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THE PROCESS OF MODELING ECONOMIC PROBLEMS PRESENTED AS A GENERALIZED NET WITH INTUITIONISTIC FUZZY LOGIC ELEMENTS

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Introduction

The paper presents a variation of the metamodel of a general process of modeling an economic problem by application of specific mathematical model over some appropriate data set presented in [1]. The new model is also described using the apparatus of Generalized Nets (GNs) [2] but is expanded with the inclusion of intuitionistic fuzzy logic [3]. The fuzziness could be introduced in several components of the GN but the GN itself is not an intuitionistic one, in the sense of [4].

Economic tasks are defined in this paper as economic problems that could be mathematically interpreted. Economic processes characterize by high complexity, due to the multiple parameters and factors that either separately or in combination influence the results, and that are difficult to predict (which leads to high levels of uncertainty). That is why economic models are built so that to reflect only a limited subset of the influencial factors. Despite that, a powerful economic model usually includes so many variables that only formalized mathematically-based techniques can be of help. Careful modeling of each process that affects the problem is needed, which can help distribute the limited available resources to the most appropriate alternative. Economic decision making is the main purpose of some classical and modern techniques such as utility theory [6] and fuzzy-rational decision analysis [7], etc. These techniques present techniques to model processes on the basis of subjective information (which is usually imprecise), taking into account the preferences and risk attitude of the decision maker. The expected utility theory is central in utility theory. Fuzzy-rational decision analysis proposes a generalization of this method, by combining classical techniques under strict uncertainty [8], [9] ,[10] and the classical expected utility criterion. Many works prove the practical applicabilty of the fuzzy-rational decision analysis [11], [12].

Economic tasks that put problems in the areas of forecasting, planning, resource allocation, investment, etc. usually require to model the behaviour of on-going, simultaneously existing processes, requiring limited quantity of resources, some of which has to be shared. The solution of such economic tasks is found usually with specific mathematical methods, using sample or real socio-economic data for a given period of time.

GNs are extensions of the ordinary and non-ordinary Petri nets. GNs however provide more and larger modelling possibilities which make them a powerful tool for modelling of parallel, real-time flowing processes. The use of GN modelling methodology is suggested because it is a formal mathematical methodology that could provide a metamodel for the different mathematical methods used in the economic modelling and there are a number of good simulators of GNs available. The specific economic models could be further on represented by respective specific GN models each of which should be viewed as a sub-net of the metamodel. The expansion of the metamodel towards a sub-net model is possible through the use of the H_1 operator from the theory of the GNs.

The metamodel

The GN that represents the metamodel of the process of modelling an economic problem is given on Fig. 1. The GN is a reduced one, without time and capacity components. The places' and the arcs' capacity is defined as infinite. The number of tokens that circle in the GN is unlimited and could be constrained due to computational capacity of a simulator that would execute the GN. All the places but l_8 have the same priority. l_8 has higher priority than l_{10}. The GN is not an intuitionistic fuzzy GN, its characteristic functions could contain intuitionistic fuzzy measures (IFM) for the extent of the validity of a token's characteristic. An IFM is changed only when tokens from different type (α, β, γ, δ, etc.) interact.

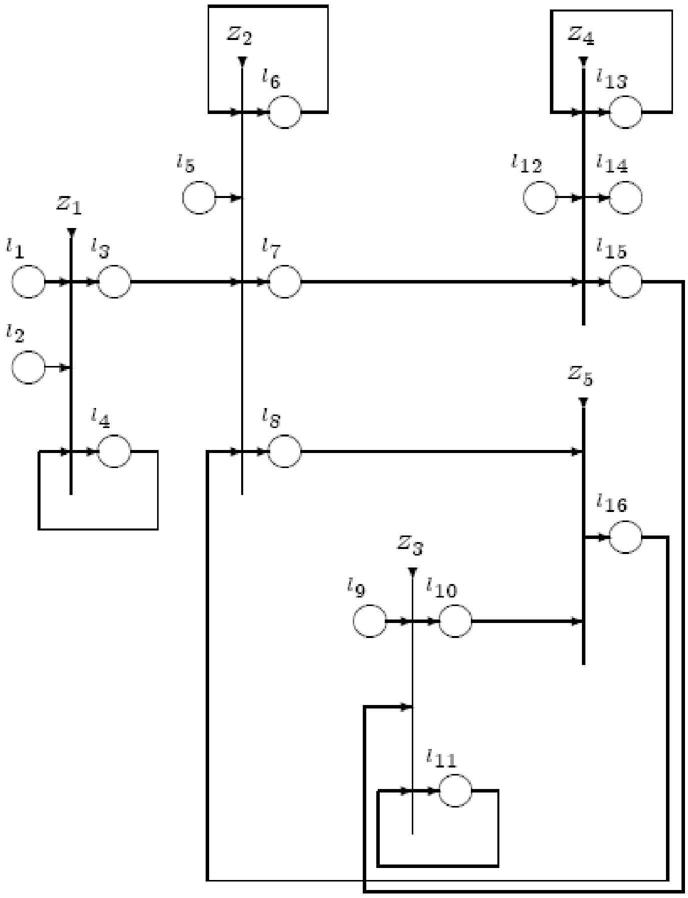

Fig. 1. The GN metamodel of the general process of modeling an economic task.

The GN contains 5 transitions: **A** = $\{Z_1, Z_2, Z_3, Z_4, Z_5\}$ which have the following forms.

$$Z_1 = \langle \{l_1, l_2, l_4\}, \{l_3, l_4\}, r_1, \wedge(l_1, \vee(l_2, l_4))\rangle,$$

represents the transformation of a real-world economic problem that is stated out in general terms, into an economic task. A token α that enters the net from place l_1 has characteristic: „a real-world economic problem".

Let the IFM for the degree to which the formulated task could be considered a correctly defined real-world economic problem be

$$IFM_\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$$

μ_α is the degree to which the formulated task is a correctly defined real-world economic problem, ν_α measures the degree to which the formulated task is not defined correctly, $1-\mu_\alpha-\nu_\alpha$ is the degree of indefiniteness.

A token β that enters the net from place l_2 has as characteristic:
„ a set of important factors that form or influence the given real-world economic problem“.
Let the IFM for the degree to which this set of factors include all the factors that influence the real-world economic problem is

$$IFM_\beta = <\mu_\beta, \nu_\beta>$$

As the GN could model many real-world tasks that could take place simultaneously, in order to distinguish these tasks, as part of the characteristic function of each couple α and β tokens is put also a number that is a unique identifier of the respective task. Additionally to the unique identifier, all types of tokens in the net have a class identifier number that assigns each of them to a respective class of economic tasks. A token from a given class of tasks with a given unique task identifier could be used for the generation of another token for the same class of tasks but with another unique task identifier.

The moment when the transition Z_1 is fired is when tokens α and β bringing same task identifier appear in l_1 and l_2. The transition's condition determining when the tokens will transfer from l_1 and l_2 respectively to l_3 and l_4 is the index matrix (IM) r_1:

$$r_1 = \begin{array}{c|cc} & l_3 & l_4 \\ \hline l_1 & W_{1,3} & false \\ l_2 & false & W_{2,4} \\ l_4 & false & W_{4,4} \end{array}$$

The predicate for transition of an α token from l_1 to l_3 is:
$W_{1,3}$ = "significant subset of factors that form or influence the given real-world economic problem is chosen"
The predicate for transition of a β token from l_2 to l_4 is:
$W_{2,4}$ = "The β token in l_2 is used for a first time".
$W_{4,4}$ = "The β token in l_4 is used for a successive time".

The token α enters l_3 and takes as next characteristic:
"significant subset of factors that form or influence the given real-world economic problem".

We could have different scenarios how to calculate the change of the IFM of the α token that enters l_3. We suggest similar logic to the idea presented in [5]. Based on the extent to which we are able to include in the model more of the available important factors, we could have five methods for calculation of the IFM of α in l_3:

1) Very optimistic: $IFM_\alpha = <\mu_\alpha+\mu_\beta - \mu_\alpha.\mu_\beta, \nu_\alpha.\nu_\beta>$
2) Optimistic: $IFM_\alpha = <\max(\mu_\alpha, \mu_\beta), \min(\nu_\alpha,\nu_\beta)>$
3) Average: $IFM_\alpha = <(\mu_\alpha+\mu_\beta)/2, (\nu_\alpha+\nu_\beta)/2>$
4) Pessimistic: $IFM_\alpha = <\min(\mu_\alpha, \mu_\beta), \max(\nu_\alpha,\nu_\beta)>$
5) Very pessimistic: $IFM_\alpha = <\mu_\alpha.\mu_\beta, \nu_\alpha+\nu_\beta- \nu_\alpha.\nu_\beta>$

Passing through Z_1 β token enters l_4 and stays there keeping the same characteristic - the initial set of factors. They could be used for generation of β token for a successive task from the same class of tasks.

The second transition is represented by:

$$Z_2 = <\{l_3, l_5, l_6, l_{16}\},\{l_6, l_7, l_8\}, r_2, \vee(\wedge(l_3,l_5), \wedge(l_3,l_6), l_{16}) >$$

The transition is activated in the moment when a task is formulated formally with variables for the significant subset of monitored factors or when a solution of the formulated task is found – i.e. when a token appears in l_3 or l_{16}. When a token from l_3 passes to l_8 it receives next characteristic:
"a specific mathematical methodology that will be employed to solve the economic task".

The mathematical apparatus used to develop and/or apply the economic model of the real-world problem is chosen on the basis of the specifics of the task and also on the subset of significant factors that will be monitored.

A token from l_{16} could pass through the transition Z_2 if in place l_5 or l_6 is present a token δ with characteristic:
„criteria that should be checked against the received solution" – i.e. if the solution is good enough in the scope of the posed economic problem. If a token δ with appropriate class identification number as that of the α token in l_{16} is not present in l_5 or l_6 then the GN will stop and wait for such a token to be produced. Let the IFM for the degree to which this set of criteria measure the solution against more of the initial factors brought by the β token with the same task identifier is IFM $_\delta = <\mu_\delta, \nu_\delta>$

So, when the α token in l_{16} passes through $Z_{2,}$ it could move to l_7 or l_8, depending on the result of check against the criteria brought by respective δ token. The calculation of this result which is actually the new IFM of the α token would be done using IFM_α and IFM_δ by one of the five scenarios described above.

If the solution is good enough acording some intuitionistic fuzzy limit value the α token in l_{16} is moved to $l_{7,}$ otherwise it is moved to l_8 for a search of solution with another methodology. In both cases α token receives as next characteristic:
"quality level of the solution".

The transition matrix for Z_2 is the following IM:

$r_2 =$

	l_6	l_7	l_8
l_3	false	false	$W_{3,8}$
l_5	$W_{5,6}$	false	false
l_6	$W_{6,6}$	false	false
l_{16}	false	$W_{16,7}$	$W_{16,8}$

$W_{3,8}$ = "a specific mathematical methodology that will be employed to solve the economic task is chosen".
$W_{5,6}$ = ""The δ token in l_5 is used for a first time".
$W_{6,6}$ = "The δ token in l_5 is used for a successive time".
$W_{16,7}$ = "a solution of the real-world economic problem with a task identifier X is found using an economic model EM_i"
$W_{16,8}$ = "a solution of the real-world economic problem with a task identifier X is found using an economic model EM_i and another solution is searched by application of an economic model EM_j"

Z_3 represents the extraction of data necessary for a given task from an intuitionistic fuzzy or a standard database with socio-economic data.

$$Z_3 = <\{l_9, l_{11}, l_{16}\}, \{l_{10}, l_{11}\}, r_3, \vee(l_9, l_{11}, l_{16})>$$

l_5 contains tokens γ_n each of which has as characteristic:
"database <n> with socio-economic data".

An intuitionistic fuzzy database could be used if the employed mathematical methodology uses intuitionistic fuzzy logic to develop the economic model. Otherwise standard data are used. In both cases the respective γ_n token in the GN has IFMγ_n = <$\mu\gamma_n$, $\nu\gamma_n$> that represents the extent to which the database could provide all requested data – i.e. a dataset that contains enough data to apply the mathematical model and no approximation of missing data is necessary.

The tokens in l_9 and/or l_{11} are as many as are the source databases available. At least one token has to be present in l_9 in order to be possible to start the GN. After an initial use each of the γ_n tokens move to l_{11} and stay there ready for next use.

Place l_8 has higher priority than l_{10}, which means that first a token with a given task identifier has to appear in l_8 and next a token with the same task identifier would be moved to l_{10}. The characteristic of a token in l_8 is used to form a query towards a database to select only those data that are necessary for the execution of the mathematical model. When the result of a select from the database <n> is produced, the γ_n token from l_9 splits – a token γ_n representing the database itself moves to l_{11} and a token γ_n' moves to l_{10} with the following characteristic:
" result-set from the select provoked from the token in l_8".

The γ_n' token next merges with the α token from l_8 and the result is an α token in l_{16}. Its characteristic contains the characteristics of the two source tokens. The new IFM of this α token could be calculated using one of the five scenarios described above.

Transition Z_3 is activated also when a token appears in l_{15} – it has as characteristic the solution of the economic task plus an IFM about the real-life reasonability status of the solution. When this token passes through the transition it is merged with the respective token in l_{11} indicating that the solution of the economic task is stored in the database.

The transition IM for Z_3 is:

$$r_3 = \begin{array}{c|cc} & l_{10} & l_{11} \\ \hline l_9 & W_{9,10} & W_{9,11} \\ l_{11} & W_{11,10} & W_{11,11} \\ l_{15} & false & W_{15,11} \end{array}$$

$W_{9,10}$ = "a query towards database <n> to select only those data that will be used in the search for a solution of the task in l_8 is executed"
$W_{9,11}$ = "a query towards database <n> is executed for a first time"
$W_{11,10}$ = "a query towards database <n> to select only those data that will be used in the search for a solution of the task in l_8 is executed"
$W_{11,11}$ = "a next query towards database <n> is executed"
$W_{15,11}$ = "the solution of the economic task is stored in the database"

$$Z_4 = \langle \{ l_7, l_{12}, l_{13}\},\{ l_{13}, l_{14}, l_{15}\}, r_4, \wedge(l_7, \vee(l_{12}, l_{13}))\rangle,$$

The transition is activated when a solution of a task is found (i.e. token α appears in l_7) and a token ε with the same task identifier as that of α is present in l_{12} or l_{13}. If the token ε with the appropriate class identification number is not available then the GN will stop and wait for such token to be produced. When a token ε passes through the transition it moves to l_{13} where it is kept and could be used to generate an ε token in l_{12} to be used for another task from the same class. The token ε has the characteristic:
„globally scoped criteria to be checked against the received solution" – i.e. criteria to check if the solution seems reasonable or appropriate according to the currently prevailing socio-economic, political, geographical, ecological conditions, etc. Let the IFM for the degree to

which this set of criteria measures the solution against more of the real-life conditions from external to the posed economic problem areas is IFM $_{\varepsilon} = <\mu_{\varepsilon}, \nu_{\varepsilon}>$

The token from l_7 has as characteristic the solution of the economic task and the quality of the solution according to the locally scoped criteria – i.e. if the proposed solution seems to be a good one taking into account only the specifics of the posed economic problem. When this α token passes through Z_4 its characteristic is changed according to the criteria brought by the respective ε token – the new value of its IFM is calculated using IFM_{α} and IFM_{ε} by one of the five scenarios described above. So, the α token from l_7 moves to l_{15} with a final new IFM as next characteristic:
"an intuitionistic fuzzy mark of the real-life reasonability status of the solution".

The transition IM for Z_4 is:

$r_4 =$

	l_{13}	l_{14}	l_{15}
l_7	false	$W_{7,14}$	$W_{7,15}$
l_{12}	$W_{12,13}$	false	false
l_{13}	$W_{13,13}$	false	false

$W_{12,13}$ = "The ε token in l_{12} is used for a first time".
$W_{13,13}$ = "The ε token in l_{13} is used for a successive time".
$W_{7,14} = W_{7,15}$ = „There is a token ε in l_{12} or l_{13} with the same class identifier as α in l_7 and according to the criteria in ε is calculated the value of quality level of the tasks' solution".

$$Z_5 = <\{l_8, l_{10}\},\{l_{16}\}, r_5, \wedge(l_8,l_{10})>,$$

Z_5 is activated when tokens with same task identification number appear in l_8 and l_{10}. l_8 has higher priority than l_{10} as described above in the description of transition Z_3.

The token γ_n ' in l_{10} merges with the α token in l_8 and a respective new IFM for α is calculated according one of the five scenarios described in the beginning of the paper. As a result the token α passes to l_{16} with next characteristic:
"slected data, appropriate for the execution of the mathematical model".

In l_{16} the respective mathematical algorithm employed to find the solution of the task is executed. By using of the H_1 operator this place could be expanded to another GN that models in details the chosen mathematical algorithm.

The transition matrix for Z_5 is:

$r_5 =$

	l_{16}
l_8	$W_{8,16}$
l_{10}	$W_{10,16}$

$W_{8,16} = W_{10,16}$ = "a result-set is produced in l_{10} that provides the data used to solve the task"

Conclusion

The constructed generalized net (GN) presents a metamodel which could be used as a wrapper of any specific mathematical methodology employed to find a solution of an economic task. In a next publication will be developed a GN model that will apply a sample mathematical methodology over a sample economic task. That model would be the sub-net represented in the metamodel by the place l_{16}. This is possible through the use of the H_1 operator from the theory of the GNs.

The intuitionistic fussy measures used are a good method to incorporate in the metamodel the different kinds of indefiniteness in the real economic environment.

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