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Relation between semipreclosed sets and beta closed sets in intuitionistic fuzzy topological spaces

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Abstract: In this paper, we have investigated the relation between semipreclosed sets and beta closed sets in intuitionistic fuzzy topological spaces even though they are equivalent sets in general topology.

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1 Introduction

Andrijevic [1] introduced semipreclosed sets in general topology. With the help of Lemma 1.1 in [1] and Theorem 1.5 in [1], he proved that the definition of a semipreclosed set is equivalent to $\mathbf{int}(\mathbf{cl}(\mathbf{int}(A))) \subseteq A$, where A is a set in a topological space. After that the set which satisfies the above condition is called as a beta closed set. But as for as intuitionistic fuzzy topology is concern, the two existing sets are not equivalent. Because the lemma which is mentioned in [1] does not exist in intuitionistic fuzzy topological spaces. We investigated it and proved this result with a suitable example.

2 Preliminaries

Definition 2.1 [2]**:** An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2 [2]**:** Let A and B be IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X\}$;

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X\}$.

The intuitionistic fuzzy sets $\tilde{0} = \{\langle x, 0, 1 \rangle \mid x \in X\}$ and $\tilde{1} = \{\langle x, 1, 0 \rangle \mid x \in X\}$ are respectively the empty set and the whole set of X. For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$.

Definition 2.3 [3]**:** An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $\tilde{0}, \tilde{1} \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 [4]**:** An IFS A is an *intuitionistic fuzzy pre closed set* (IFPCS for short) if $\mathbf{cl}(\mathbf{int}(A)) \subseteq A$.

Definition 2.5 [5]**:** An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be

(i) *intuitionistic fuzzy semi-pre closed set* (IFSPCS for short) if there exists an IFPCS B such that $\mathbf{int}(B) \subseteq A \subseteq B$.

(ii) *intuitionistic fuzzy semi-pre open set* (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that $B \subseteq A \subseteq \mathbf{cl}(B)$.

Definition 2.6 [4]**:** An IFS A is an *intuitionistic fuzzy beta closed set* (IFβCS for short) if $\mathbf{int}(\mathbf{cl}(\mathbf{int}(A))) \subseteq A$.

Lemma 2.7: [1] For every closed set F in a topological space X and every $A \subset X$, we have **int**$(A \subset F) \subset$ **int**$(A) \subset F$.

Theorem 2.8: [1] For any subset A of a topological space X, the following conditions are equivalent:

(i) A is a semipreclosed set;

(ii) **int**(**cl**(**int**(A))) $\subseteq A$;

(iii) **int**(A) is a regular open set.

3 Relation between semipreclosed sets and beta closed sets in intuitionistic fuzzy topological spaces

In this section, we have investigated the relation between semipreclosed sets and beta closed sets in intuitionistic fuzzy topological spaces and proved that intuitionistic fuzzy beta closed set does not imply an intuitionistic fuzzy semipreclosed set.

Remark 3.1: For every intuitionistic fuzzy closed set F in an intuitionistic fuzzy topological space X and every $C \subset X$, we have int$(C \cup F) \not\subset$ int$(C) \cup F$.

Example 3.2: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ be an IFT on X. Here, $A = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$, where $\mu_A(a) = 0.5$, $\mu_A(b) = 0.3$, $\nu_A(a) = 0.5$ and $\nu_A(b) = 0.7$, and $B = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$, where $\mu_B(a) = 0.8$, $\mu_B(b) = 0.7$, $\nu_B(a) = 0.2$ and $\nu_B(b) = 0.3$. Let $C = \langle x, (0.8, 0.5), (0.2, 0.2)\rangle$ be an intuitionistic fuzzy set in X. Now **int**$(C \cup F) = B$, where $F = A^c$ is an intuitionistic fuzzy closed set in X and **int**$(C) \cup F = A \cup F = F$. But $B \not\subset F$. That is **int**$(C \cup F)$ $\not\subset$ **int**$(C) \cup F$. Hence, Lemma 2.7 proved by Andrijevic is not satisfied in intuitionistic fuzzy topological spaces.

Theorem 3.3: Every IFSPCS is an IFβCS in an IFTS (X, τ).

Proof: Let A be an IFSPCS. Then by definition 2.5, there exists an IFPCS B such that **int**$(B) \subseteq A \subseteq B$ and **cl**(**int**(B)) $\subseteq B$. Now **int**(**cl**(**int**(A))) $\subseteq$ **int**(**cl**(**int**(B))) $\subseteq$ **int**$(B) \subseteq A$. Hence, **int**(**cl**(**int**(A))) $\subseteq A$ and thus A is an IFβCS in X. □

Remark 3.4: Not every IFβCS is an IFSPCS in an IFTS (X, τ). This can be seen from the following example.

Example 3.5: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, C, D, \tilde{1}\}$ be an IFT on X. Here $A = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$, $B = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$, $C = \langle x, (0.5, 0.5), (0.4, 0.5)\rangle$ and $D = \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$. Let $G = \langle x, (0.5, 0.8), (0.4, 0.2)\rangle$ be an IFS in X. Now **int**(**cl**(**int**(G))) $= C \subseteq G$. Therefore G is an IFβCS in X. But G is not an IFSPCS in X, as we cannot find any IFPCS H such that **int**$(H) \subseteq G \subseteq H$ in X. That is:

$$\text{IFSPCS} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{IF}\beta\text{CS}$$

4 Conclusions

In an intuitionistic fuzzy topological space an intuitionistic fuzzy β closed set need not be an intuitionistic fuzzy semipreclosed set, even though they are equivalent in general topological spaces.

References

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