Notes on Intuitionistic Fuzzy Sets Print ISSN 1310-4926, Online ISSN 2367-8283

2025, Volume 31, Number 3, 332–345

DOI: 10.7546/nifs.2025.31.3.332-345

Evaluation and forecasting of technical conditions in complex systems using intuitionistic fuzzy logics

Toncho Ivanov Boyukov [©]



Faculty of Technical Sciences, Burgas State University "Prof. Dr. Asen Zlatarov" 1 Prof. Yakimov Str., 8010 Burgas, Bulgaria

e-mail: toncho_b@abv.bg

Received: 2 June 2025 **Revised:** 7 July 2025 Accepted: 19 August 2025 Online First: 22 August 2025

Abstract: This article describes methods for monitoring and studying the technical conditions and diagnostics of systems and objects, which should provide us with easy access to information about the state of a given system or object from a monitored system. Evaluations using intuitionistic fuzzy sets provide us with a framework for implementing new approaches during system operation in real time, which gives us the opportunity to improve the efficiency and accuracy of the processes.

Here we integrate factors such as measurements (e.g., temperature, deformation, degradation) from system components and map them into intuitionistic fuzzy assessments. The method takes into account the occurring changes in the state, external and internal influencing factors and allows us to predict critical states using analytical and simulation tools. The structure thus built allows us to more accurately and specifically monitor and analyze the observed system.

Keywords: Diagnostics, Efficiency, Accuracy, Evaluation, Uncertainty, Information technology, Intuitionistic fuzzy sets.

2020 Mathematics Subject Classification: 03E72.



1 Introduction

The development of diagnostic applications that allow real-time access to information about the operating status of various types of systems, as well as the implementation of new methods in the workflow, has a key role and potential to significantly improve the efficiency and accuracy of the monitored process. Intuitionistic fuzzy sets provide us with the necessary framework for modeling the relationships in the workflow, which will reveal the uncertainty at each stage of the process [13, 15] and [2, Section 2.4].

2 Intuitionistic fuzzy sets as a means for estimating an observed system state

Fuzzy sets are introduced by Zadeh (1965) [17]. In them each element x contains a degree of membership $\mu(x)$, determining the degree to which it belongs to the fuzzy set. Intuitionistic fuzzy sets (IFs), introduced by Atanassov in 1983) [1,3], extend the fuzzy sets with the presence of two functions:

- a degree of membership $\mu(x) \in [0, 1]$,
- a degree of non-membership $\nu(x) \in [0,1]$,

so that for each x: $0 \le \mu(x) + \nu(x) \le 1$.

Those function define a third function – the degree of uncertainty: $\pi(x) = 1 - \mu(x) - \nu(x)$, which expresses the uncertainty or indeterminacy regarding the element x's status of membership or non-membership to the set.

This method allows for accurate and clear interpretation of uncertainty in data, especially in areas such as sensor systems and the diagnostics of complex systems.

The IFS is defined by the triple:

$$A(x) = \{ \langle \mu_A(x), \nu_A(x), \pi_A(x) \rangle \mid x(t) \in E \}, \tag{1}$$

where E is a fixed universe.

The intuitionistic fuzzy pair (IFP) is defined in [5] by $\langle a, b \rangle$, where $a, b, a + b \in [0, 1]$.

Let $U = \{u_1, u_2, \dots, u_n\}$ be the set of input factors. Each element $u_i \in U$ is illustrated by an IFP:

$$I_U(u_i) = \langle \mu_U(u_i), \nu_U(u_i) \rangle.$$

Let $P = \{p_1, p_2, \dots, p_m\}$ be the set of internal processes. Each element $p_j \in P$ is illustrated by an IFP:

$$F_P(p_j) = \langle \mu_P(p_j), \nu_P(p_j) \rangle.$$

Let $O = \{o_1, o_2, \dots, o_q\}$ be the set of outputs. Each element o_k of O is illustrated by an IFP:

$$O_O(o_k) = \langle \mu_O(o_k), \nu_O(o_k) \rangle.$$

We calculate the weighted activation for each element, membership and non-membership as:

$$WA_{U}(u_{i}) = \mu_{U}(u_{i}) \cdot (1 - \nu_{U}(u_{i})),$$

$$WA_{P}(p_{j}) = \mu_{P}(p_{j}) \cdot (1 - \nu_{P}(p_{j})),$$

$$WA_{O}(o_{k}) = \mu_{O}(o_{k}) \cdot (1 - \nu_{O}(o_{k})).$$
(2)

It should be noted that in Section 2 the notation u_i is used for input factors, while in Section 5 the notation U_j is used for possible causes. Although both sets are denoted by U, they represent different levels in the system description: input-level factors vs. abstracted causes.

Index matrix representation with state estimation of a complex system

Let us define a system state index matrix (IM, see [18]), where each entry contains a triplet:

Interpretation of the index matrix

In the constructed index matrix, the **rows** correspond to the "Observed system condition" (as evaluated from measurement or monitoring), while the **columns** correspond to the "Reference" or "Expected system condition". Thus, each entry $\langle \mu, \nu \rangle$ represents the degree to which an observed state (e.g., "Average") matches a given reference label (e.g., "Good").

For example, the entry in row "Good", column "Average", $\langle 0.8, 0.1 \rangle$, should be interpreted as follows: the observed system condition is good, while it is being compared against the reference condition "Average"; the degree of correspondence is $\mu=0.8$, the degree of non-correspondence is $\nu=0.1$, and the remaining $\pi=0.1$ reflects uncertainty in the result of the comparison.

It should be emphasized that the values used are illustrative and serve to demonstrate how such an index matrix could be structured. In a real application, the numerical values would be obtained as a result from either expert judgment or statistical data analysis.

The purpose of the index matrix is to provide a compact formal tool for comparing observed and expected states under uncertainty. Later, it can be aggregated with forecast information to produce rule-based evaluations of system condition and to detect deviations (e.g., early warning when "Observed" = "Average" aligns poorly with "Expected" = "Good").

The matrix thus constructed allows us to derive the system conditions through intuitionistic fuzzy estimation, where each cell takes into account the expected degree of correspondence between the observed and reference states when working under uncertainty.

3 Modeling and analysis of the factors contributing to changes in the technical condition of the studied objects

Here we apply the (IFs) framework introduced by Atanasov [3]. This approach allows us to represent both the degree of membership (μ) and non-membership (ν) , and to derive the degree of uncertainty (π) , with the aim of capturing the uncertainty of the estimate more accurately.

3.1 Process modeling

Criteria: We introduce a certain number of criteria, denoted as $C = \{C_1, C_2, \dots, C_n\}$, used as an assessment of the causes of changes in the technical condition of the studied systems:

- Measurability How accurate is the quantitative impact of given causes?
- **Consequences** What impact does it have on the state of the system?
- Manageability The ability of these causes to be controlled or reduced?
- Actionability Is it possible to derive specific actions from these causes?

Each cause U_i is described over these criteria as:

$$R_i = [(\mu_{i1}, \nu_{i1}), (\mu_{i2}, \nu_{i2}), \dots, (\mu_{in}, \nu_{in})],$$

where μ_{ij} denotes the *degree of truth* expressing to what extent the cause U_i is consistent with the criterion C_j , ν_{ij} denotes the *degree of falsity* expressing to what extent the cause U_i fails to satisfy the criterion C_j , and $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$ represents the *degree of uncertainty* (or indeterminacy) related to the evaluation.

Example 1. This example illustrates the representation of two possible causes of deterioration in the technical state of an object — *fatigue* and *corrosion* — expressed as intuitionistic fuzzy evaluations over three predefined criteria. Each pair (μ, ν) in R_{fatigue} and $R_{\text{corrosion}}$ denotes, respectively, the degree of truth and the degree of falsity of the given cause with respect to the corresponding criterion.

$$R_{\text{fatigue}} = [(0.9, 0.05), (0.7, 0.2), (0.8, 0.1)], \quad R_{\text{corrosion}} = [(0.6, 0.3), (0.5, 0.4), (0.7, 0.15)]$$

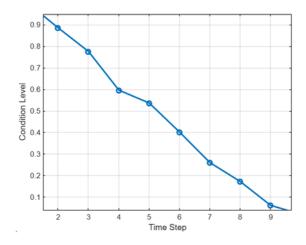


Figure 1. System condition over time

Similarly, we define factors influencing the intensity of changes F_k by:

$$F_k = [(\mu_{k1}, \nu_{k1}), (\mu_{k2}, \nu_{k2}), \dots, (\mu_{kn}, \nu_{kn})].$$

Example 2. Here we provide intuitionistic fuzzy evaluations of two *factors* that influence the *intensity* of changes in the technical state: *usage conditions* and *environmental influence*. Each pair (μ, ν) in F_{use} and $F_{\text{environment}}$ expresses the strength of influence (truth) and the lack of influence (falsity) with respect to the selected criteria.

$$F_{\text{use}} = [(0.8, 0.1), (0.6, 0.25), (0.75, 0.15)], \quad F_{\text{environment}} = [(0.9, 0.05), (0.7, 0.1), (0.85, 0.05)]$$

Reliability indicators are presented as intuitionistic fuzzy values on the corresponding indicators by:

$$C_{\text{reliability}} = (\mu, \nu), \quad \pi = 1 - \mu - \nu,$$

where:

- μ is the degree of reliability (e.g., high Mean Time Between Failures (MTBF));
- ν is the degree of unreliability (e.g., frequent Mean Time to Repair (MTTR)).

Example 3. This example shows the representation of a reliability indicator in the intuitionistic fuzzy form. The pair (μ, ν) corresponds to the estimated degree of reliability and unreliability of the system (e.g., based on MTBF and MTTR), while π quantifies the unreliability that remains after accounting for both.

$$C_{\text{reliability}} = (0.85, 0.1), \quad \pi = 0.05.$$

Index matrix of evaluations and unification

Let $U = \{U_1, U_2, \dots, U_m\}$ be the set of possible causes. Each element U_i is evaluated against a certain criterion C_j $(j = 1, \dots, n)$ using following matrix:

$$\begin{array}{c|cccc}
 & C_1 & \cdots & C_n \\
\hline
U_1 & (\mu_{11}, \nu_{11}) & \cdots & (\mu_{1n}, \nu_{1n}) \\
\vdots & \vdots & \ddots & \vdots \\
U_m & (\mu_{m1}, \nu_{m1}) & \cdots & (\mu_{mn}, \nu_{mn})
\end{array} \tag{4}$$

Interpretation

The matrix thus constructed allows us to discover which of the causes have the greatest influence on the change in the parameters of the system, as well as which are the most uncertain [18].

The process of state change can be described as a gradual transition through intuitionistic fuzzy states of the system. For example, warning conditions can be detected when the degree of fluctuation π increases significantly (e.g., $\pi > 0.2$), and critical failure may be associated with $\mu < 0.5$ and $\nu > 0.4$. Figure 2 below illustrates this evolution.

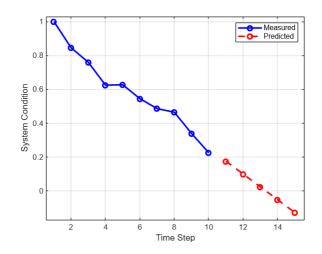


Figure 2. Measured and predicted system condition

4 Prediction and assessment of changes in the state of the studied system, based on simulation

Here we present a MATLAB-based simulation framework for assessing and predicting the technical condition of a complex system. A methodology combining degradation modeling, polynomial extrapolation, and fuzzy rule-based estimation is used, serving as a tool to support decision-making under uncertainty, [16].

We simulated the state of three components at 10 time steps. Each component was assigned a normalized operating value, where 1.0 indicates optimal state and values approaching 0.0 correspond to deterioration and failure. The changes were modeled progressively and second-order polynomial regression was used to predict five future time steps. This approach allows for early prediction of critical points before failure [7, 8, 10–13].

To incorporate uncertainty into the model, we interpreted the predicted values as intuitionistic fuzzy estimates: μ , ν , π , as above.

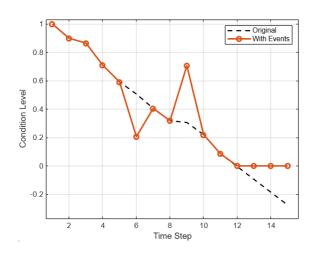


Figure 3. System dynamics with simulated fault and repair events

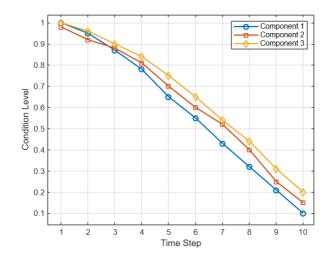


Figure 4. Evolution of system component condition

In Figure 3, each curve represents the polynomial forecast of the state of a system component. A critical condition is defined when the value drops below 0.3. The projected time steps in Figure 4, in which each component reaches this threshold, are identified in Table 5, which supports timely maintenance decisions.

Because there is no available toolkit including intuitionistic fuzzy evaluation, we implemented a manually defined rule-based evaluation, mimicking fuzzy logic classification.

Additionally, simulated failure and recovery events were modeled as additive deviations with normalization. An automated alerting mechanism was implemented based on threshold exceedance.

4.1 Results and analysis

Evaluation of the dynamics and condition of components

Condition values for each component are presented in Tables 3 and 4, respectively. The predicted values shown in Figure 5 are mapped to condition scores (0-100) using a nonlinear transformation based on certain rules (Table 2).

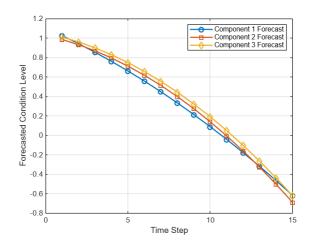


Figure 5. Forecast of system component condition

Critical condition forecasting

Table 5 identifies the predicted critical time step for each component based on a threshold of 0.3. Component 1 shows the fastest degradation, reaching critical status by Step 9, followed closely by Components 2 and 3.

Interpretation of forecast data

To represent state changes under uncertainty, we modeled the system state as a temporal intuitionistic fuzzy set (see [3]) with time-scale $T = \{t_1, t_2, \dots t_n\}$:

$$S(T) = \{ \langle \langle s, t \rangle, \mu(s, t), \nu(s, t) \rangle \mid \langle s, t \rangle \in E \times T \}, \tag{5}$$

where:

- $\mu(s,t)$ proportional to the predicted value, at s in the moment t,
- $\pi(s,t)$ representing the degree of uncertainty, which in some simplified cases is set to a constant $\epsilon=0.05$ for demonstration purposes.
- $\nu(s,t)$ calculated from the predicted degree of membership with a fixed margin ϵ to account for modeling tolerance,

$$\nu(s,t) = 1 - \mu(s,t) - \pi(s,t).$$

In the current model, the tolerance parameter $\epsilon = 0.05$ is interpreted as the degree of uncertainty π . Consequently, the sum of the degrees of membership and non-membership is fixed at:

$$\mu(s,t) + \nu(s,t) = 0.95,$$

which reflects the fact that 5% of the assessment is reserved for indeterminacy due to system limitations and modeling assumptions. This allows us to dynamically represent and track confidence in the state of the system and raises areas of uncertainty, typically around $0.3 < \mu < 0.6$.

Maintenance

Taking into account the forecast data and threshold analysis, corrective actions are proposed in Table 6. These recommendations reflect the logic of condition-based decisions, emphasizing early intervention.

Discussion

The model thus created offers us the following conclusions:

- All components of the system show gradual change; early indicators appear around time Step 6.
- The rule-based scoring transforms numerical trends into intuitive scales suitable for interpretation.
- The nonlinear scoring is more sensitive in early-warning zones ($\mu \in [0.4, 0.7]$).

- Simulated fault and recovery events demonstrate the system's dynamic response capability.
- Alerting logic based on $\mu(t) < 0.3$ and $\pi(t) > 0.2$ successfully triggers critical condition warnings.

Using intuitionistic fuzzy estimates in forecasting and condition assessment improves the monitoring and adaptability of predictive maintenance systems. By modeling both certainty and uncertainty, we better align system diagnostics in real complex systems.

4.2 Risk indexing and reporting threshold based on intuitionistic fuzzy sets

To improve state monitoring under uncertainty, we introduce a risk assessment model based on IFS IFs. Each system component at time (t) is represented by the pair:

$$IFs(t) = \langle \mu(t), \nu(t) \rangle.$$

Introduce a Risk Index

We propose to measure the system risk through a **Risk Index** R(t), as a weighted combination of certainty in failure and uncertainty:

$$R(t) = \alpha \cdot \nu(t) + \beta \cdot \pi(t), \tag{6}$$

where:

- $\alpha \in [0, 1]$ is the weight for non-membership (failure),
- $\beta \in [0, 1]$ is the weight for uncertainty,
- Recommended values: $\alpha = 0.7$, $\beta = 0.3$.

Alarm classification function

To aid in decision making, we define an alarm classification function based on R(t):

$$Alarm(t) = \begin{cases} Critical, & \text{if } R(t) \ge 0.5\\ Warning, & \text{if } 0.3 \le R(t) < 0.5\\ Normal, & \text{if } R(t) < 0.3 \end{cases} \tag{7}$$

This function enables real-time risk assessment and supports timely maintenance actions.

4.2.1 Example: Risk table

Table 1 provides a sample values for different time steps, the calculated risk index R(t), and the corresponding alarm level.

As seen in Table 1, the risk index R(t) increases as the system degrades. The alarm classification shifts accordingly from Normal to Warning and finally to Critical. This supports risk management and fault prevention.

Table 1. Example of risk index calculation

Time step	$\mu(t)$	u(t)	$\pi(t)$	R(t)	Alarm(t)
1	0.90	0.05	0.05	0.070	Normal
2	0.80	0.10	0.10	0.130	Normal
3	0.65	0.25	0.10	0.205	Normal
4	0.55	0.30	0.15	0.285	Normal
5	0.50	0.35	0.15	0.325	Warning
6	0.45	0.40	0.15	0.355	Warning
7	0.35	0.50	0.15	0.425	Warning
8	0.20	0.65	0.15	0.535	Critical
9	0.10	0.75	0.15	0.585	Critical
10	0.05	0.85	0.10	0.625	Critical

Analysis of system condition dynamics and rule-based evaluation

The presented analysis evaluates both the observed and forecasted condition of the system and its components over discrete time steps. The predicted values in Table 2 represent the normalized overall status of the system, derived from the aggregation of condition indicators.

Table 2. Evaluation of system state based on predicted values

Time	Predicted	Status (0-100)
1	1.00	100.00
2	0.95	95.00
3	0.87	87.00
4	0.75	75.00
5	0.60	70.00
6	0.50	60.00
7	0.40	50.00
8	0.30	37.50
9	0.20	25.00
10	0.10	12.50

Tables 3 and 4 detail the evolution of individual component states. Forecasting is performed using polynomial regression to extrapolate degradation trends. Negative forecasted values, as noted, indicate a complete loss of operational capability beyond acceptable limits.

A rule-based evaluation approach is applied to identify critical states:

- If the predicted condition level of a component drops below 0.3, it is classified as *Critical*.
- If the decline rate Δ is less than -0.1 per time step, the system issues a Warning signal to initiate preventive measures.

• High uncertainty levels (e.g., $\pi > 0.2$) trigger additional diagnostic verification before intervention.

Table 3. Observed condition levels of system components over time

Time Step	Component 1	Component 2	Component 3
1	1.00	0.98	1.00
2	0.95	0.92	0.96
3	0.87	0.88	0.90
4	0.78	0.81	0.84
5	0.65	0.70	0.75
6	0.55	0.60	0.65
7	0.43	0.52	0.54
8	0.32	0.40	0.44
9	0.21	0.25	0.31
10	0.10	0.15	0.20

Table 4. Forecasted condition levels of system components

Time Step	Component 1	Component 2	Component 3
1	1.000	0.980	1.000
2	0.947	0.929	0.956
3	0.872	0.868	0.899
4	0.774	0.799	0.828
5	0.655	0.720	0.745
6	0.514	0.633	0.648
7	0.351	0.537	0.538
8	0.166	0.432	0.416
9	-0.041	0.318	0.280
10	-0.270	0.196	0.131
11	-0.521	0.065	-0.031
12	-0.794	-0.075	-0.206
13	-1.089	-0.225	-0.393
14	-1.405	-0.385	-0.593
15	-1.743	-0.554	-0.806

Note: Some values in Table 4 are negative due to polynomial regression, they indicate complete loss of work capacity outside the acceptable limits.

Table 5 summarizes the estimated time steps at which each component reaches the critical threshold. Based on these forecasts, Table 6 outlines recommended corrective measures, such as preventive maintenance, increased monitoring, recalibration, and component replacement scheduling.

This combined methodology allows for dynamic tracking of the degradation process, timely detection of critical conditions, and formulation of targeted maintenance strategies that reduce the risk of unplanned downtime.

Table 5. Predicted critical condition threshold (Level < 0.3)

Component	Critical time step
Component 1	Step 9
Component 2	Step 10
Component 3	Step 9

Table 6. Suggested corrective measures based on forecast analysis

Component	Suggested Action
Component 1	Initiate preventive maintenance at Step 7. Prepare for partial replacement by
	Step 9 due to projected critical degradation.
Component 2	Increase monitoring frequency around Step 8. Schedule diagnostic testing and
	replacement planning before Step 10.
Component 3	Recommend system recalibration at Step 7. Full inspection and redundancy
	activation advised before Step 9.

5 Conclusion

Here we present a structured methodology for assessing and predicting the technical condition of complex systems, through the use of intuitionistic fuzzy sets, predictive simulation, and fuzzy logic.

The inclusion of intuitionistic fuzzy logic allows the system to detect deviations and predict the occurrence of critical events more accurately. Through the use of IFS triples (μ, ν, π) , the condition of each component is expressed not only in terms of current health but also in terms of risk and uncertainty. The proposed risk index R(t) and alarm function enhance the decision-making process by quantifying thresholds for action.

The integration of simulation and rule-based assessment provides us with a comprehensive framework for real-time monitoring, diagnostics, and maintenance planning. In addition, the system enables early intervention and dynamic resource allocation, which supports safety and efficiency.

Thus, the proposed approach can be extended with machine learning models, with the aim of further improving the adaptability and resilience of future control systems.

Acknowledgements

The author expresses gratitude to all individuals and institutions that contributed to the advancement of this research. Particular appreciation is extended to Burgas State University "Prof. Dr. Asen Zlatarov", Burgas, Bulgaria, for its valuable support and collaboration. This work was supported by Contract No. NNP MUP-P7, within the framework of the research project, funded under the program for scientific research and artistic-creative activities.

References

- [1] Atanassov, K. T. (1983). Intuitionistic fuzzy sets. *VII ITKR Session*, Sofia, 20–23 June 1983 (Deposed in Centr. Sci.-Techn. Library of the Bulg. Acad. of Sci., 1697/84) (in Bulgarian). Reprinted: *International Journal Bioautomation*, 2016, 20(S1), S1–S6.
- [2] Atanassov, K. T. (1997). *Generalized Nets and Systems Theory*. "Prof. M. Drinov" Academic Publishing House, Sofia.
- [3] Atanassov, K. T. (2012). On Intuitionistic Fuzzy Sets Theory. Springer, Berlin, pp. 1–16.
- [4] Atanassov, K. T. (2014). *Index Matrices: Towards an Augmented Matrix Calculus*. Studies in Computational Intelligence, Vol. 573. Springer, Cham.
- [5] Atanassov, K., Szmidt, E., & Kacprzyk, K. (2013). On intuitionistic fuzzy pairs. *Notes on Intuitionistic Fuzzy Sets*, 19(3), 1–13.
- [6] Atanassov, K., & Vassilev, P. (2020). Intuitionistic fuzzy sets and other fuzzy sets extensions representable by them. *Journal of Intelligent & Fuzzy Systems*, 38(1), 525–530.
- [7] Babuška, R. (1998). Fuzzy Modeling for Control. Kluwer Academic Publishers, Boston.
- [8] Dubois, D., & Prade, H. (1980). Fuzzy Sets and Systems: Theory and Applications. Academic Press, New York.
- [9] Ismaili, S., & Fidanova, S. (2019). Application of intuitionistic fuzzy sets for conflict resolution modeling and agent based simulation. *International Journal Bioautomation*, 23(2), 175–184.
- [10] Klir, G. J., & Yuan, B. (1995). Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall, Upper Saddle River.
- [11] Mendel, J. M. (2001). *Uncertain Rule-based Fuzzy Logic Systems: Introduction and New Directions*. Prentice Hall, Upper Saddle River.
- [12] Novák, V., Perfilieva, I., Močkoř, J. (1999). *Mathematical Principles of Fuzzy Logic*. Kluwer Academic Publishers, Dordrecht.

- [13] Pedrycz, W., & Gomide, F. (2021). Fuzzy Systems Engineering: Toward Human-centric Computing. Wiley-IEEE Press, Hoboken.
- [14] Ross, T. J. (2010). Fuzzy Logic with Engineering Applications. (3rd edn). Wiley, Chichester.
- [15] Sotirov, S., Sotirova, E., Werner, M., Simeonov, S., Hardt, W., & Simeonova, N. (2016). Ituitionistic fuzzy estimation of the generalized nets model of spatial-temporal group scheduling problems. *Imprecision and Uncertainty in Information Representation and Processing*. Studies in Fuzziness and Soft Computing, vol 332. Springer, Cham. pp. 401–414.
- [16] Yager, R. R., & Zadeh, L. A. (Eds.). (1992). An Introduction to Fuzzy Logic Applications in Intelligent Systems. Springer, Boston.
- [17] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.
- [18] Zimmermann, H. J. (2001). Fuzzy Set Theory—And Its Applications. (4th edition). Springer, Boston.