper γ-cuts of $\mathcal{E}_{\langle \Phi,\Psi \rangle}$ are defined as:

$$\left[\mathcal{E}_{\langle \Phi,\Psi \rangle}\right]^{\gamma} = \{z \in \mathbb{R} : \Psi(z) \leq 1 - \gamma\} \quad \textit{and} \quad \left[\mathcal{E}_{\langle \Phi,\Psi \rangle}\right]_{\gamma} = \{z \in \mathbb{R} : \Phi(z) \geq \gamma\}.$$

Remark 2.1. *If $\mathcal{E}_{\langle \Phi,\Psi \rangle} \in IF_1$, then we get $\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_\gamma$ as $[\Phi]^\gamma$ and $\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]^\gamma$ as $[1-\Psi]^\gamma$ in the boundary case of the fuzzy set.*

Consider $\mathcal{E}_{\langle \Phi_1,\Psi_1 \rangle}$, $\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}$ and $\lambda \in \mathbb{R}$, the scaler-multiplication and addition are represented as

$$\Big[\mathcal{E}_{\langle \Phi_1,\Psi_1 \rangle} \oplus \mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]^\gamma = \Big[\mathcal{E}_{\langle \Phi_1,\Psi_1 \rangle}\Big]^\gamma + \Big[\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]^\gamma, \quad \Big[\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]^\gamma = \lambda\Big[\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]^\gamma,$$
$$\Big[\mathcal{E}_{\langle \Phi_1,\Psi_1 \rangle} \oplus \mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]_\gamma = \Big[\mathcal{E}_{\langle \Phi_1,\Psi_1 \rangle}\Big]_\gamma + \Big[\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]_\gamma, \quad \Big[\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]_\gamma = \lambda\Big[\mathcal{J}_{\langle \Phi_2,\Psi_2 \rangle}\Big]_\gamma.$$

Definition 2.3. *Considering $\mathcal{E}_{\langle \Phi,\Psi \rangle}$ an element of IF_1 and $\gamma \in [0,1]$, we introduce the subsequent sets:*

$$\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_l^+(\gamma) = \inf\{z \in \mathbb{R} \mid \Phi(z) \geq \gamma\}, \qquad \Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_r^+(\gamma) = \sup\{z \in \mathbb{R} \mid \Phi(z) \geq \gamma\},$$
$$\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_l^-(\gamma) = \inf\{z \in \mathbb{R} \mid \Psi(z) \leq 1-\gamma\}, \quad \Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_r^-(\gamma) = \sup\{z \in \mathbb{R} \mid \Psi(z) \leq 1-\gamma\}.$$

Remark 2.2. *It is important to note that the subsequent sets defined above represent the left and right extremities of the lower and upper γ-cuts expressed as the real intervals below:*

$$\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_\gamma = \bigg[\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_l^+(\gamma), \Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_r^+(\gamma)\bigg],$$
$$\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]^\gamma = \bigg[\Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_l^-(\gamma), \Big[\mathcal{E}_{\langle \Phi,\Psi \rangle}\Big]_r^-(\gamma)\bigg].$$

Definition 2.4. [3] *Consider an intuitionistic fuzzy valued function $\mathcal{K}_{\langle \Phi,\Psi \rangle} : \mathbb{I} \to IF_1$ and $s_0 \in \mathbb{I}$. Then $\mathcal{K}_{\langle \Phi,\Psi \rangle}$ is said to be intuitionistic fuzzy continuous in s_0 if and only if:*

$$(\forall \epsilon > 0)(\exists \eta > 0)\Big(\forall s \in \mathbb{I} \textit{ where } \mid s - s_0 \mid < \eta\Big) \Rightarrow \mathcal{D}_\infty\Big(\mathcal{K}_{\langle \Phi,\Psi \rangle}(s), \mathcal{K}_{\langle \Phi,\Psi \rangle}(s_0)\Big) < \epsilon.$$

Definition 2.5. [3] *We say that the function $\mathcal{K}_{\langle \Phi,\Psi \rangle} : \mathbb{I} \to IF_1$ is differentiable at $s_0 \in (b,c)$ if there exists $\mathcal{K}'_{\langle \Phi,\Psi \rangle}(s_0) \in IF_1$ such that*

$$\lim_{h\to 0^+} \frac{\mathcal{K}_{\langle \Phi,\Psi \rangle}(t_0+h) \ominus \mathcal{K}_{\langle \Phi,\Psi \rangle}(s_0)}{h} \quad \textit{and} \quad \lim_{h\to 0^+} \frac{\mathcal{K}_{\langle \Phi,\Psi \rangle}(s_0) \ominus \mathcal{K}_{\langle \Phi,\Psi \rangle}(s_0-h)}{h},$$

exist and they are equal to $\mathcal{K}'_{\langle \Phi,\Psi \rangle}(s_0)$.

If $\mathcal{K}_{\langle \Phi,\Psi \rangle} : \mathbb{I} \to \mathrm{IF}_1$ is differentiable at $s_0 \in \mathbb{I}$, then we say that $\mathcal{K}'_{\langle \Phi,\Psi \rangle}(s_0)$ is the intuitionistic fuzzy derivative of $\mathcal{K}_{\langle \Phi,\Psi \rangle}(s)$ at the point s_0.

Definition 2.6. *We say that the function $\mathcal{K}_{\langle \Phi,\Psi \rangle} : \mathbb{I} \to IF_1$ is differentiable of second order at $t_0 \in (b,c)$ if there exists $\mathcal{K}''_{\langle \Phi,\Psi \rangle}(t_0) \in IF_1$ such that limits*

$$\lim_{h\to 0^+} \frac{\mathcal{K}'_{\langle \Phi,\Psi \rangle}(t_0+h) \ominus \mathcal{K}'_{\langle \Phi,\Psi \rangle}(t_0)}{h} \quad \textit{and} \quad \lim_{h\to 0^+} \frac{\mathcal{K}'_{\langle \Phi,\Psi \rangle}(t_0) \ominus \mathcal{K}'_{\langle \Phi,\Psi \rangle}(t_0-h)}{h}$$

exist and they are equal to $\mathcal{K}''_{\langle \Phi,\Psi \rangle}(t_0)$.

The limit is taken in the compete metric space $(\mathrm{IF}_1, \mathcal{D}_\infty)$ [10]. At the end points of $\mathbb{I}$, we take only the one-sided derivatives.

Theorem 2.1. [4] *Suppose that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ and $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)$ are differentiable, such that*

$$[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)]_\gamma = [\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)],\ [\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)]^\gamma = [\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)].$$

Then, $\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), \mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ are differentiable and we get

$$[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)]_\gamma = [\mathcal{K}''^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), \mathcal{K}''^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)],\ [\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)]^\gamma = [\mathcal{K}''^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}''^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)].$$

Definition 2.7. [6] *Consider that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ is an IFVF and continuous on $[0,\infty)$. Assume that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(wt)e^{-t}$ is improper intuitionistic fuzzy Riemann integrable on $[0,\infty)$, then $\int_0^\infty \mathcal{K}_{\langle\Phi,\Psi\rangle}(wt)e^{-t}dt$ is called intuitionistic fuzzy Sumudu transform (in short, IFST) and is given as*

$$G(w) = \boldsymbol{S}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] = \int_0^\infty \mathcal{K}_{\langle\Phi,\Psi\rangle}(wt)e^{-t}dt,\ \ w \in [-\tau_1, \tau_1].$$

We have

$$\int_0^\infty \mathcal{K}_{\langle\Phi,\Psi\rangle}(wt,\gamma)e^{-t}dt = \Bigg(\int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)e^{-t}dt, \int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)e^{-t}dt,$$
$$\int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)e^{-t}dt, \int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)e^{-t}dt \Bigg),$$

and through the definition of ST,

$$\mathcal{S}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)) = \int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)e^{-t}dt,$$
$$\mathcal{S}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)) = \int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)e^{-t}dt,$$
$$\mathcal{S}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)) = \int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)e^{-t}dt,$$
$$\mathcal{S}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)) = \int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)e^{-t}dt,$$

then, we obtain

$$\boldsymbol{S}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(wt,\gamma)] = \Bigg(\mathcal{S}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)), \mathcal{S}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)), \mathcal{S}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(wt,\gamma)), \mathcal{S}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(wt,\gamma)) \Bigg). \tag{2.1}$$

Theorem 2.2. [6] *Consider that $\mathcal{G}_{\langle\Phi_1,\Psi_1\rangle}(t), \mathcal{K}_{\langle\Phi_2,\Psi_2\rangle}(t)$ are two continuous IFVFs on $[b,c]$ and c_1, c_2 in $\mathbb{R}$, then*

$$\boldsymbol{S}\big[c_1\mathcal{G}_{\langle\Phi_1,\Psi_1\rangle}(t) \oplus c_2\mathcal{K}_{\langle\Phi_2,\Psi_2\rangle}(t)\big] = c_1\boldsymbol{S}(\mathcal{G}_{\langle\Phi_1,\Psi_1\rangle}(t)) \oplus c_2\boldsymbol{S}(\mathcal{K}_{\langle\Phi_2,\Psi_2\rangle}(t)).$$

Theorem 2.3. [6] *Suppose that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ are IFVFs and continuous on $[0,\infty)$, and $\mathcal{K}_{\langle\Phi,\Psi\rangle}$ is the primitive of $\mathcal{K}'_{\langle\Phi,\Psi\rangle}$ on $[0,\infty)$. Then*

$$\boldsymbol{S}[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)] = \frac{1}{w}\boldsymbol{S}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] \ominus \frac{1}{w}\mathcal{K}_{\langle\Phi,\Psi\rangle}(0).$$

Theorem 2.4. *Suppose that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ and $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)$ are the continuous IFVFs on $[0,\infty)$ where $\mathcal{K}_{\langle\Phi,\Psi\rangle}(wt)e^{-t}$ and $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(wt)e^{-t}$ exist, are continuous and are improper intuitionistic fuzzy Riemann integrable on $[0,\infty)$, then we get*

$$\boldsymbol{S}[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)] = \frac{1}{w^2}\boldsymbol{S}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] \ominus \frac{1}{w^2}\mathcal{K}_{\langle\Phi,\Psi\rangle}(0) \ominus \frac{1}{w}\mathcal{K}'_{\langle\Phi,\Psi\rangle}(0).$$

Proof. Due to Definition 2.7 we have

$$\mathbf{S}[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)] \;\; = \;\; \int_0^\infty \mathcal{K}''_{\langle\Phi,\Psi\rangle}(wt)e^{-t}dt, \;\; w \in [-\tau_1,\tau_1].$$

Now, applying the formula for integration by parts, we get

$$\begin{aligned}\mathbf{S}[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)] &= \frac{1}{w}\int_0^\infty \mathcal{K}'_{\langle\Phi,\Psi\rangle}(wt)e^{-t}dt \oplus \left[\frac{1}{w}\mathcal{K}'_{\langle\Phi,\Psi\rangle}(wt)e^{-t}\right]_0^\infty, \;\; w \in [-\tau_1,\tau_1] \\ &= \frac{1}{w}\left[\int_0^\infty \mathcal{K}'_{\langle\Phi,\Psi\rangle}(wt)e^{-t}dt \ominus \mathcal{K}'_{\langle\Phi,\Psi\rangle}(0)\right] \\ &= \frac{1}{w}\mathbf{S}[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)] \ominus \frac{1}{w}\mathcal{K}'_{\langle\Phi,\Psi\rangle}(0).\end{aligned}$$

By means of Theorem (2.4), we obtain $\mathbf{S}[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)] = \frac{1}{w}\mathbf{S}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] \ominus \frac{1}{w}\mathcal{K}_{\langle\Phi,\Psi\rangle}(0)$. Consequently, we deduce that

$$\mathbf{S}[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)] = \frac{1}{w^2}\mathbf{S}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] \ominus \frac{1}{w^2}\mathcal{K}_{\langle\Phi,\Psi\rangle}(0) \ominus \frac{1}{w}\mathcal{K}'_{\langle\Phi,\Psi\rangle}(0). \qquad \square$$

3 Analytical approaches for solving initial value problem of second order in intuitionistic fuzzy environment

In this section, we explain how to apply the intuitionistic fuzzy Sumudu transform method for solving the initial value problem of the second order in the intuitionistic fuzzy framework, expressed in the subsequent form:

$$\begin{cases} \mathrm{y}''_{\langle\Phi,\Psi\rangle}(t) = f(t, \mathrm{y}_{\langle\Phi,\Psi\rangle}(t), \mathrm{y}'_{\langle\Phi,\Psi\rangle}(t)), \\ \mathrm{y}_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(\mathrm{y}^+_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \mathrm{y}^+_{\langle\Phi,\Psi\rangle,r}(0,\gamma), \mathrm{y}^-_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \mathrm{y}^-_{\langle\Phi,\Psi\rangle,r}(0,\gamma)\Big), \\ \mathrm{y}'_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(\mathrm{y}'^+_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \mathrm{y}'^+_{\langle\Phi,\Psi\rangle,r}(0,\gamma), \mathrm{y}'^-_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \mathrm{y}'^-_{\langle\Phi,\Psi\rangle,r}(0,\gamma)\Big). \end{cases} \tag{3.1}$$

By using the IFST we get

$$\mathbf{S}[\mathrm{y}''_{\langle\Phi,\Psi\rangle}(t)] = \mathbf{S}[f(t, \mathrm{y}_{\langle\Phi,\Psi\rangle}(t), \mathrm{y}'_{\langle\Phi,\Psi\rangle}(t))]. \tag{3.2}$$

Thanks to Theorem 4.2, Equation (3.2) becomes

$$\frac{1}{w^2}\mathbf{S}[\mathrm{y}_{\langle\Phi,\Psi\rangle}(t)] \ominus \frac{1}{w^2}\mathrm{y}_{\langle\Phi,\Psi\rangle}(0) \ominus \frac{1}{w}\mathrm{y}'_{\langle\Phi,\Psi\rangle}(0) = \mathbf{S}[f(t, \mathrm{y}_{\langle\Phi,\Psi\rangle}(t), \mathrm{y}'_{\langle\Phi,\Psi\rangle}(t))].$$

Thus,

$$\begin{cases} \frac{1}{w^2}\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \frac{1}{w^2}y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - \frac{1}{w}y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) = \mathcal{S}[f^{+}_{l}(t, y_{\langle\Phi,\Psi\rangle}(t), y'_{\langle\Phi,\Psi\rangle}(t),\gamma)], \\ \frac{1}{w^2}\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \frac{1}{w^2}y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - \frac{1}{w}y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) = \mathcal{S}[f^{+}_{r}(t, y_{\langle\Phi,\Psi\rangle}(t), y'_{\langle\Phi,\Psi\rangle}(t),\gamma)], \\ \frac{1}{w^2}\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \frac{1}{w^2}y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - \frac{1}{w}y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) = \mathcal{S}[f^{-}_{l}(t, y_{\langle\Phi,\Psi\rangle}(t), y'_{\langle\Phi,\Psi\rangle}(t),\gamma)], \\ \frac{1}{w^2}\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \frac{1}{w^2}y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - \frac{1}{w}y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) = \mathcal{S}[f^{-}_{r}(t, y_{\langle\Phi,\Psi\rangle}(t), y'_{\langle\Phi,\Psi\rangle}(t),\gamma)]. \end{cases} \tag{3.3}$$

To facilitate resolution of the system (3.3), we suppose that

$$\begin{cases} \mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = H_1(s,\gamma), \\ \mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = H_2(s,\gamma), \\ \mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = H_3(s,\gamma), \\ \mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = H_4(s,\gamma). \end{cases} \tag{3.4}$$

such that $H_1(s,\gamma), H_2(s,\gamma), H_3(s,\gamma)$ and $H_4(s,\gamma)$ are solutions of system (3.3).

Applying the inverse ST, $y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$, $y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$, $y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ are determined by the following formulas:

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \mathcal{S}^{-1}[H_1(s,\gamma)], \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \mathcal{S}^{-1}[H_2(s,\gamma)], \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \mathcal{S}^{-1}[H_3(s,\gamma)], \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \mathcal{S}^{-1}[H_4(s,\gamma)]. \end{cases} \tag{3.5}$$

4 Example

This section clarifies our work by attacking the following initial value problem of the second order in an intuitionistic fuzzy environment.

$$\begin{cases} y''_{\langle\Phi,\Psi\rangle}(t) = 2y'_{\langle\Phi,\Psi\rangle}(t) + 3y_{\langle\Phi,\Psi\rangle}(t), \\ y_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(3+\gamma, 5-\gamma, 2+2\gamma, 6-2\gamma\Big), \\ y'_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(2+\gamma, 4-\gamma, 1+2\gamma, 5-2\gamma\Big). \end{cases} \tag{4.1}$$

the PF of the problem (4.1) is written as

$$\begin{cases} y''^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = 2y'^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) + 3y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \\ y''^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = 2y'^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) + 3y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), \\ y''^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = 2y'^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) + 3y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \\ y''^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = 2y'^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) + 3y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), \end{cases} \tag{4.2}$$

Using the IFST, we obtain

$$\begin{cases} \mathcal{S}[y''^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = 2\mathcal{S}[y'^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] + 3\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{S}[y''^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = 2\mathcal{S}[y'^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] + 3\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \\ \mathcal{S}[y''^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = 2\mathcal{S}[y'^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] + 3\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{S}[y''^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = 2\mathcal{S}[y'^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] + 3\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]. \end{cases} \tag{4.3}$$

Therefore,

$$\begin{cases} \frac{1}{w^2}\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \frac{1}{w^2}y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - \frac{1}{w}y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)) \\ \qquad = \frac{2}{w}\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \frac{2}{w}y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + 3\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \frac{1}{w^2}\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \frac{1}{w^2}y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - \frac{1}{w}y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)) \\ \qquad = \frac{2}{w}\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \frac{2}{w}y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + 3\mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \\ \frac{1}{w^2}\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \frac{1}{w^2}y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - \frac{1}{w}y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)) \\ \qquad = \frac{2}{w}\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \frac{2}{w}y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + 3\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \frac{1}{w^2}\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \frac{1}{w^2}y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - \frac{1}{w}y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)) \\ \qquad = \frac{2}{w}\mathcal{S}[y_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \frac{2}{w}y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + 3\mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]. \end{cases} \tag{4.4}$$

Thus,

$$\begin{cases} \mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{w}{-3w^2-2w+1}y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1-2w}{-3w^2-2w+1}y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \\ \mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{w}{-3w^2-2w+1}y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1-2w}{-3w^2-2w+1}y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma), \\ \mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{w}{-3w^2-2w+1}y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1-2w}{-3w^2-2w+1}y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \\ \mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{w}{-3w^2-2w+1}y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1-2w}{-3w^2-2w+1}y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma). \end{cases} \tag{4.5}$$

At the next step, we substitute the value of the PF of $y'_{\langle\Phi,\Psi\rangle}(0)$ and $y_{\langle\Phi,\Psi\rangle}(0)$ in the system (4.5), and we get

$$\begin{cases} \mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{(-4-\gamma)w+(3+\gamma)}{1-2w-3w^2}, \\ \mathcal{S}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{(-6+\gamma)w+(5-\gamma)}{1-2w-3w^2}, \\ \mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{(-3-2\gamma)w+(2+2\gamma)}{1-2w-3w^2}, \\ \mathcal{S}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{(-7+2\gamma)w+(6-2\gamma)}{1-2w-3w^2}. \end{cases} \tag{4.6}$$

Now, we apply the inverse ST and obtain

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\frac{2\gamma+7}{4})\mathcal{S}^{-1}[\frac{1}{w+1}] + (\frac{5+2\gamma}{4})\mathcal{S}^{-1}[\frac{1}{1-3w}], \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (\frac{-2\gamma+11}{4})\mathcal{S}^{-1}[\frac{1}{w+1}] + (\frac{-2\gamma+9}{4})\mathcal{S}^{-1}[\frac{1}{1-3w}], \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\frac{4\gamma+5}{4})\mathcal{S}^{-1}[\frac{1}{w+1}] + (\frac{4\gamma+3}{4})\mathcal{S}^{-1}[\frac{1}{1-3w}], \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (\frac{-4\gamma+13}{4})\mathcal{S}^{-1}[\frac{1}{w+1}] + (\frac{-4\gamma+11}{4})\mathcal{S}^{-1}[\frac{1}{1-3w}]. \end{cases} \tag{4.7}$$

Hence,

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\frac{2\gamma+7}{4})e^{-t} + (\frac{5+2\gamma}{4})e^{3t}, \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (\frac{-2\gamma+11}{4})e^{-t} + (\frac{-2\gamma+9}{4})e^{3t}, \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\frac{4\gamma+5}{4})e^{-t} + (\frac{4\gamma+3}{4})e^{3t}, \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (\frac{-4\gamma+13}{4})e^{-t} + (\frac{-4\gamma+11}{4})e^{3t}. \end{cases} \tag{4.8}$$

It should be noted that $y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) \leq y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$; $y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) \leq y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$, the functions $y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$, $y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ are increasing and the functions $y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ are decreasing with respect to γ, respectively.

Consequently, we have established that $\Big(y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)\Big)$ represents the PF of the solution of the problem (4.2).

5 Conclusion

In this study, the crucial properties related to intuitionistic fuzzy Sumudu transform is proved. The main finding of this research paper is to solve second order differential equation by Sumudu transform method in intuitionistic fuzzy environment. The efficiency of the approach is illustrated by attacking a numerical example.

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