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SOME PROPERTIES OF TWO OPERATIONS OVER INTUITIONISTIC FUZZY SETS

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1 Introduction

Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Different relations and operations are introduced over the IFSs. Some of them are the following

$$
\begin{array}{lcl}
A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\
A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)), \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\
A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\
A + B & = & \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\}, \\
A.B & = & \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle | x \in E\}, \\
A@B & = & \{\langle x, \dfrac{\mu_A(x) + \mu_B(x)}{2}, \dfrac{\nu_A(x) + \nu_B(x)}{2}\rangle | x \in E\}.
\end{array}
$$

In [2] we introduced two new operators, defined over IFSs. They are analogous of operations "substraction" and "division" and have the forms for every two given IFSs A and B:

$$A - B = \{\langle x, \mu_{A-B}(x), \nu_{A-B}(x)\rangle | x \in E\},$$

where

$$
\mu_{A-B}(x) = \begin{cases} \dfrac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \text{ and } \nu_B(x) > 0 \\ & \text{and } \nu_A(x)\pi_B(x) \leq \pi_A(x)\nu_B(x) \\ 0, & \text{otherwise} \end{cases}
$$

and

$$
\nu_{A-B}(x) = \begin{cases} \dfrac{\nu_A(x)}{\nu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \text{ and } \nu_B(x) > 0 \\ & \text{and } \nu_A(x)\pi_B(x) \leq \pi_A(x)\nu_B(x) \\ 1, & \text{otherwise} \end{cases} ;
$$

and

$$A : B = \{\langle x, \mu_{A:B}(x), \nu_{A:B}(x)\rangle | x \in E\},$$

where

$$
\mu_{A:B}(x) = \begin{cases} \dfrac{\mu_A(x)}{\mu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \text{ and } \mu_B(x) > 0 \\ & \text{and } \dfrac{\mu_A(x)}{\mu_B(x)} \leq \dfrac{\pi_A(x)}{\pi_B(x)} \text{ and } \mu_A(x)\pi_B(x) \leq \pi_A(x)\mu_B(x) \\ 0, & \text{otherwise} \end{cases}
$$

and

$$\nu_{A:B}(x) = \begin{cases} \dfrac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \text{ and } \mu_B(x) > 0 \\ & \text{and } \mu_A(x)\pi_B(x) \leq \pi_A(x)\mu_B(x) \\ 1, & \text{otherwise} \end{cases};$$

2 On some properties of operations "substraction" and "division"

Following [?], firstly we shall mention that in a result of each one of the two operation we obtain an IFS.

In [?] are proved the following assertions for every two IFSs A and B:

(a) $A - A = O^*$,

(b) $A : A = E^*$,

(c) $A - O^* = A$,

(d) $A : E^* = A$,

(e) $(A : B).B = A$.

The following assertions hold.

Theorem 1: For every two IFSs A and B:

$$A : B = \{\langle x, \mu_{A:B}(x), \nu_{A:B}(x)\rangle | x \in E\},$$

where

$$\mu_{A:B}(x) = \begin{cases} \dfrac{\mu_A(x)}{\mu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \text{ and } \mu_B(x) > 0 \\ & \text{and } \dfrac{\mu_A(x)}{\mu_B(x)} \leq \dfrac{\pi_A(x)}{\pi_B(x)} \\ & \text{and } \mu_A(x)\pi_B(x) \leq \pi_A(x)\mu_B(x) \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{A:B}(x) = \begin{cases} \dfrac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}, & \text{if } \mu_A(x) \geq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \text{ and } \mu_B(x) > 0 \\ & \text{and } \mu_A(x)\pi_B(x) \leq \pi_A(x)\mu_B(x) \\ 1, & \text{otherwise} \end{cases};$$

Theorem 2: For every two IFSs A and B:

(e) $A - U^* = O^*$,

(f) $A : U^* = O^*$,

(g) $(A - B) + B = A$,

(i) $(A - B) - C = (A - C) - B$,

(j) $(A : B) : C = (A : C) : B$.

Now, we see that for the two IFSs A and B, so that $B \subset A$ and
(a) for each $x \in E$

$$\nu_B(x) > 0,$$

$$\nu_A(x)\pi_B(x) \leq \pi_A(x)\nu_B(x),$$

then

$$A - B = \{\langle x, \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)}, \frac{\nu_A(x)}{\nu_B(x)}\rangle | x \in E\},$$

(b) for each $x \in E$

$$\mu_B(x) > 0,$$

$$\mu_A(x)\pi_B(x) \leq \pi_A(x)\mu_B(x),$$

then

$$A : B = \{\langle x, \frac{\mu_A(x)}{\mu_B(x)}, \frac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}\rangle | x \in E\}.$$

Obviously

$$O^* - U^* = O^*,$$

$$O^* - E^* = O^*,$$

$$U^* - O^* = U^*,$$

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$$E^* - O^* = E^*,$$

$$E^* - U^* = O^*,$$

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$$O^* : E^* = O^*,$$

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$$U^* : E^* = O^*,$$

$$E^* : O^* = O^*,$$

$$E^* : U^* = O^*.$$

Two open problems at the moment are the following:
1. Are there relations between operations " $-$ " and " : " from one side and opeastions " $\cup$ " and " $\cap$ " from other?
2. Are there other relations between operations "$-$" and " : " from one side and operations " $+$ " and "." from other?

Theorem 3: For every three IFSs A, B and C:
(a) $(A@B) - C = (A - C)@(B - C)$,

(b) $(A@B) : C = (A : C)@(B : C)$,

(c) $\overline{A - B} = \overline{A} : \overline{B}$.

Having in mind relations between operations "$-$" and "$+$" and between operations ":" and ".", we can call the new operations algebraic operations.

References

[1] K. Atanassov, Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[2] R. Riećan and K. Atanassov, On two operations over intuitionistic fuzzy sets. Journal of Applied Mathematics, Statistics and Informatics, Vol. 2, 2006, No. 2, 145-148.