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A REMARK ON AN INTUITIONISTIC FUZZY APPROACH TO A PROBLEM IN STAR DYNAMICS

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Abstract: The short remark contains a discussion and an example on the possibility for using of apparatus of the intuitionistic fuzzy sets for the conceptial solution of a problem in star dynamics and formation of large-scale structure.

Keywords: Intuitionistic fuzzy set, Star dynamics

The $n-$body problem is well known and with a venerable history. Different approaches and solutions are used, but, practically, no one can solve it fully. A lot of solutions are based on probability-statistical methods.

In particular, the techniques have been updated in view of the massive computational efforts to model the large-scale structure of the Universe and galaxy formation and evolution [1]. While the dynamics is not the slow part in such computations, these routines have hardly seen any updates in the past decade. Speeding them up by a factor of 5 or more would make a significant improvement in the overall computation and allow resolution of an astrophysically new level of detail [2].

Here we shall try to use the apparatus of the Intuitionistic Fuzzy Sets (IFSs; see [3]) - extensions of the ordinary fuzzy sets, to describe a special case of the above mentioned problem and to get an idea how to solve of this problem in practice. The present paper must be perceived as a first step of a more detailed future work by the authors.

Let us have a point A which lies between two (point) systems S_1 and S_2 (see Fig. 1), which essentially have an influence on it. Let $Z_1, Z_2, ..., Z_{2k}$ be other systems, which influence on A in smaller degree. Therefore, S_1 and S_2 are the two systems which influence on A with highest degree. In another note we shall discuss the situation in which the systems which influence on A are greater in number. We must note also, that the number $2k$ of the additional systems changes with respect to the next recursive algorithm. These $2k$ systems can be determined in such a way that exactly k of them to be on the left side of A and the rest k of them to be on the right side of A. Of course, this can be obtained by suitable numeration and union of the additional systems. For the needs of the algorithm it is necessary that the number of the additional systems from the two sides of A to be equal.

In the present paper we shall assume that point A is static and here we shall determine the future direction of point A and its value, in a result of the acting of the systems.

Let us mark the powers acting on A from S_1 and S_2 by σ_1 and σ_2 and the sum of all powers acting on A from $Z_1, Z_2, ..., Z_{2k}$ by σ. Now, let us define the numbers:

$$\mu_A = \frac{\sigma_1}{\sigma_1 + \sigma_1 + \sigma}$$

$$\nu_A = \frac{\sigma_2}{\sigma_1 + \sigma_1 + \sigma}.$$

Therefore

$$\pi_A = \frac{\sigma}{\sigma_1 + \sigma_1 + \sigma}.$$

The point where is placed A in the intuitionistic fuzzy interpretation triangle is shown on Fig. 2

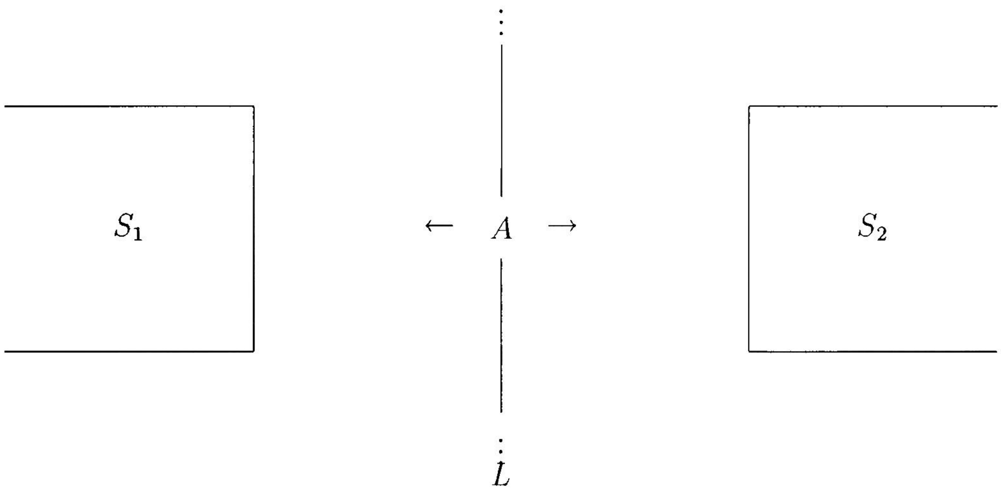

Fig. 1

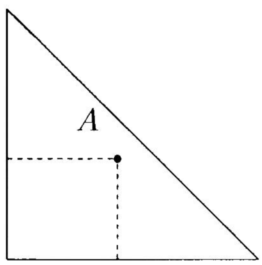

Fig. 2.

We determine the two systems among $Z_1, Z_2, ..., Z_{2k}$ which influence the most power on A and let these systems lie on the two sides of line L (see Fig. 1. Of course, systems S_1 and S_2 are excluded from this procedure. Let these two systems are Z_1 and Z_2.

Let us mark the powers ackting on A from $Z_1, Z_2, ..., Z_{2k}$ by $\rho_1, \rho_2, ..., \rho_{2k}$, respectively.

Now, we determine the numbers

$$\alpha_1 = \frac{\rho_1}{\rho_1 + \rho_1 + \overline{\rho}_{3,...,2k}}$$

$$\beta_1 = \frac{\rho_2}{\rho_1 + \rho_1 + \overline{\rho}_{3,...,2k}},$$

where $\overline{\rho}_{3,...,2k}$ is the sum of the values of influence of the rest systems $Z_3, Z_4, ..., Z_{2k}$.

If f is the function which determine the place of point A in the interpretation triangle, i.e., if

$$f(A) =< \mu_A, \nu_A >,$$

now we determine the fictitious point $A_1 = F_{\alpha_1,\beta_1}(A)$ and its place in the interpretation triangle:

$$f(A_1) = F_{\alpha_1,\beta_1}(A_0) = F_{\alpha_1,\beta_1}(< \mu_{A_0}, \nu_{A_0} >) =< \mu_{A_1}, \nu_{A_1} >,$$

where A_0 coincides with A.

Let n $(k - 1 \geq n \geq 1)$ is a natural number. Then, following our algorithm, on the $n-$th step we shall determine the numbers

$$\alpha_n = \frac{\rho_{2n-1}}{\rho_{2n-1} + \rho_{2n} + \overline{\rho}_{1,2,...,2n-2,2n+1,...,2k}}$$

$$\beta_n = \frac{\rho_{2n}}{\rho_{2n-1} + \rho_{2n} + \overline{\rho}_{1,2,...,2n-2,2n+1,...,2k}},$$

where $\overline{\rho}_{1,2,...,2n-2,2n+1,...,2k}$ is the sum of the values of influence of the systems $Z_1, Z_2, ...,$ $Z_{2n-2}, Z_{2n+1}, ..., Z_{2k}$ which do not take part in the above formulas, but they influence on the point A and of its representator for the present step - point A_{n-1}.

Now we determine the point

$$A_n = F_{\alpha_n,\beta_n}(F_{\alpha_{n-1},\beta_{n-1}}(...(F_{\alpha_1,\beta_1}(A)...))$$

and its place in the interpretation triangle:

$$f(A_n) = F_{\alpha_n,\beta_n}(A_{n-1}) = F_{\alpha_n,\beta_n}(< \mu_{A_{n-1}}, \nu_{A_{n-1}} >) =< \mu_{A_{n-1}}, \nu_{A_{n-1}} > .$$

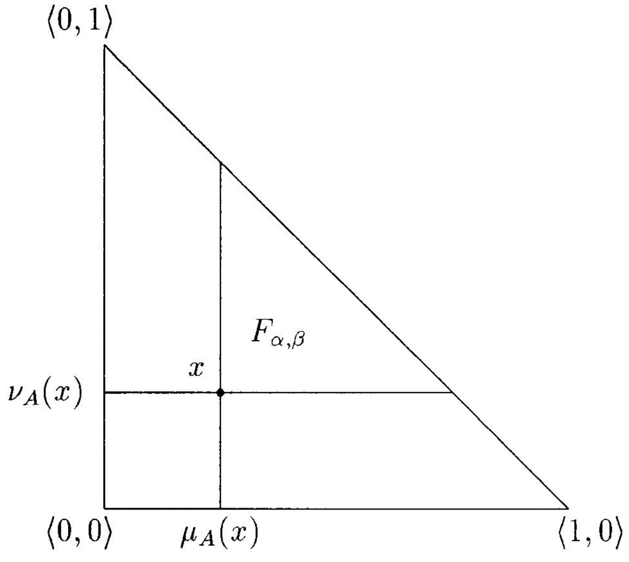

Fig. 3

This procedure contains k steps.

Because operator $F_{\alpha,\beta}$ acts as it is shown on Fig. 3, we obtain a sequence of points $A_0, A_1, ..., A_k$ with determines the place of the final from of A in a result of the actions of the systems.

The interpretation triangle can be transform to the form from Fig. 4.

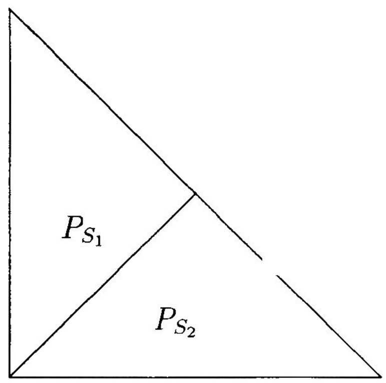

Fig. 4

The perpendicular from point $< 0, 0 >$ to the hypotenuse cuts the triangle into two (equal) parts, marked on Fig. 4 by P_{S_1} and P_{S_2}. Of the point A_k places in region P_{S_i}, where $i \in \{1, 2\}$, then we can assert that point will be transformed to the direction of system S_i.

References:

[1] Lewis, G. F. et al., 2000, "The Effects of Gasdynamics, Cooling, Star Formation, and Numerical Resolution in Simulations of Cluster Formation", Astrophys. J., 536, 623.

[2] Katz, N., Weinberg, D., & Hernquist, L., 1996, "Cosmological Simulations with TreeSPH", Astrophys. J. Suppl., 105, 19.

[3] Atanassov K., Intuitionistic Fuzzy Sets, Springer-Verlag, Heidelberg, 1999.