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DETERMINING THE SPECIFICITY, SENSITIVITY, POSITIVE AND NEGATIVE PREDICTIVE VALUES IN INTUITIONISTIC FUZZY LOGIC

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Introduction

During the last 20-30 years the theory of pattern recognition underwent rapid development and found a really wide application in various areas of life, including in medicine [9, 12, 13]. This theory encompasses a broad spectrum of approaches and methods – deterministic, statistical, fuzzy, and structural. There exist different taxonomies of the methods for pattern recognition [8, 17]. The classification of patterns with distance functions is one of the earliest concepts in the pattern recognition. The rule of the κ nearest neighbors (K-NN) is one of the most used and precise methods, based on distance functions. Voronoi diagrams may be viewed as a particular case of the κ-NN method and because of that they find wide application in the development of algorithms for determining the nearest and furthest neighbors [2]. On the other hand, one of the peculiarities of the data acquired in medical examination is that one and the same disease may develop with different severity in different patients. This may be represented with different degrees of membership to a given class corresponding to the disease in question. Because of that increasingly the apparatus of fuzzy sets, and in particular – of the intuitionistic fuzzy sets, is being used in the processing of medical information [5, 6, 7, 10, 14]. In [15, 16] were proposed intuitionistic fuzzy Voronoi diagrams, which combine the advantages of the Voronoi diagrams and intuitionistic fuzzy sets.

In the present paper a classification of a set of patterns with the aid of intuitionistic fuzzy Voronoi diagrams, in view of the possibility for more adequate description of the considered objects, allowing for recognition of patterns with non-strict membership is considered.

Intuitionistic fuzzy sets (IFS) were proposed in 1983 г. by K. Atanassov [1] as an extension of the theory of the fuzzy sets, developed by L. Zadeh [19].

Let there be a fixed set E. The set A* is called intuitionistic fuzzy (IFS), if there is [1]:

$$A^*=\{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in E\},$$

where the functions μ_A: $E\rightarrow[0,1]$ and ν_A: $E\rightarrow[0,1]$ determine respectively the degree of membership and non-membership of the element $x\in E$ to the set A, which is a subset of E and for every $x\in E$:

$$0\leq \mu_A(x) + \nu_A(x) \leq 1.$$

For the purpose of the present work it is assumed that $\mu_A(x)$ and $\nu_A(x)$ are acquired based on expert evaluations.

The function π_A, determined by the formula:

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x)$$

gives the degree of indeterminacy of the membership of the element $x \in E$ to the set A. Obviously, in the case of fuzzy set $\pi_A(x)=0$ for every $x \in E$.

Intuitionistic Fuzzy Voronoi Diagrams:

Let the set P consisting of n points $p_i \in P$ be given in the plane.

Let us denote by $p_i q'$ the orthogonal projection of the segment qp_i (q is an arbitrary point in the plane) on the straight line defined by the points p_i and p_j, i.e.

$$\prod_{p_i p_j} qp_i = p_i q'$$

Let us denote with ST the segment of the symmetrical axis of $p_i p_j$ (see Fig.1).

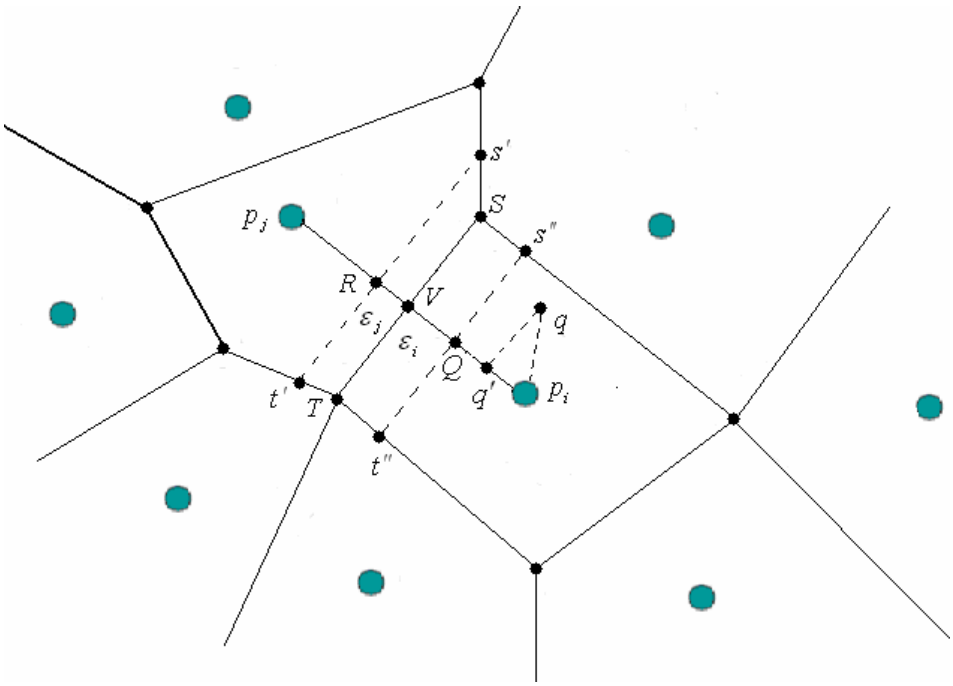

Fig. 1. Intuitionistic fuzzy modification of Voronoi diagrams

Using one of the algorithms in [15, 16] we determine the ε – neighbourhood of ST, i.e. we determine the position of the points Q and R. This neighbourhood which defines the width of the strip around ST is bounded by two parallel lines ($s't'$ and $s''t''$).

Then by definition: Intuitionistic fuzzy Voronoi diagrams over an arbitrary point-set P in the plane is defined as the part of the plane, located in n cells. Each cell is unique for the point $p_i \in P$ and is such that point q lies in the cell, corresponding to the point p_i, if the folowing conditions are simultaneously fulfilled:

$$dist(q, p_i) < dist(q, p_j)$$

and

when $q' \in p_i p_j$ and

$$\prod_{p_i p_j} qp_i < p_i Q \tag{1}$$

for every p_j from P and $j \neq i$.

If the condition (1) is not fulfilled, i.e.

when $q' \in p_i p_j$ and $p_i R > \prod_{p_i p_j} q p_i > p_i Q$, then point q will lie inside the strip between the cells, corresponding to the point p_i and point p_j, i.e. point q will lie in the area of indeterminacy.

The concept of intuitionistic fuzzy modification of Voronoi diagrams, comes to transforming part of the plane to convex regions (fig. 1) that are separated by strips. These strips define the areas of indeterminacy. The presence of such areas is especially suitable, even necessary, when dealing with classification problems in the field of medicine, since there are numerous cases in the medical practice when the patient cannot be diagnosed, or a particular treatment cannot be prescribed without additional examinations, consultations with specialists or waiting. In the terms of decision-making theory this corresponds to a situation under which no decision is made, i.e. the patient has not been classified. When using classification with IFVD, this means the pattern (vector, point) has fallen in the areas (strips) of indeterminacy.

In this case the classification is done as follows:

1. Point z with unknown classification is associated to class ω_i, to which the point x_l with known classification belongs and in whose corresponding cell the point z falls.
2. If z falls in an area of indeterminacy, it is not classified

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The sensitivity, specificity, positive predictive value, and negative predictive value are calculated according to the formulas:

$$SEN = \frac{TP}{TP + FN},$$

$$SPE = \frac{TN}{TN + FP},$$

$$PPV = \frac{TP}{TP + FP},$$

$$NPV = \frac{TN}{TN + FN},$$

where TP is true positives; FP, false positives; TN, true negatives; FN, false negatives.

In the current paper we introduce four parameters - sensitivity, specificity, positive predictive value and negative predictive value in the sense of the intuitionistic fuzzy logic. These parameters are defined further below.

Definition. Sensitivity in the sense of intuitionistic fuzzy logic (SEN_{IFS}) will be called the ratio:

$$SEN_{IFS} = \frac{TP}{TP + IP + FN}, \quad (2)$$

Specificity in the sense of intuitionistic fuzzy logic (SPE_{IFS}) will be called the ratio:

$$SPE_{IFS} = \frac{TN}{TN + IN + FP}, \quad (3)$$

Positive predictive value in the sense of intuitionistic fuzzy logic (PPV_{IFS}) will be called the ratio:

$$PPV_{IFS} = \frac{TP}{TP + FP + IP + IN}. \quad (4)$$

Negative predictive value in the sense of the intuitionistic fuzzy logic (NPV_{IFS}) will be called the ratio:

$$NPV_{IFS} = \frac{TN}{TN + FN + IN + IP}. \quad (5)$$

where:

TN (*true negatives*) – the number of healthy patients determined as such;
TP (*true positives*) – the number of sick patients determined as such;
FP (*false positives*) – the number of healthy patients determined as sick;
FN (*false negatives*) - the number of sick patients determined as healthy;
IN (*indeterminate negatives*) - the number of healthy patients left unclassified;
IP (*indeterminate positives*) - the number of sick patients left unclassified

Then the generalized values for the degree of membership μ_i, the degree of non-membership ν_i and the degree of indeterminacy π_i, $i = 1,2$ for each of the considered classes is calculated according to the formulas:

$$\mu_i = \frac{TP}{TP + IP + FN}; \quad (6)$$

$$\nu_i = \frac{FN}{TP + IP + FN}; \quad (7)$$

$$\pi_i = \frac{IP}{TP + IP + FN}; \quad (8)$$

$$\mu_{3-i} = \frac{TN}{TN + IN + FP}; \quad (9)$$

$$\nu_{3-i} = \frac{FP}{TN + IN + FP}; \quad (10)$$

$$\pi_{3-i} = \frac{IN}{TN + IN + FP}. \quad (11)$$

Obviously, for *i*=1,2:

$$\mu_i + \nu_i + \pi_i = 1$$

and

$$\mu_{3-i} + \nu_{3-i} + \pi_{3-i} = 1.$$

Since

$$0 \le \pi_i \le 1 \text{ и } 0 \le \pi_{3-i} \le 1,$$

then

$$0 \le \mu_i + \nu_i \le 1$$

and

$$0 \le \mu_{3-i} + \nu_{3-i} \le 1,$$

i.e. (μ_i, ν_i) and (μ_{3-i}, ν_{3-i}) are intuitionistic fuzzy couples for *i*=1,2.

EXPERIMENTAL SETUP AND RESULTS

A retrospective study, that covers 151 patients, admitted and discharged in the Centre of Acute Respiratory Insufficiancy, University Hospital “Alexandrovska” - Sofia, Bulgaria, for a

period of 11 years is examined. All the patients were on a long-term mechanical ventilation for more than 7 days (26.12 ± 11.09). The principle is carried for definitive withdrawal of mechanical ventilation, so that the patient to be able to breathe spontaneously without ventilatory support for more than 24 h [3, 4, 11, 18]. In the history of these patients, we could not find data for preceding chronic pulmonary disease or serious systematic illness for anybody, deteriorating the primary disease process.

Each patient is represented by two n-dimensional vectors (in our case, $n = 17$), i.e.:

$x = (x_1, x_2, ..., x_n)$,

where:

x_1 - fever; x_2 - hemoglobine; x_3 - hematocrit; x_4 - Leuc; x_5 - *RUE*; x_6 - total blood protein (tbp); x_7 - blood albumin (alb); x_8 - blood sugar (bs); x_9 - lactate; x_{10} - fraction of inspired oxygen FiO_2; x_{11} - arterial oxygen partial pressure PaO_2; x_{12} - arterial carbon dioxide tension $PaCO_2$; x_{13} - ratio PaO_2/FiO_2; x_{14} - heart rate (Ps); x_{15} - systolic arterial pressure (RRs); x_{16} - diastolic arterial pressure (RRd); x_{17} - mean arterial pressure (RRm.). One of the vectors is composed of the values of the respective features in day before the beginning of weaning. In this case we agree to assume that the patient belongs to class ω_1 — "not ready for weaning". The second vector consists of the values of the features in the day the weaning begins, i.e. the values on the basis of which the treating doctor has evaluated the patient as "ready for weaning". In this case we agree to assume that the patient belongs to class ω_2 — "ready for weaning".

An intuitionistic fuzzy Voronoi diagram is constructed over a set of patterns with known classification (Fig.2). Each pattern corresponds to a single patient. The degrees of membership, non-membership and indeterminacy to the two classes of the intuitionistic fuzzy Voronoi diagram over the considered set have been calculated for each image

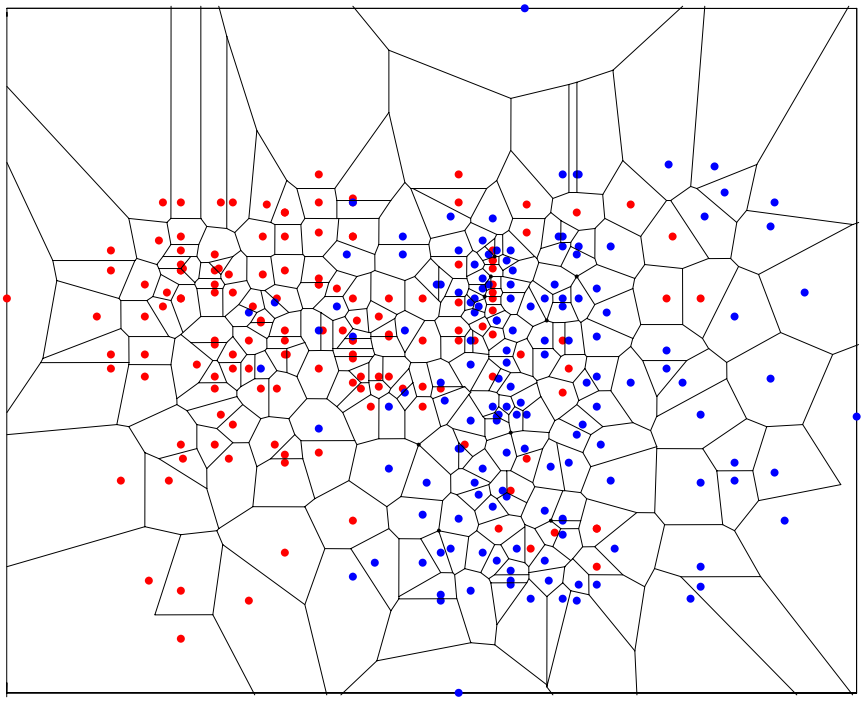

Fig.2

In Table 1 are shown the values of *TN, TP, FP, FN, IP* and *IN*, as well as the calculated through them values of the newly introduced parameters SPE_{IFS}, SEN_{IFS}, PPV_{IFS} and NPV_{IFS}. The values for SPE_{IFS}, SEN_{IFS}, PPV_{IFS} and NPV_{IFS} are calculated according to the formulas (2) to (5).

Table 1.

TN	*TP*	*FP*	*FN*	*IP*	*IN*	SPE_{IFS}	SEN_{IFS}	PPV_{IFS}	NPV_{IFS}
84	83	58	46	22	9	0.55	0.56	0.48	0.52

The generalized values for the degree of membership μ_i, degree of non-membership ν_i and the degree of indeterminacy π_i, $i = 1,2$ have been calculated for each of the considered classes according to formulas (6) to (11). The results are shown in Table 2.

Table 2.

μ_i	ν_i	π_i	μ_{3-i}	ν_{3-i}	π_{3-i}
0.5497	*0.3046*	*0.1457*	*0.5563*	*0.3841*	*0.0596*

The obtained results have been used in computer system for analyzing the patient's condition for weaning from long-term mechanical ventilation.

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