Notes on Intuitionistic Fuzzy Sets
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Interval valued intuitionistic fuzzy primary ideal

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Abstract: The concept of fuzzy semiprimary ideal is extended by introducing intuitionistic fuzzy primary ideals as well as intuitionistic fuzzy semiprimary ideals in rings. Using this concept, Interval valued intuitionistic fuzzy primary ideal and Interval valued intuitionistic fuzzy semiprimary ideals is defined. Various properties of interval valued intuitionistic fuzzy primary ideals and interval valued intuitionistic fuzzy semiprimary ideals are proved. Finally, interval valued intuitionistic fuzzy Lie primary ideals and interval valued intuitionistic fuzzy lie semi primary ideals are defined, some properties are established.
Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy primary ideal, Intuitionistic fuzzy semi-primary ideal, Interval valued intuitionistic fuzzy primary ideals, Interval valued intuitionistic fuzzy semi primary ideals, Interval valued intuitionistic fuzzy Lie primary ideals, Interval valued intuitionistic fuzzy lie semi primary ideals.
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1 Introduction

Zadeh, in his paper [10], introduced the concept of fuzzy sets and fuzzy set operations. The notion of intuitionistic fuzzy set and its operations were introduced by Atanassov [1], as a generalization of the notion of fuzzy set. Atanassov [2, 3] discussed the operators over interval valued intuitionistic fuzzy sets. Palanivelrajan and Nandakumar [7] introduced the definition and some properties of intuitionistic fuzzy primary and semiprimary ideals. Some operations on intuitionistic fuzzy primary and semiprimary ideals are also discussed by Palanivelrajan and Nandakumar [8]. Based on concept discussed by Atanassov [2, 3], the idea of interval valued being applied on intuitionistic fuzzy primary and intuitionistic semiprimary ideals. Finally, results based on interval valued intuitionistic fuzzy Lie primary ideals and intuitionistic fuzzy lie semiprimary ideal are established.

2 Preliminaries

In this section, some basic definitions that are essential for this paper are assembled.

Definition 2.1. Let S be any nonempty set. A mapping $\mu : S \rightarrow [0, 1]$ is called a fuzzy subset of S.

Definition 2.2. A fuzzy ideal μ of a ring R is called fuzzy primary ideal, if for all $a, b \in R$ either $\mu(ab) = \mu(a)$ or else $\mu(ab) \leq \mu(b^m)$ for some $m \in \mathrm{Z}_+$.

Definition 2.3. A fuzzy ideal μ of a ring R is called fuzzy semiprimary ideal, if for all $a, b \in R$ either $\mu(ab) \leq \mu(a^n)$, for some $n \in \mathrm{Z}_+$, or else $\mu(ab) \leq \mu(b^m)$ for some $m \in \mathrm{Z}_+$.

Definition 2.4. An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$, where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.5. A fuzzy ideal A of a ring R is called Intuitionistic fuzzy primary ideal if for all $a, b \in R$ either $\mu(ab) = \mu(a)$ and $\nu_A(ab) = \nu_A(a)$, or $\mu(ab) \leq \mu_A(b^m)$ and $\nu_A(ab) \geq \nu_A(b^m)$, for some $m \in \mathrm{Z}_+$.

Definition 2.6. A fuzzy ideal A of a ring R is called intuitionistic fuzzy semiprimary ideal if for all $a, b \in R$ either $\mu_A(ab) \leq \mu_A(a^n)$ and $\nu_A(ab) \geq \nu_A(a^n)$, for some $n \in \mathrm{Z}_+$ or else $\mu(ab) \leq \mu_A(b^m)$ and $\nu_A(ab) \geq \nu_A(b^m)$ for some $m \in \mathrm{Z}_+$.

Definition 2.7. An interval valued intuitionistic fuzzy set A is specified by a function $M_A : E \rightarrow \mathrm{INT}[0, 1]$, where INT[0, 1] is the set of all intervals within [0, 1], for all $x \in E$, $M_A(x)$ is an interval $[a, b]$, where $0 \leq a \leq b \leq 1$.

Remark 2.1. An interval valued intuitionistic fuzzy sets A over E is defined as an object of the form $A = \{x, M_A(x), N_A(x) \mid x \in E\}$, where $M_A(x) \subseteq [0, 1]$ and $N_A(x) \subseteq [0, 1]$ are interval and for all $x \in E$ $\sup M_A(x) + \sup N_A(x) \leq 1$.

Definition 2.8. Let A be an interval valued intuitionistic fuzzy sets. A fuzzy ideal A of a ring R is said to be interval valued intuitionistic fuzzy primary ideal of R if for all $a, b \in R$ then either $\mu_A(ab) = \inf M_A(ab) = \inf M_A(a) = \mu_A(a)$ and $\nu_A(ab) = \inf N_A(ab) = \inf N_A(a) = \nu_A(a)$ or else $\mu_A(ab) = \inf M_A(ab) \leq \inf M_A(b^n) = \mu_A(b^n)$ and $\nu_A(ab) = \inf N_A(ab) \geq \inf N_A(b^n) = \nu_A(b^n)$, for some $n \in Z_+$.

Definition 2.9. Let A be an interval valued intuitionistic fuzzy set A fuzzy ideal A of a ring R is said to be interval valued intuitionistic fuzzy semiprimary ideal of R if for all a, $b \in R$ then either $\mu_A(ab) = \inf M_A(ab) \leq \inf M_A(a^n) = \mu_A(a^n)$ and $\nu_A(ab) = \inf N_A(ab) \geq \inf N_A(a^n) = \nu_A(a^n)$, for some $n \in Z_+$ or $\mu_A(ab) = \inf M_A(ab) \leq \inf \mu_A(b^m) = \mu_A(b^m)$ and $\nu_A(ab) = \inf N_A(ab) \geq \inf N_A(b^m) = \nu_A(b^m)$, for some $m \in Z_+$.

Example 2.1.

$$\mu_A(x) = \inf M_A(x) = \begin{cases} [1, 1] & if\ x = 0 \\ [0.4, 0.6] & otherwise \end{cases}$$

$$\nu_A(x) = \inf N_A(x) = \begin{cases} [0, 0] & if\ x = 0 \\ [0.2, 0.3] & otherwise \end{cases}$$

Definition 2.10. A Lie algebra is a vector space L over the field F (equal to R or C) on which $L \times L \to L$ denoted by $(x, y) \to [x, y]$ is defined and satisfying the following axioms

i. $[x, y]$ is bilinear
ii. $[x, x] = 0$ for all $x \in L$
iii. $[[x, y], z] + [[y, z], x] + [[z, x], y] = 0$ for all $x, y, z \in L$ (Jacobi identity)

Definition 2.11. A fuzzy set $\mu : L \to [0, 1]$ is called a fuzzy Lie subalgebra of L if

i. $\mu(x + y) \geq \min(\mu(x), \mu(y))$
ii. $\mu(\alpha x) \geq \mu(x)$
iii. $\mu([xy]) \geq \min(\mu(x), \mu(y))$ holds for all $x, y \in L$ and $\alpha \in F$.

Definition 2.1.2 A fuzzy set $\mu : L \to [0, 1]$ is called a fuzzy Lie ideal of L if

i. $\mu(x + y) \geq \min(\mu(x), \mu(y))$
ii. $\mu(\alpha x) \geq \mu(x)$
iii. $\mu([xy]) \geq \mu(x)$ holds for all $x, y \in L$ and $\alpha \in F$.

Definition 2.13. An interval valued intuitionistic fuzzy set $A = (\overline{\mu}_A, \overline{\nu}_A)$ in L is called an interval valued intuitionistic fuzzy Lie ideal of L if the following conditions are satisfied

i. $\overline{\mu}_A(x + y) \geq \min(\overline{\mu}_A(x), \overline{\mu}_A(y))$ and $\overline{\nu}_A(x+y) \leq \max(\overline{\nu}_A(x), \overline{\nu}_A(y))$
ii. $\overline{\mu}_A(\alpha x) \geq \overline{\mu}_A(x)$ and $\overline{\nu}_A(\alpha x) \leq \overline{\nu}_A(x)$
iii. $\overline{\mu}_A([x, y]) \geq \overline{\mu}_A(x)$ and $\overline{\nu}_A([x, y]) \leq \overline{\nu}_A(x)$, for all $x, y \in L$ and $\alpha \in F$.

Definition 2.14. Let A be an interval valued intuitionistic fuzzy Lie ideal of a Lie algebra L then A is said to be an interval valued intuitionistic fuzzy Lie primary ideal of L if for all $x, y \in L$ then either $\overline{\mu}_A(xy) = \overline{\mu}_A(x)$ and $\overline{\nu}_A(xy) = \overline{\nu}_A(x)$ or $\overline{\mu}_A(xy) \leq \overline{\mu}_A(x^n)$ and $\overline{\nu}_A(xy) \geq \overline{\nu}_A(x^n)$, for some $n \in Z_+$.

Definition 2.15. Let A be an interval valued intuitionistic fuzzy Lie primary ideal of a Lie algebra L then A is said to interval valued intuitionistic fuzzy Lie semiprimary ideal of L if for all x, $y \in L$ and for some $n \in Z_+$ either $\overline{\mu}_A(xy) \leq \overline{\mu}_A(x^n)$ and $\overline{\nu}_A(xy) \geq \overline{\nu}_A(x^n)$ or else $\overline{\mu}_A(xy) \leq \overline{\mu}_A(y^m)$ and $\overline{\nu}_A(xy) \geq \overline{\nu}_A(y^m)$, for some $m \in Z_+$.

3 Interval valued intuitionistic fuzzy primary ideals and Interval valued intuitionistic fuzzy semiprimary ideals

Theorem 3.1. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A \cap B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \leq \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \geq \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of the ring R, then $\mu_B(\mathrm{xy}) = \inf \mu_B(xy) \leq \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \geq \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \cap B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A\cap B}(xy) &= \inf M_{A\cap B}(xy)\\ &= \min\left(\inf M_A(xy), \inf M_B(xy)\right)\\ &\leq \min\left(\inf M_A(x^n), \inf M_B(x^n)\right)\\ &= \inf M_{A\cap B}(x^n)\\ &= \mu_{A\cap B}(x^n).\end{aligned}$$

Therefore, $\mu_{A\cap B}(xy) \leq \mu_{A\cap B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A\cap B}(xy) &= \inf N_{A\cap B}(xy)\\ &= \max\left(\inf N_A(xy), \inf N_B(xy)\right)\\ &\geq \max\left(\inf N_A(x^n), \inf N_B(x^n)\right)\\ &= \inf N_{A\cap B}(x^n)\\ &= \nu_{A\cap B}(x^n).\end{aligned}$$

Therefore, $\nu_{A\cap B}(xy) \geq \nu_{A\cap B}(x^n)$.

Therefore, $A \cap B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. □

Theorem 3.2. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A \cup B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \leq \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \geq \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of the ring R, then $\mu_B(xy) = \inf M_B(xy) \leq \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \geq \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \cup B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A\cup B}(xy) &= \inf M_{A\cup B}(xy)\\ &= \max(\inf M_A(xy), \inf M_B(xy))\\ &\le \max\left(\inf M_A(x^n), \inf M_B(x^n)\right)\\ &= \inf M_{A\cup B}(x^n)\\ &= \mu_{A\cup B}(x^n).\end{aligned}$$

Therefore, $\mu_{A\cup B}(xy) \le \mu_{A\cup B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A\cup B}(xy) &= \inf N_{A\cup B}(xy)\\ &= \min(\inf N_A(xy), \inf N_B(xy))\\ &\ge \min\left(\inf N_A(x^n), \inf N_B(x^n)\right)\\ &= \inf N_{A\cup B}(x^n)\\ &= \nu_{A\cup B}(x^n).\end{aligned}$$

Therefore, $\nu_{A\cup B}(xy) \ge \nu_{A\cup B}(x^n)$.

Therefore, $A \cup B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. ☐

Theorem 3.3. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A + B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \le \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \ge \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_B(xy) = \inf M_B(xy) \le \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \ge \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A + B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A+B}(xy) &= \inf M_{A.B}(xy)\\ &= \inf M_A(xy) + \inf M_B(xy) - \inf M_A(xy).\inf M_B(xy)\\ &\le \inf M_A(x^n) + \inf M_B(x^n) - \inf M_A(x^n).\inf M_B(x^n)\\ &= \mu_A(x^n) + \mu_B(x^n) - \mu_A(x^n).\mu_B(x^n)\\ &= \mu_{A+B}(x^n).\end{aligned}$$

Therefore, $\mu_{A+B}(xy) \le \mu_{A+B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A+B}(xy) &= \inf N_{A.B}(xy)\\ &= \inf N_A(xy).\inf N_B(xy)\\ &\ge \inf N_A(x^n).\inf N_B(x^n)\\ &= \nu_A(x^n).\nu_B(x^n)\\ &= \nu_{A+B}(x^n).\end{aligned}$$

Therefore, $\nu_{A+B}(xy) \geq \nu_{A+B}(x^n)$.

Therefore, $A + B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. □

Theorem 3.4. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R then $A.B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R then $\mu_A(xy) = \inf M_A(xy) \leq \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \geq \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R then $\mu_B(xy) = \inf M_B(xy) \leq \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \geq \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A.B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A.B}(xy) &= \inf M_{A.B}(xy)\\ &= \inf M_A(xy).\inf M_B(xy)\\ &\leq \inf M_A(x^n).\inf M_B(x^n)\\ &= \mu_A(x^n).\mu_B(x^n)\\ &= \mu_{A.B}(x^n).\end{aligned}$$

Therefore, $\mu_{A.B}(xy) \leq \mu_{A.B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A.B}(xy) &= \inf N_{A.B}(xy)\\ &= \inf N_A(xy) + \inf N_B(xy) - \inf N_A(xy).\inf N_B(xy)\\ &\geq \inf N_A(x^n) + \inf N_B(x^n) - \inf N_A(x^n).\inf N_B(x^n)\\ &= \nu_A(x^n) + \nu_B(x^n) - \nu_A(x^n).\nu_B(x^n)\\ &= \nu_{A.B}(x^n).\end{aligned}$$

Therefore, $\nu_{A.B}(xy) \geq \nu_{A.B}(x^n)$.

Therefore, $A.B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. □

Theorem 3.5. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A@B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \leq \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \geq \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_B(xy) = \inf M_B(xy) \leq \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \geq \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A@B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A@B}(xy) &= \inf M_{A@B}(xy)\\ &= \frac{\inf M_A(xy) + \inf M_B(xy)}{2}\\ &\le \frac{\inf M_A(x^n) + \inf M_B(x^n)}{2}\\ &= \frac{\mu_A(x^n) + \mu_B(x^n)}{2}\\ &= \mu_{A@B}(x^n).\end{aligned}$$

Therefore, $\mu_{A@B}(xy) \le \mu_{A@B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A@B}(xy) &= \inf N_{A@B}(xy)\\ &= \frac{\inf N_A(xy) + \inf N_B(xy)}{2}\\ &\ge \frac{\inf N_A(x^n) + \inf N_B(x^n)}{2}\\ &= \frac{\nu_A(x^n) + \nu_B(x^n)}{2}\\ &= \nu_{A@B}(x^n).\end{aligned}$$

Therefore, $\nu_{A@B}(xy) \ge \nu_{A@B}(x^n)$.

Therefore, $A@B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. □

Theorem 3.6. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A \$ B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \le \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \ge \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of the ring R, then $\mu_B(xy) = \inf M_B(xy) \le \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \ge \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \$ B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A\$B}(xy) &= \inf M_{A\$B}(xy)\\ &= \sqrt{\inf M_A(xy) \inf M_B(xy)}\\ &\le \sqrt{\inf M_A(x^n) \inf M_B(x^n)}\\ &= \sqrt{\mu_A(x^n)\mu_B(x^n)}\\ &= \mu_{A\$B}(x^n).\end{aligned}$$

Therefore, $\mu_{A\$B}(xy) \le \mu_{A\$B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A\$B}(xy) &= \inf N_{A\$B}(xy)\\ &= \sqrt{\inf N_A(xy)\inf N_B(xy)}\\ &\geq \sqrt{\inf N_A(x^n)\inf N_B(x^n)}\\ &= \sqrt{\nu_A(x^n)\nu_B(x^n)}\\ &= \nu_{A\$B}(x^n).\end{aligned}$$

Therefore, $\nu_{A\$B}(xy) \geq \nu_{A\$B}(x^n)$.

Therefore, A \$ B is an interval valued intuitionistic fuzzy semiprimary ideal of R. □

Theorem 3.7. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A\#B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \leq \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \geq \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_B(xy) = \inf M_B(xy) \leq \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \geq \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \# B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A\#B}(xy) &= \inf M_{A\#B}(xy)\\ &= \frac{2\inf M_A(xy)\inf M_B(xy)}{\inf M_A(xy)+\inf M_B(xy)}\\ &\leq \frac{2\inf M_A(x^n)\inf M_B(x^n)}{\inf M_A(x^n)+\inf M_B(x^n)}\\ &= \frac{2\mu_A(x^n)\mu_B(x^n)}{\mu_A(x^n)+\mu_B(x^n)}\\ &= \mu_{A\#B}(x^n).\end{aligned}$$

Therefore, $\mu_{A\#B}(xy) \leq \mu_{A\#B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A\#B}(xy) &= \inf N_{A\#B}(xy)\\ &= \frac{2\inf N_A(xy)\inf N_B(xy)}{\inf N_A(xy)+\inf N_B(xy)}\\ &\geq \frac{2\inf N_A(x^n)\inf N_B(x^n)}{\inf N_A(x^n)+\inf N_B(x^n)}\\ &= \frac{2\nu_A(x^n)\nu_B(x^n)}{\nu_A(x^n)+\nu_B(x^n)}\\ &= \nu_{A\#B}(x^n).\end{aligned}$$

Therefore, $\nu_{A\#B}(xy) \geq \nu_{A\#B}(x^n)$.

Therefore, $A \# B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. ☐

Theorem 3.8. If A and B are interval valued intuitionistic fuzzy semiprimary ideal of R, then $A*B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R.

Proof. Let A be an interval valued intuitionistic fuzzy semiprimary ideal of a ring R, then $\mu_A(xy) = \inf M_A(xy) \leq \inf M_A(x^n) = \mu_A(x^n)$ and $\nu_A(xy) = \inf N_A(xy) \geq \inf N_A(x^n) = \nu_A(x^n)$ for some $n \in Z_+$ and $x, y \in R$.

Let B be an interval valued intuitionistic fuzzy semiprimary ideal of the ring R, then $\mu_B(xy) = \inf M_B(xy) \leq \inf M_B(x^n) = \mu_B(x^n)$ and $\nu_B(xy) = \inf N_B(xy) \geq \inf N_B(x^n) = \nu_B(x^n)$, for some $n \in Z_+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A * B$ implies $x, y \in A$ and $x, y \in B$.

Consider

$$\begin{aligned}\mu_{A*B}(xy) &= \inf M_{A*B}(xy)\\ &= \frac{\inf M_A(xy) + \inf M_B(xy)}{2(\inf M_A(xy)\inf M_B(xy)+1)}\\ &\leq \frac{\inf M_A(x^n) + \inf M_B(x^n)}{2(\inf M_A(x^n)\inf M_B(x^n)+1)}\\ &= \frac{\mu_A(x^n)+\mu_B(x^n)}{2(\mu_A(x^n)\mu_B(x^n)+1)}\\ &= \mu_{A*B}(x^n).\end{aligned}$$

Therefore, $\mu_{A*B}(xy) \leq \mu_{A*B}(x^n)$.

Consider

$$\begin{aligned}\nu_{A*B}(xy) &= \inf N_{A*B}(xy)\\ &= \frac{\inf N_A(xy) + \inf N_B(xy)}{2(\inf N_A(xy)\inf N_B(xy)+1)}\\ &\geq \frac{\inf N_A(x^n) + \inf N_B(x^n)}{2(\inf N_A(x^n)\inf N_B(x^n)+1)}\\ &= \frac{\nu_A(x^n)+\nu_B(x^n)}{2(\nu_A(x^n)\nu_B(x^n)+1)}\\ &= \nu_{A*B}(x^n).\end{aligned}$$

Therefore, $\nu_{A*B}(xy) \geq \nu_{A*B}(x^n)$.

Therefore, $A * B$ is an interval valued intuitionistic fuzzy semiprimary ideal of R. ☐

Theorem 3.9. If A and B are interval valued intuitionistic fuzzy primary ideal of R then $A \cap B$ is an interval valued intuitionistic fuzzy primary ideal of R.

Theorem 3.10. If A and B are interval valued intuitionistic fuzzy primary ideal of R then $A \cup B$ is an interval valued intuitionistic fuzzy primary ideal of R.

Theorem 3.11. If *A* and *B* are interval valued intuitionistic fuzzy primary ideal of *R* then *A* + *B* is an interval valued intuitionistic fuzzy primary ideal of *R*.

Theorem 3.12. If *A* and *B* are interval valued intuitionistic fuzzy primary ideal of *R* then *A* . *B* is an interval valued intuitionistic fuzzy primary ideal of *R*.R

Theorem 3.13. If *A* and *B* are interval valued intuitionistic fuzzy primary ideal of *R* then *A* @ *B* is an interval valued intuitionistic fuzzy primary ideal of *R*.

Theorem 3.14. If *A* and *B* are interval valued intuitionistic fuzzy primary ideal of *R* then *A* $ *B* is an interval valued intuitionistic fuzzy primary ideal of *R*.

Theorem 3.15. If *A* and *B* are interval valued intuitionistic fuzzy primary ideal of *R* then *A* # *B* is an interval valued intuitionistic fuzzy primary ideal of *R*.

Theorem 3.16. If *A* and *B* are interval valued intuitionistic fuzzy primary ideal of *R* then *A* * *B* is an interval valued intuitionistic fuzzy primary ideal of *R*.

4 Interval valued intuitionistic fuzzy Lie primary ideal

Theorem 4.1. If $A=(\overline{\mu}_A,\overline{\nu}_A)$ is an interval valued intuitionistic fuzzy Lie primary ideal of a Lie algebra *L*, then the level subset $U(\overline{\mu}_A,\overline{\alpha})=\{\alpha\in L\mid \mu_A(x)\geq\overline{\alpha}\}$ and $L(\overline{\nu}_A,\overline{\alpha})=$ $=\{x\in L\mid \overline{\nu}_A(x)\leq\overline{\alpha}\}$ are Lie primary ideals of *L* for every $\overline{\alpha}\in I_m(\overline{\mu}_A)\cap I_m(\overline{\nu}_A)\subseteq$ D[0, 1], where $I_m(\overline{\mu}_A)$ and $I_m(\overline{\nu}_A)$ are sets of values of $\overline{\mu}_A$ and $\overline{\nu}_A$, respectively.

Proof. Let $\overline{\alpha}\in I_m(\overline{\mu}_A)\cap I_m(\overline{\nu}_A)\subseteq$ D[0, 1] and let $x,y\in U(\overline{\mu}_A,\overline{\alpha})$ and $\alpha\in F$, then $\overline{\mu}_A(x)\geq I$, where I = [0, 1] and $\overline{\mu}_A(x)\geq\overline{\alpha}$, it follows that $\overline{\mu}_A(xy)=\overline{\mu}_A(x)\geq\overline{\alpha}$, so that $xy\in U(\overline{\mu}_A,\overline{\alpha})$, consequently $U(\overline{\mu}_A,\overline{\alpha})$ is an interval valued intuitionistic fuzzy Lie primary ideal of *L*. Let $x,y\in L(\overline{\nu}_A,\overline{\alpha})$ and $\alpha\in F$, then $\overline{\nu}_A(x)\leq I$, where I = [0, 1] and $\overline{\nu}_A(x)\leq\overline{\alpha}$ it follows that $\overline{\nu}_A(xy)=\overline{\nu}_A(x)\leq\alpha$, so that $xy\in L(\overline{\nu}_A,\overline{\alpha})$. Consequently $L(\overline{\nu}_A,\overline{\alpha})$ is an interval valued intuitionistic fuzzy Lie primary ideal of *L*.

Theorem 4.2. If $A=(\overline{\mu}_A,\overline{\nu}_A)$ and $B=(\overline{\mu}_B,\overline{\nu}_B)$ be two interval valued intuitionistic fuzzy Lie primary ideal of a Lie algebra *L*, then $A\times B$ is an interval valued intuitionistic fuzzy Lie primary ideal of $L\times L$.

Proof. We know that $A\times B=\{\overline{\mu}_A\times\overline{\mu}_B,\overline{\nu}_A\times\overline{\nu}_B\}$ where $(\overline{\mu}_A\times\overline{\mu}_B)(x,y)=\min(\overline{\mu}_A(x),\ \overline{\mu}_B(x))$ and $(\overline{\nu}_A\times\overline{\nu}_B)(x,y)=\max(\overline{\nu}_A(x),\ \overline{\nu}_B(y))$.

Let $x=(x_1,x_2)$ and $y=(y_1,y_2)\in L\times L$.

Now

$$(\overline{\mu}_A\times\overline{\mu}_B)(xy)=\overline{\mu}_A\times\overline{\mu}_B\ ((x_1,x_2)(y_1,y_2))$$
$$=\overline{\mu}_A\times\overline{\mu}_B\ (x_1y_1,x_2y_2)$$
$$=\min(\overline{\mu}_A(x_1y_1),\ \overline{\mu}_B(x_2y_2))$$

$= \min(\bar{\mu}_A(x_1), \bar{\mu}_B(x_2))$

$= (\bar{\mu}_A \times \bar{\mu}_B)(x_1x_2)$

$= (\bar{\mu}_A \times \bar{\mu}_B)(x)$.

Therefore, $(\bar{\mu}_A \times \bar{\mu}_B)(xy) = (\bar{\mu}_A \times \bar{\mu}_B)(x)$.

Now

$$(\bar{\nu}_A \times \bar{\nu}_B)(xy) = (\bar{\nu}_A \times \bar{\nu}_B)((x_1, x_2)(y_1, y_2))$$
$$= (\bar{\nu}_A \times \bar{\nu}_B)(x_1 y_1, x_2 y_2)$$
$$= \max(\bar{\nu}_A(x_1 y_1), \bar{\nu}_B(x_2 y_2))$$
$$= \max(\bar{\nu}_A(x_1), \bar{\nu}_B(x_2))$$
$$= (\bar{\nu}_A \times \bar{\nu}_B)(x_1 x_2)$$
$$= (\bar{\nu}_A \times \bar{\nu}_B)(x).$$

Therefore, $(\bar{\nu}_A \times \bar{\nu}_B)(xy) = (\bar{\nu}_A \times \bar{\nu}_B)(x)$.

Therefore, $A \times B$ is an interval valued intuitionistic fuzzy Lie primary ideal of L. □

Theorem 4.3. If $A = (\bar{\mu}_A, \bar{\nu}_A)$ and $B = (\bar{\mu}_B, \bar{\nu}_B)$ are interval valued intuitionistic fuzzy Lie primary ideal on L then $[A, B]$ is also an interval valued intuitionistic fuzzy Lie primary ideal of L.

Proof. Let A be an interval valued intuitionistic fuzzy lie ideal of a Lie algebra L then $\bar{\mu}_A(xy) = \bar{\mu}_A(x)$ and $\bar{\nu}_A(xy) = \bar{\nu}_A(x)$, for some every $x, y \in L$.

Consider $x, y \in L$.

Now

$$\langle\langle \bar{\mu}_A, \bar{\mu}_B \rangle\rangle(xy) = \sup(\min(\bar{\mu}_A(xy), \bar{\mu}_B(xy)) \mid xy, x, y \in L_1, [xy, xy] = xy\}$$
$$= \sup(\min(\bar{\mu}_A(x), \bar{\mu}_B(x)))$$
$$= \langle\langle \bar{\mu}_A, \bar{\mu}_B \rangle\rangle(x).$$

Therefore, $\langle\langle \bar{\mu}_A, \bar{\mu}_B \rangle\rangle(xy) = \langle\langle \bar{\mu}_A, \bar{\mu}_B \rangle\rangle(x)$.

Now

$$\langle\langle \bar{\nu}_A, \bar{\nu}_B \rangle\rangle(xy) = \inf(\max(\bar{\nu}_A(xy), \bar{\nu}_B(xy)) \mid xy, x, y \in L_1, [xy, xy] = xy\}$$
$$= \inf(\max(\bar{\nu}_A(x), \bar{\nu}_B(x)))$$
$$= \langle\langle \bar{\nu}_A, \bar{\nu}_B \rangle\rangle(x).$$

Therefore, $\langle\langle \bar{\nu}_A, \bar{\nu}_B \rangle\rangle(xy) = \langle\langle \bar{\nu}_A, \bar{\nu}_B \rangle\rangle(x)$.

Therefore, $[A, B]$ is an interval valued intuitionistic fuzzy Lie primary ideal of L. □

Theorem 4.4. If A_1, A_2, B_1, B_2 be interval valued intuitionistic fuzzy Lie primary ideal in L such that $A_1 \subseteq A_2$ and $B_1 \subseteq B_2$ then $[A_1, B_1] \subseteq [A_2, B_2]$.

Proof. Consider $x, y \in L$.

Now

$$\langle\langle \bar{\mu}_{A_1}, \bar{\mu}_{B_1} \rangle\rangle(xy) = \sup(\min(\bar{\mu}_{A_1}(xy), \bar{\mu}_{B_1}(xy)) \mid xy, x, y \in L_1, [xy, xy] = xy\}$$
$$= \sup(\min(\bar{\mu}_{A_1}(xy), \bar{\mu}_{B_1}(xy))$$
$$\geq \sup(\min(\bar{\mu}_{A_2}(\text{xy}), \bar{\mu}_{B_2}(xy))) \mid xy, x, y \in L_1, [xy, xy] = xy$$

$= \sup (\min (\overline{\mu}_{A_2}(x), \overline{\mu}_{B_2}(x)))$

$= \langle\langle \overline{\mu}_{A_2}, \overline{\mu}_{B_2} \rangle\rangle(x)$.

Therefore, $\langle\langle \overline{\mu}_{A_1}, \overline{\mu}_{B_1} \rangle\rangle(xy) \geq \langle\langle \overline{\mu}_{A_2}, \overline{\mu}_{B_2} \rangle\rangle(x)$.

Now

$\langle\langle \overline{\nu}_{A_1}, \overline{\nu}_{B_1} \rangle\rangle(xy) = \inf (\max (\overline{\nu}_{A_1}(xy), \overline{\nu}_{B_1}(xy)) \mid xy, x, y \in L_1, [xy, xy] = xy \}$

$\leq \inf (\max (\overline{\nu}_{A_2}(xy), \overline{\nu}_{B_2}(xy)) \mid xy, x, y \in L_1, [xy, xy] = xy \}$

$= \inf (\max (\overline{\nu}_{A_2}(x), \overline{\nu}_{B_2}(x))$

$= \langle\langle \overline{\nu}_{A_2}, \overline{\nu}_{B_2} \rangle\rangle(x)$

Therefore, $\langle\langle \overline{\nu}_{A_1}, \overline{\nu}_{B_1} \rangle\rangle (xy) \leq \langle\langle \overline{\nu}_{A_2}, \overline{\nu}_{B_2} \rangle\rangle(x)$

Hence, $[A_1, B_1] \subseteq [A_2, B_2]$. □

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