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On an operator, mapping intuitionistic fuzzy sets into fuzzy sets

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Abstract: In the present paper a possible way for defining a hesitancy-dependent operator mapping the class of intuitionistic fuzzy sets to the class of fuzzy sets is proposed. It is proved that the so-defined operator preserves ordering (in the sense of inclusion).
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1 Introduction

The Fuzzy set theory was introduced by Zadeh (see [7]) as an appropriate mathematical instrument for the description of uncertainty observed in nature (see e.g. [8], [5]). Later the notion intuitionistic fuzzy set introduced by K. Atanassov (see [1]). Sometimes, these sets are also referred to as Atanassov sets [4].

We will quickly remind some basic definitions and notions.

Let X be a universe set , $A \subset X$. Then a mapping $\mu_A : X \to [0, 1]$ is called membership of the element x from X to the set A. Thus a fuzzy set A^* is defined as the set of ordered couples:

$$A^* = \{\langle x, \mu_A(x)\rangle | x \in X\}$$

An intuitionistic fuzzy set is defined with the help of two mappings $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ such that for all $x \in X$,

$$\mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

Definition 1. Following [1], we call the set of ordered triples

$$A^* \stackrel{\text{def}}{=} \{x, \mu_A(x), \nu_A(x) | x \in E\}$$

an intuitionistic fuzzy set (IFS) and the mapping π_A, which is given for all $x \in X$ by

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x), \tag{2}$$

a hesitancy function.

When

$$\pi_A \equiv 0,$$

the intuitionistic fuzzy set coincides with a fuzzy set.

Let us further denote the class of all intuitionistic fuzzy sets defined over the universe X by IFS(X). For the class of all fuzzy sets defined over X we will use the denotation FS(X).

We will require the following fact and definitions (valid for all $x \in X$):

$$\nu_A(x) = 1 - \pi_A(x) - \mu_A(x). \tag{3}$$

Definition 2 (cf. [2, p.17, Eq. (2.1)])**.** Let $A, B \in$ IFS(X). We say that A is included in B (or A is a subset of B) if and only if for all $x \in X$ it is fulfilled:

$$\begin{aligned} &\mu_A(x) \leq \mu_B(x) \\ &1 - \pi_A(x) - \mu_A(x) \geq 1 - \pi_B(x) - \mu_B(x) \end{aligned} \tag{4}$$

and we denote that by

$$A \subseteq B.$$

Definition 3 (cf. [6, p.28, Eq. (1.17)])**.** Let $A, B \in$ FS(X). We say that A is included in B (or A is a subset of B) if and only if for all $x \in X$ it is fulfilled:

$$\mu_A(x) \leq \mu_B(x) \tag{5}$$

and we denote that by

$$A \subseteq B.$$

2 The proposed operator

Our purpose here is to define an operator $T : IFS(X) \to FS(X)$, such that for any $A, B \in IFS(X)$ for which $A \subset B$, it produces $T(A) \subset T(B)$. Also, if $A \in FS(X)$ we would like to have $T(A) = A$, i.e. it acts as identity operator.

Definition 4. Let $A \in$ IFS(X), then we can define T as follows

$$T(A) \stackrel{\text{def}}{=} \{\langle x, \mu^*(A) \rangle | x \in X\},$$

where

$$\mu_A^*(x) = \begin{cases} 0 & \text{if } \pi_A(x) = 1 \\ \frac{\mu_A(x)}{1 - \pi_A(x)} & \text{otherwise} \end{cases} \tag{6}$$

First we will check that this definition is correct, i.e. that $T(A)$ is indeed fuzzy set. In order to establish that we have to verify that for all $x \in X$

$$\mu_A^*(x) \leq 1.$$

But we have (see e.g. (3))

$$\mu_A(x) \leq 1 - \pi_A(x)$$

Hence if $\pi_A(x) \neq 1$, we have:

$$\mu_A^*(x) = \frac{\mu_A(x)}{1 - \pi_A(x)} \leq 1.$$

Finally, if $\pi_A(x) = 1$, then $\mu_A^*(x) = \mu_A(x) = 0$. Therefore, $T(A)$ is indeed a fuzzy set.

Theorem 1. *For any* $A, B \in \mathrm{IFS}(X)$ *such that* $A \subseteq B$ *we have* $T(A) \subseteq T(B)$.

Proof. Without loss of generality we may assume $\pi_A(x) > 0$ and $\pi_B(x) > 0$. From Definition 2 we have:

$$\mu_A(x) \leq \mu_B(x);\ 1 - \pi_A(x) - \mu_A(x) \geq 1 - \pi_B(x) - \mu_B(x)$$

We want to show that

$$\mu_A^*(x) \leq \mu_B^*(x)$$

i.e.

$$\frac{\mu_A(x)}{1 - \pi_A(x)} \leq \frac{\mu_B(x)}{1 - \pi_B(x)} \tag{7}$$

Let us first consider the case when $\pi_A(x) \leq \pi_B(x)$.

This is equivalent to $1 - \pi_A(x) \geq 1 - \pi_B(x)$. Then (7) would be equivalent to

$$(1 - \pi_B(x))\mu_A(x) \leq (1 - \pi_A(x))\mu_B(x)$$

which is obviously true.

Hence it remains to check what happens when $\pi_A(x) > \pi_B(x)$.

This means that:

$$1 - \pi_A(x) < 1 - \pi_B(x).$$

On the other hand from (4) we obtain

$$\varepsilon(x) = \mu_B(x) - \mu_A(x) \geq \pi_A(x) - \pi_B(x) = \delta(x).$$

Again we rewrite (7) as

$$(1 - \pi_B(x))\mu_A(x) \leq (1 - \pi_A(x))\mu_B(x)$$

which is equivalent to:

$$\mu_B(x) - \mu_A(x) + \pi_B(x)\mu_A(x) \geq \pi_A(x)\mu_B(x)$$

which may be rewritten as

$$\varepsilon(x) + \pi_B(x)\mu_A(x) \geq (\pi_B(x) + \delta(x))(\mu_A(x) + \varepsilon(x))$$

which after simplification is reduced to:

$$\varepsilon(x)(1 - \pi_B(x)) \geq \delta(x)\mu_B(x)$$

But the last is obviously true since:

$$\varepsilon(x)(1 - \pi_B(x)) \geq \delta(x)(1 - \pi_B(x)) \geq \delta(x)\mu_B(x).$$

Thus for all $x \in X$, we have $\mu_A^*(x) \leq \mu_B^*(x)$, which completes the proof. $\square$

3 Conclusion

Here, we have presented a new operator which transforms an intuitionistic fuzzy set into a fuzzy set, while preserving the existing inclusions. It is noteworthy that this operator is different from the operator for de-i-fuzzification defined in [3].

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