Cuts properties of IF-nearness

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1. Outline

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2. Introduction

Definition 1 (Kalina, 1997) *The binary fuzzy relation N on a real line with values in the interval $[0,1]$ is called a fuzzy nearness if for each $x, y, z \in E$ there is*

- $xNx = 1$,
- $xNy = yNx$,
- *if z is between x and y then $xNz \geq xNy$.*

Other properties of fuzzy nearness

- $\lim_{y \to \infty} xNy = 0$,
- $xNy = (x + \delta)N(y + \delta)$ for $\delta > 0$.

Examples of fuzzy nearness

Example 1 (crisp case)

$$xNy = \begin{cases} 1; & \textit{if } x = y \\ 0; & \textit{if } x \neq y \end{cases}$$

Example 2

$$xNy = \begin{cases} 1 - |x-y|; & \textit{if } |x-y| \leq 1 \\ 0; & \textit{if } |x-y| > 1 \end{cases}$$

Example 3

$$xNy = \begin{cases} 1 - \frac{|x-y|}{c}; & \textit{if } |x-y| \leq c \\ 0; & \textit{if } |x-y| > c \end{cases}$$

Definition 2 (DeCock, Kerre, 2003) *Let* (X, ρ) *be a metric (pseudometric) space. Then* $R : X^2 \to [0, 1]$ *is a resemblance relation, if*

- $R(x, x) = 1$ *for all* $x \in X$,
- $R(x, y) = R(y, x)$ *for all* $x, y \in X$,
- *if* $\rho(x, y) \leq \rho(x, z)$ *then* $R(x, y) \geq R(x, z)$.

Example 4

$$R(x, y) = \begin{cases} 1 - \rho(x, y); & \text{if } \rho(x, y) \leq 1 \\ 0; & \text{if } \rho(x, y) > 1 \end{cases}$$

Example 5 (Fuzzy nearness, which is not a fuzzy resemblance)

Let N be define by the following formula:

$$(x_1, y_1)N(x_2, y_2) = \begin{cases} 1 - |x_1 - x_2|; & \textit{if } y_1 = y_2, |x_1 - x_2| \leq 1 \\ 0; & \textit{else} \end{cases}$$

Definition 3 (Janiš, 2008) *Let* (X, ρ, N, M) *be a quadruple such that* (X, ρ) *be a metric space, let* (N, M) *satisfy the following*

- $N(x, x) = 1$ *for all* $x \in X$,
- $N(x, y) = N(y, x)$ *for all* $x, y \in X$,
- $N(x, y) \geq N(x, z)$ *whenever* $\rho(x, y) \leq \rho(x, z)$,
- $M(x, x) = 0$ *for all* $x \in X$,
- $M(x, y) = M(y, x)$ *for all* $x, y \in X$,
- $M(x, y) \leq M(x, z)$ *whenever* $\rho(x, y) \leq \rho(x, z)$,
- $N(x, y) + M(x, y) \leq 1$ *for all* $x, y \in X$.

Then (N, M) *is called an IF nearness on* X.

In a similar way we can define IF-nearness in a linear space.

Examples of IF nearness

Example 6 *Let N be a usual fuzzy nearness in a metric space, let $M = 1 - N$. Then (N, M) is an IF-nearness.*

Example 7 *Let (X, ρ) be a metric space, let $x, y \in X$ and let k, l be real numbers. Put $N(x, y) = max\{0, 1 - k\rho(x, y)\}$ and $M(x, y) = min\{1, l\rho(x, y)\}$. Then (M, N) is an IF-nearness if and only if $k \geq l > 0$.*

Example 8 *Let (X, ρ) be a metric space, let $a, b \in X$ and let k, l be real numbers. Put $N(x, y) = a^{\rho(x,y)}$ and $M(x, y) = 1 - b^{\rho(x,y)}$. Then (M, N) is an IF-nearness if and only if $a \geq b > 0$.*

3. Cuts and cuts properties

$\alpha-$ **cuts of fuzzy relation (resemblance)** If $\alpha \in [0, 1)$ then by

R_α we will understand the $\alpha-$ cut of fuzzy relation R, which is the following (crisp) relation

$$(x, y) \in R_\alpha \Leftrightarrow R(x, y) > \alpha.$$

Proposition 1 **(**Tepavčevič, Šešelja, Janiš, Renčová, 2008) *Let* (X, ρ) *be a metric space. A fuzzy relation* R *is a resemblance on* (X, ρ) *if and only if its cuts fulfill the following conditions*

(i) $(x, x) \in R_\alpha$ *for all* $x \in X$ *and all* $\alpha \in [0, 1)$,

(ii) if $(x, y) \in R_\alpha$ *then* $(y, x) \in R_\alpha$ *for all* $x, y \in X$ *and all* $\alpha \in [0, 1)$,

(iii) $(x, z) \in R_\alpha$ *implies* $(x, y) \in R_\alpha$ *whenever* $\rho(x, y) \leq \rho(x, z)$.

$\alpha-$ cuts of IF-sets

Definition 4 *Let* $\alpha \in [0,1)$ *then by* $(M,N)_\alpha$ *we will understand the* $\alpha-$*cut of IF-nearness (M,N) which is the following*

$$(x,y) \in (M,N)_\alpha \Leftrightarrow M(x,y) + N(x,y) > \alpha.$$

Definition 5 *Let* T *be some t-norm,* $\alpha \in [0,1)$ *then by* $(M,N)_\alpha^T$ *we will understand the* $\alpha-$*cut (with respect to the t-norm* T*) of IF-nearness* (M,N) *which is the following*

$$x \in (M,N)_\alpha^T \Leftrightarrow T(M(x), 1 - N(x)) > \alpha.$$

Thank you for attention!