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ON INTUITIONISTIC FUZZY LOGIC INTERPRETATION OF SMIRNOV'S AXIOMATIC SYSTEM OF THE SYLOGISTIC

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In [1] V. Smirnov discussed different logical systems, one of which is the following:

A1. $\mathrm{A}MP \& \mathrm{A}SM \supset \mathrm{A}SP$
A2. $\mathrm{E}MP \& \mathrm{A}SM \supset \mathrm{E}SP$
A3. $\mathrm{A}SP \supset \mathrm{I}SP$
A4. $\mathrm{E}SP \supset \mathrm{E}PS$
A5. $\mathrm{O}SP \equiv \neg \mathrm{A}SP$
A6. $\mathrm{I}SP \equiv \neg \mathrm{E}SP$
A7. $\mathrm{I}SP \supset \mathrm{A}SS$,

where operators A, E, O and I have the following senses:

A XY denotes "All X are Y",
E XY denotes "No one X is Y",
I XY denotes "Some X is Y",
O XY denotes "Some X is not Y".

First, we shall define the predicate $\mathcal{A}_X(Y)$ by "X is Y".
Let us assume for every three objects X, Y, Z:

$$V(\mathcal{A}_X(X)) = <1, 0>, \tag{1}$$

$$V(\mathcal{A}_X(Y)) = V(\mathcal{A}_Y(X)), \tag{2}$$

$$V(\mathcal{A}_X(Y)) \wedge V(\mathcal{A}_Y(Z)) \leq V(\mathcal{A}_X(Z)), \tag{3}$$

whete for every two propositional forms P and Q

$$V(P) \leq V(Q) \text{ iff } \mu(P) \leq \mu(Q) \text{ and } \nu(P) \geq \nu(Q).$$

In the frames of the Intuitionistic Fuzzy Logic (IFL) (see [2-6]), to each propositional form p (cf. [7]) we assign two real numbers, $\mu(p)$ and $\nu(p)$, called truth- and falsity-degrees, respectively, with the following constraint:

$$\mu(p) + \nu(p) \leq 1.$$

The notion of *intuitionistic fuzzy tautology* (IFT) is defined in [2,6] through:

The propositional form A for which $V(A) = \langle a, b \rangle$ is an IFT if and only if $a \geq b$.

We shall prove the following

LEMMA For every propositional forms P and Q, if $V(P) \leq V(Q)$, then $P \supset Q$ is an IFT. The opposite is not true.

Proof: Let $V(P) =< a, b >$ and $V(Q) =< c, d >$. Let $V(P) \leq V(Q)$. Then $a \leq c$ and $b \geq d$ and therefore from

$$V(P \supset Q) =< max(b, c), min(a, d) >$$

we obtain that

$$max(b, c) - min(a, d) \geq b - d \geq 0,$$

i.e., $P \supset Q$ is an IFT. In the opposite case, for example, if $V(P) =< 0.1, 0.2 >$ and $V(Q) = < 0.3, 0.4 >$, then if $P \supset Q$ is an IFT, because $V(P \supset Q) =< 0.3, 0.1 >$, but, obviously, $V(P) \leq V(Q)$ is not valid.

Second, we shall show the intuitionistic fuzzy interpretations of the above four operators and of the seven axioms.

The operators are interpreted as:

A XY	by	$\forall X \mathcal{A}_X(Y)$,
E XY	by	$\neg \exists X \mathcal{A}_X(Y)$,
I XY	by	$\exists X \mathcal{A}_X(Y)$,
O XY	by	$\neg \forall X \mathcal{A}_X(Y)$.

Now, the axioms have the following interpretations:

B1. $\forall M \mathcal{A}_M(P) \& \forall S \mathcal{A}_S(M) \supset \forall S \mathcal{A}_S(P)$

B2. $\neg \exists M \mathcal{E}_M(P) \& \forall S \mathcal{A}_S(M) \supset \neg \exists S \mathcal{E}_S(P)$

B3. $\forall S \mathcal{A}_S(P) \supset \exists S \mathcal{I}_S(P)$

B4. $\neg \exists S \mathcal{E}_S(P) \supset \neg \exists P \mathcal{E}_P(S)$

B5. $\exists S \neg \mathcal{O}_S(P) \equiv \neg \forall S \mathcal{A}_S(P)$

B6. $\exists S \mathcal{I}_S(P) \equiv \neg \forall S \mathcal{E}_S(P)$

B7. $\forall S \mathcal{I}_S(P) \supset \forall S \mathcal{A}_S(S)$.

The following assertion is valid.

THEOREM For every three objects M, P and S B1-B7 are IFTs.

Proof: For B1:

$$V(\forall M \mathcal{A}_M(P) \& \forall S \mathcal{A}_S(M) \supset \forall S \mathcal{A}_S(P))$$

$$= (V(\forall M \mathcal{A}_M(P)) \wedge V(\forall S \mathcal{A}_S(M))) \rightarrow V(\forall S \mathcal{A}_S(P)))$$

$$= \langle \min_S (\mathcal{A}_S(M)), \max_S (\mathcal{A}_S(M))\rangle \wedge \langle \min_M (\mathcal{A}_M(P)), \max_M (\mathcal{A}_M(P))\rangle$$

$$= \langle min(\min_S \mu(\mathcal{A}_S(M)), \min_M \mu(\mathcal{A}_S(M))), max(\max_S \nu(\mathcal{A}_S(M)), \max_M \nu(\mathcal{A}_M(P)))\rangle$$

$$= \langle min(\min_S \mu(\mathcal{A}_S(M)), \min_M \mu(\mathcal{A}_S(M))), max(\max_S \nu(\mathcal{A}_S(M)), \max_M \nu(\mathcal{A}_M(P)))\rangle$$

$$= \langle \min_S (min(\mu(\mathcal{A}_S(M)), \mu(\mathcal{A}_S(M))), \max_S (max(\nu(\mathcal{A}_S(M)), \nu(\mathcal{A}_M(P)))\rangle$$

From the Lemma and from (3) it follows that $\mathcal{A}_M(P) \wedge \mathcal{A}_S(M) \to \mathcal{A}_S(P)$ is an IFT.

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