NIFS 16 (2010), 3, 42-44

On intuitionistic fuzzy level operators

Beloslav Riečan[1,2] and Krassimir Atanassov[3]

[1] Faculty of Natural Sciences, Matej Bel University, Department of Mathematics
Tajovského 40, 974 01 Banská Bystrica, Slovakia

[2] Mathematical Institute of Slovak Acad. of Sciences, Štefánikova 49, SK–81473 Bratislava
e-mails: *riecan@mat.savba.sk* and *riecan@fpv.umb.sk*

[3] Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria,
e-mail: *krat@bas.bg*

The concept of an Intuitionistic Fuzzy Set (IFS) is defined (see, e.g., [1]) as follows.

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A | E \to [0,1]$ and $\nu_A : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$: $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

For every two IFSs A and B a variety of relations and operations have been defined (see, e.g. [1]). The most important of them are:

$$\begin{array}{lcl} A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\ A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\ A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\ \overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}. \end{array}$$

Let A be an IFS. The level operator $N_{\alpha,\beta}$ is defined by (see [1]):

$$N_{\alpha,\beta}(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | (x \in E) \& (\mu_A(x) \geq \alpha) \& (\nu_A(x) \leq \beta)\}.$$

We shall extend this definition, following the idea from [2], where some extended modal operators are defined. Four of them, that we will use below, are

$$H_B(A) = \{\langle x, \mu_B(x).\mu_A(x), \nu_A(x) + \nu_B(x).\pi_A(x)\rangle \mid x \in E\},$$
$$H_B^*(A) = \{\langle x, \mu_B(x).\mu_A(x), \nu_A(x) + \nu_B(x).(1 - \mu_B(x).\mu_A(x) - \nu_A(x))\rangle \mid x \in E\},$$
$$J_B(A) = \{\langle x, \mu_A(x) + \mu_B(x).\pi_A(x), \nu_B(x).\nu_A(x)\rangle \mid x \in E\},$$
$$J_B^*(A) = \{\langle x, \mu_A(x) + \mu_B(x).(1 - \mu_A(x) - \nu_B(x).\nu_A(x)), \nu_B(x).\nu_A(x)\rangle \mid x \in E\},$$

Let A and B be two IFSs. Then

$$N_B(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | (x \in E) \& (\mu_A(x) \geq \mu_B(x)) \& (\nu_A(x) \leq \nu_B(x))\},$$

$$N_B^*(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | (x \in E) \& (\mu_A(x) \leq \mu_B(x)) \& (\nu_A(x) \geq \nu_B(x))\}.$$

It is very important to note that sets $N_{\alpha,\beta}(A), N_B(A)$ and $N^*_B(A)$ are IFSs, but now over new universes, that are subsets of universe E.

Obviously, for each IFS A:

$$A = N_{O^*}(A) = N^*_{E^*}(A),$$

where

$$O^* = \{\langle x, 0, 1\rangle | x \in E\}, \quad \text{and} \quad E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Theorem 1: For every two IFSs A and B:

a) $A = N_B(A)$ iff $B \subset A$,

b) $A = N^*_B(A)$ iff $A \subset B$,

c) $N_B(A) = \emptyset$ iff $(\forall x \in E)(\mu_A(x) < \mu_B(x)) \vee (\nu_A(x) > \nu_B(x))$,

d) $N^*_B(A) = \emptyset$ iff $(\forall x \in E)(\mu_A(x) > \mu_B(x)) \vee (\nu_A(x) < \nu_B(x))$,

e) $A \subset N_B(A) \cup N^*_B(A)$.

Following [3] we shall introduce the operators

$$P_B(A) = \{\langle x, \max(\mu_B(x), \mu_A(x)), \min(\nu_B(x), \nu_A(x))\rangle | x \in E\},$$

$$Q_B(A) = \{\langle x, \min(\mu_B(x), \mu_A(x)), \max(\nu_B(x), \nu_A(x))\rangle | x \in E\},$$

$$\mathcal{C}(A) = \{\langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y)\rangle \mid x \in E\},$$

$$\mathcal{I}(A) = \{\langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y)\rangle \mid x \in E\}.$$

Theorem 2: For every three IFSs A, B, C:

a) $N_C(N_B(A)) = N_{B \cup C}(A)$.

b) $N^*_C(N^*_B(A)) = N^*_{B \cap C}(A)$.

c) $N_C(N_B(A)) = N_{P_C(B)}(A)$,

d) $N^*_C(N^*_B(A)) = N^*_{Q_C(B)}(A)$.

e) $N^*_C(N^*_B(A)) = N^*_{Q_C(B)}(A)$,

f) $N^*_C(N^*_B(A)) = N^*_{Q_C(B)}(A)$.

Proof: a) Let the three IFSs A, B, C be given. Then

$$\begin{aligned} N_C(N_B(A)) &= N_C(\{\langle x, \mu_A(x), \nu_A(x)\rangle | (x \in E)\&(\mu_A(x) \geq \mu_B(x))\&(\nu_A(x) \leq \nu_B(x))\}) \\ &= \{\langle x, \mu_A(x), \nu_A(x)\rangle | (x \in E)\&(\mu_A(x) \geq \mu_B(x)) \\ &\quad \&(\mu_A(x) \geq \mu_C(x))\&(\nu_A(x) \leq \nu_B(x))\&(\nu_A(x) \leq \nu_C(x))\}) \\ &= \{\langle x, \mu_A(x), \nu_A(x)\rangle | (x \in E)\&(\mu_A(x) \geq \max(\mu_B(x), \mu_C(x)) \\ &\quad \&(\nu_A(x) \leq \min(\nu_B(x)), \nu_C(x))\}) = N_{B \cup C}(A). \end{aligned}$$

The next assertions are proved analogously and we will omit their proofs.

Theorem 3: For every three IFSs A, B, C:

a) $N_C(A \cap B) = N_C(A) \cap^* N_C(B)$.

b) $N^*_C(A \cup B) = N^*_C(A) \cup^* N^*_C(B)$,

where here and below $\cup^*$ and $\cap^*$ are set theoretic operations *"union"* and *"intersection"*.

Theorem 4: For every two IFSs A and B: $N_B^*(A) = \overline{N_{\overline{B}}(\overline{A})}$.
Theorem 5: For every two IFSs A and B:

$$\text{a) } N_{\mathcal{C}(B)}(A) = \bigcap^*_{C \subset \mathcal{C}(B)} N_C(A),$$

$$\text{b) } N^*_{\mathcal{I}(B)}(A) = \bigcup^*_{\mathcal{I}(B) \subset C} N^*_C(A).$$

Theorem 6: For every two IFSs A and B:

$$\text{a) } N_C(\mathcal{I}(A)) = \bigcup^*_{B \subset \mathcal{I}(A)} N_C(B),$$

$$\text{b) } N^*_C(\mathcal{C}(A)) = \bigcap^*_{\mathcal{C}(A) \subset B} N^*_C(B).$$

Theorem 7: For every three IFSs A, B, C:

a) $N_{J_C(B)}(A) \subset^* N_B(A)$,

b) $N_{H_C(B)}(A) \supset^* N_B(A)$,

c) $N_{J^*_C(B)}(A) \subset^* N_B(A)$,

d) $N_{H^*_C(B)}(A) \supset^* N_B(A)$,

e) $N^*_{J_C(B)}(A) \supset^* N^*_B(A)$,

f) $N^*_{H_C(B)}(A) \subset^* N^*_B(A)$,

g) $N^*_{J^*_C(B)}(A) \supset^* N^*_B(A)$,

h) $N^*_{H^*_C(B)}(A) \subset^* N^*_B(A)$,

where $\subset^*$ and $\supset^*$ are set theoretical relations *"inclusion"*.

Theorem 8: For every two IFSs A and B:

a) $N_{\mathcal{C}(B)}(A) = N_{\sup_{y \in E} \mu_C(y), \inf_{y \in E} \nu_C(y)}(A)$,

b) $N^*_{\mathcal{I}(B)}(A) = N_{\inf_{y \in E} \mu_C(y), \sup_{y \in E} \nu_C(y)}(A)$.

Theorem 9: For every three IFSs A, B, C:

a) $N_{P_C(B)}(A) = N_C(A) \cap^* N_B(A)$.

b) $N_{Q_C(B)}(A) = N_C(A) \cup^* N_B(A)$.

Aknowledgements: The second author is grateful for the support provided by the project BIn-2/09 "Design and development of intuitionistic fuzzy logic tools in information technologies" funded by the National Science Fund, Bulgarian Ministry of Education, Youth, and Science.

References

[1] Atanassov, K. Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Heidelberg, 1999.

[2] Atanassov, K. New operators over intuitionistic fuzzy sets. Proc. of IPMU'04, Perugie, Italy, June 2004, Vol. 2, 1383-1387.

[3] Atanassov, K. On Intuitionistic Fuzzy Sets Theory, Springer (in press).