Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy perfectly semi-γ^* generalized continuous mappings

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Abstract: We introduce intuitionistic fuzzy perfectly semi-γ^* generalized continuous mappings, for which the inverse image of every intuitionistic fuzzy semi-γ^* generalized closed set is clopen. We locate this class within the hierarchy of intuitionistic fuzzy continuity notions and consolidate its immediate implications into a single structural theorem. We provide a sufficient codomain condition under which the new notion coincides with intuitionistic fuzzy perfect continuity. Two explicit constructions on finite underlying sets are given. The first is a nonconstant mapping satisfying the new definition; the second is simultaneously intuitionistic fuzzy continuous and intuitionistic fuzzy semi-γ^* generalized continuous but fails to be perfectly semi-γ^* generalized continuous. Thus the implications from the new notion to ordinary intuitionistic fuzzy continuity and to intuitionistic fuzzy semi-γ^* generalized continuity are shown to be strict.
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1 Introduction

Fuzzy sets, introduced by Zadeh [12], provided a foundation for fuzzy topological spaces developed by Chang [6]. In classical topology, semi-generalized closed sets were investigated

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by Bhattacharyya and Lahiri [5]. Intuitionistic fuzzy sets, introduced by Atanassov [4], provide graded membership and non-membership information. Çoker [7] subsequently developed intuitionistic fuzzy topological spaces. Within this setting, generalized closed sets and generalized continuity permit one to compare distinct ways in which inverse images preserve topological structure. In particular, intuitionistic fuzzy semi-γ^* generalized closed sets are recalled in [1, Definition 2.7], where they are attributed to earlier work, and the associated continuity is studied in that paper. Related intuitionistic fuzzy semi-γ^* generalized irresolute mappings were investigated in [3].

The question motivating the present work is the following: what additional structure is obtained if the inverse image of every intuitionistic fuzzy semi-γ^* generalized closed set is required to be simultaneously open and closed? This condition combines a large test class in the codomain with the clopen inverse-image requirement of perfect continuity, as defined in Definition 2.7. It is therefore stronger than ordinary intuitionistic fuzzy continuity and than intuitionistic fuzzy semi-γ^* generalized continuity; the failure of these two converse implications is established by an explicit example.

The contributions are fourfold. First, we introduce intuitionistic fuzzy perfectly semi-γ^* generalized continuity and consolidate its immediate implications into one theorem, supported by a preliminary inclusion lemma. Second, we provide a sufficient codomain condition for its coincidence with intuitionistic fuzzy perfect continuity: every semi-γ^* generalized closed set in the codomain is closed. This is a reduction result, not a necessary-and-sufficient characterization of all codomains on which the mapping classes coincide. Third, we construct a nonconstant example on finite underlying sets and determine the relevant generalized closed sets and inverse images. Fourth, a second example proves that the implications toward ordinary intuitionistic fuzzy continuity and intuitionistic fuzzy semi-γ^* generalized continuity are proper. No assertion of strictness is made for the other implications in Theorem 3.2.

2 Preliminaries

Definition 2.1 ([4]). *An intuitionistic fuzzy set (IFS) A on X is an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A, \nu_A : X \to [0,1]$ are its membership and non-membership functions and for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$. The family of all such sets is denoted by $\mathrm{IFS}(X)$. We also write $A = \langle x, \mu_A, \nu_A\rangle$.

Definition 2.2 ([4]). *For $A, B \in \mathrm{IFS}(X)$, the standard operations are*

(a) $A \subseteq B$ *if* $\mu_A(x) \le \mu_B(x)$ *and* $\nu_A(x) \ge \nu_B(x)$ *for all* $x \in X$;
(b) $A = B$ *if* $A \subseteq B$ *and* $B \subseteq A$;
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$;
(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The sets $\widetilde{0} = \langle x, 0, 1\rangle$ and $\widetilde{1} = \langle x, 1, 0\rangle$ are the empty and whole IFSs, respectively.

Definition 2.3 ([7]). *An intuitionistic fuzzy topology (IFT) on X is a family $\tau \subseteq \mathrm{IFS}(X)$ containing $\widetilde{0}$ and $\widetilde{1}$ and closed under finite intersections and arbitrary unions. The pair (X, τ) is an intuitionistic fuzzy topological space (IFTS). Its members are intuitionistic fuzzy open sets (IFOSs), and their complements are intuitionistic fuzzy closed sets (IFCSs). An IFS is clopen if it is both open and closed.*

We use $\mathrm{int}(A)$ for the union of all IFOSs contained in A, and $\mathrm{cl}(A)$ for the intersection of all IFCSs containing A. For $f : X \to Y$ and $B \in \mathrm{IFS}(Y)$,

$$f^{-1}(B) = \{\langle x, \mu_B(f(x)), \nu_B(f(x))\rangle : x \in X\}.$$

Inverse images preserve complements, arbitrary unions, and finite intersections.

Definition 2.4 ([9]). *An IFS A in an IFTS is intuitionistic fuzzy γ closed if*

$$\mathrm{cl}(\mathrm{int}(A)) \cap \mathrm{int}(\mathrm{cl}(A)) \subseteq A,$$

and intuitionistic fuzzy γ open if $A \subseteq \mathrm{cl}(\mathrm{int}(A)) \cup \mathrm{int}(\mathrm{cl}(A))$.

Definition 2.5 ([1]). *We recall the following definition from* [1, Definition 2.7]*, where it is attributed to earlier work rather than introduced as a new notion here. An IFS A is intuitionistic fuzzy semi-γ^* generalized closed if*

$$\mathrm{int}(\mathrm{cl}(A)) \cap \mathrm{cl}(\mathrm{int}(A)) \subseteq U$$

whenever $A \subseteq U$ and U is intuitionistic fuzzy semi-open, meaning $U \subseteq \mathrm{cl}(\mathrm{int}(U))$. A semi-$\gamma^*$ generalized open set is the complement of a semi-γ^* generalized closed set. We denote the family of semi-γ^* generalized closed sets in X by $\mathcal{SG}_{\gamma^*}(X)$.

Definition 2.6 ([1]). *A mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy semi-γ^* generalized continuous if $f^{-1}(V)$ is semi-γ^* generalized closed in X for every IFCS V in Y.*

Definition 2.7 (Intuitionistic fuzzy analogue of perfect continuity). *In the context of classical strong forms of continuity discussed by Noiri* [11]*, we use the following intuitionistic fuzzy analogue of perfect continuity. A mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy perfectly continuous if $f^{-1}(V)$ is intuitionistic fuzzy clopen in X for every IFCS V in Y. Equivalently, the same condition may be imposed on every IFOS in Y.*

Reference [11] is cited for the classical context, not as a source defining this intuitionistic fuzzy analogue. Throughout this paper, "perfectly continuous" has exactly the meaning in Definition 2.7; we do not use "super continuous" as an interchangeable term.

For clarity, the closed-set conventions used below are as follows: an IFS A is α closed if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$, semi-closed if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$, and preclosed if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$ [10]. The corresponding continuity notions require inverse images of IFCSs to belong to the indicated classes. An IFS R is regular closed if $R = \mathrm{cl}(\mathrm{int}(R))$. Almost semi-$\gamma^*$ generalized continuity requires inverse images of regular closed sets to be semi-γ^* generalized closed [2].

Lemma 2.8 (closed-set inclusions)**.** *Every IFCS is intuitionistic fuzzy α closed, semi-closed, preclosed, γ closed, and semi-γ^* generalized closed. In particular, every regular closed IFS is semi-γ^* generalized closed.*

Proof. Let C be an IFCS. Since $\mathrm{cl}(C) = C$ and $\mathrm{int}(C) \subseteq C$, monotonicity of closure gives $\mathrm{cl}(\mathrm{int}(C)) \subseteq \mathrm{cl}(C) = C$. Hence

$$\mathrm{cl}(\mathrm{int}(\mathrm{cl}(C))) = \mathrm{cl}(\mathrm{int}(C)) \subseteq C, \qquad \mathrm{int}(\mathrm{cl}(C)) = \mathrm{int}(C) \subseteq C.$$

These inclusions establish α closedness, semi-closedness, and preclosedness. They also imply

$$\mathrm{cl}(\mathrm{int}(C)) \cap \mathrm{int}(\mathrm{cl}(C)) \subseteq C,$$

which establishes γ closedness. For every semi-open U with $C \subseteq U$, the same intersection is contained in $C \subseteq U$, proving semi-γ^* generalized closedness by Definition 2.5. Finally, a regular closed set is closed, because it is the closure of an IFS, so the last assertion follows. □

3 Perfectly semi-γ^* generalized continuous mappings

We use the abbreviation IFPSγ^*G-continuous for the term "intuitionistic fuzzy perfectly semi-γ^* generalized continuous."

Definition 3.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is IFPSγ^*G-continuous if $f^{-1}(A)$ is intuitionistic fuzzy clopen in X for every $A \in \mathcal{SG}_{\gamma^*}(Y)$.*

Recall that intuitionistic fuzzy perfect continuity tests all closed sets. Definition 3.1 enlarges this test class to all semi-γ^* generalized closed sets.

Theorem 3.2 (Consolidated implication theorem)**.** *Every IFPSγ^*G-continuous mapping is intuitionistic fuzzy perfectly continuous and, consequently, intuitionistic fuzzy continuous* [8]*, α-continuous, semi-continuous, precontinuous* [10]*, γ-continuous* [9]*, semi-γ^* generalized continuous* [1]*, and almost semi-γ^* generalized continuous* [2].

Proof. Let V be an IFCS in Y. Lemma 2.8 makes V semi-γ^* generalized closed, so Definition 3.1 gives that $f^{-1}(V)$ is clopen. Thus f is perfectly continuous and, since $f^{-1}(V)$ is closed, ordinarily continuous. Applying Lemma 2.8 in X to the closed set $f^{-1}(V)$ proves each of the α-, semi-, pre-, γ-, and semi-γ^* generalized continuity assertions. Finally, every regular closed set R in Y is closed by Lemma 2.8. Its inverse image is therefore clopen, hence closed and, again by Lemma 2.8, semi-γ^* generalized closed. This proves almost semi-γ^* generalized continuity. □

The implications relevant to the counterexample are

$$\text{IFPS}\gamma^*\text{G-continuous} \implies \text{IF perfectly continuous} \implies \text{IF continuous},$$

and

$$\text{IFPS}\gamma^*\text{G-continuous} \implies \text{IF semi-}\gamma^*\text{ generalized continuous}.$$

Example 3.7 proves that ordinary intuitionistic fuzzy continuity and intuitionistic fuzzy semi-γ^* generalized continuity do not imply IFPSγ^*G-continuity. It does not assert that each individual implication in the first chain is strict.

Theorem 3.3 (Open-set characterization)**.** *A mapping $f:(X,\tau)\to(Y,\sigma)$ is IFPSγ^*G-continuous if and only if the inverse image of every intuitionistic fuzzy semi-γ^* generalized open set in Y is intuitionistic fuzzy clopen in X.*

Proof. If A is semi-γ^* generalized open, then A^c is semi-γ^* generalized closed. Since $f^{-1}(A^c)=(f^{-1}(A))^c$ and the complement of a clopen set is clopen, necessity follows. Applying the same argument to the complement of a semi-γ^* generalized closed set proves sufficiency. □

Definition 3.4. An IFTS (Y,σ) is called an $S\gamma^*C$-space if every intuitionistic fuzzy semi-γ^* generalized closed set in Y is intuitionistic fuzzy closed.

By Lemma 2.8, the closed and semi-γ^* generalized closed classes coincide on an $S\gamma^*C$-space.

Theorem 3.5 (Reduction to perfect continuity)**.** *Let (Y,σ) be an $S\gamma^*C$-space. A mapping $f:(X,\tau)\to(Y,\sigma)$ is IFPSγ^*G-continuous if and only if it is intuitionistic fuzzy perfectly continuous.*

Proof. The forward implication is Theorem 3.2. Conversely, suppose that f is intuitionistic fuzzy perfectly continuous and let A be semi-γ^* generalized closed in Y. Since Y is an $S\gamma^*C$-space, A is closed. Perfect continuity gives that $f^{-1}(A)$ is clopen, as required. □

The $S\gamma^*C$ condition is sufficient for this coincidence. The theorem does not establish its necessity. Outside this class, Definition 3.1 tests additional generalized closed sets, whose inverse images must also be clopen.

Finite constructions

We now give two explicit constructions on finite underlying sets. The first shows that the class of IFPSγ^*G-continuous mappings contains nonconstant examples, while the second demonstrates that the principal converse implications discussed above fail in general.

Example 3.6 (A nonconstant IFPSγ^*G-continuous mapping)**.** Let $X=\{1,2,3\}$ and $Y=\{a,b\}$. Define $f:X\to Y$ by $f(1)=b$ and $f(2)=f(3)=a$. Equip Y with the intuitionistic fuzzy indiscrete topology

$$\sigma=\{\widetilde{0}_Y,\widetilde{1}_Y\},$$

and equip X with the pullback topology

$$\tau=\{f^{-1}(B):B\in\mathrm{IFS}(Y)\}.$$

This is an IFT because inverse images preserve $\widetilde{0}$, $\widetilde{1}$, arbitrary unions, and finite intersections. It is not discrete: every member of τ has identical membership and non-membership values at 2 and 3, so an IFS distinguishing these points is not in τ.

For every $A\in\mathrm{IFS}(Y)\setminus\{\widetilde{0}_Y,\widetilde{1}_Y\}$, indiscreteness gives

$$\mathrm{int}(A)=\widetilde{0}_Y,\qquad \mathrm{cl}(A)=\widetilde{1}_Y.$$

The only semi-open IFSs are $\widetilde{0}_Y$ and $\widetilde{1}_Y$: for any other U, $\mathrm{cl}(\mathrm{int}(U)) = \widetilde{0}_Y$, so $U \not\subseteq \widetilde{0}_Y$. Furthermore,

$$\mathrm{int}(\mathrm{cl}(A)) \cap \mathrm{cl}(\mathrm{int}(A)) = \widetilde{1}_Y \cap \widetilde{0}_Y = \widetilde{0}_Y.$$

The defining inclusion holds for the only semi-open superset $\widetilde{1}_Y$. The two trivial IFSs satisfy the definition as well. Consequently,

$$\mathcal{SG}_{\gamma^*}(Y) = \mathrm{IFS}(Y).$$

For every such A, $f^{-1}(A) \in \tau$ by construction and $(f^{-1}(A))^c = f^{-1}(A^c) \in \tau$. Hence every inverse image is clopen. Therefore f is a nonconstant IFPSγ^*G-continuous mapping between non-discrete IFTSs with explicitly specified finite underlying sets. Their families of IFSs need not be finite.

Example 3.7 (Failure of the principal converses)**.** On the same indiscrete space (Y, σ), consider the identity $i : (Y, \sigma) \to (Y, \sigma)$. It is intuitionistic fuzzy continuous because the only closed IFSs are $\widetilde{0}_Y$ and $\widetilde{1}_Y$. It is also intuitionistic fuzzy semi-γ^* generalized continuous because the inverse images of these two sets are semi-γ^* generalized closed.

Choose

$$B = \{\langle a, 0.6, 0.2\rangle, \langle b, 0.3, 0.5\rangle\}.$$

By Example 3.6, B is semi-γ^* generalized closed. Nevertheless $i^{-1}(B) = B$ is neither open nor closed in σ, so it is not clopen. Thus i is not IFPSγ^*G-continuous. This proves that neither ordinary intuitionistic fuzzy continuity nor intuitionistic fuzzy semi-γ^* generalized continuity implies IFPSγ^*G-continuity.

Proposition 3.8 (Composition)**.** *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be IFPSγ^*G-continuous. Then $g \circ f$ is IFPSγ^*G-continuous.*

Proof. If A is semi-γ^* generalized closed in Z, then $g^{-1}(A)$ is clopen in Y, hence closed and, by Lemma 2.8, semi-γ^* generalized closed. Definition 3.1 applied to f yields $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ clopen in X. □

4 Conclusion

We introduced IFPSγ^*G-continuity as a clopen inverse-image condition imposed on all intuitionistic fuzzy semi-γ^* generalized closed sets. Its immediate consequences are collected in a single implication theorem, with the required closed-set inclusions proved in Lemma 2.8. Theorem 3.5 provides a sufficient codomain condition for coincidence with intuitionistic fuzzy perfect continuity: the codomain is an $S\gamma^*C$-space. No necessity claim is made for this condition. Example 3.6 supplies a nonconstant construction on finite underlying sets, and Example 3.7 establishes strictness of the implications from IFPSγ^*G-continuity to ordinary intuitionistic fuzzy continuity and to intuitionistic fuzzy semi-γ^* generalized continuity. Strictness is not claimed for the remaining implications in Theorem 3.2. Further work may investigate separation properties, products, and other codomain conditions for coincidence of the mapping classes.

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