

Intuitionistic fuzzy integrals

Radoslav T. Tsvetkov

Technical University of Sofia

8 Kliment Ohridski Boul., 1000 Sofia, Bulgaria

e-mails: `radotzv8@gmail.com`, `rado_tzv8@hotmail.com`

Abstract: We define intuitionistic fuzzy integral through concept for Sugeno's integral. We describe inequalities generated by intuitionistic fuzzy sets and topological operators.

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1 Introduction

Definition 1. Let for every $x \in X$ $f(x) \in [0, 1]$. We assume that (X, Σ, m) is a measure space with $m(X) = 1$, $A \in \Sigma$.

$$B = \{< x, \mu_B(x), \nu_B(x) > / x \in X\}$$

is IFS, where $\mu_B(x)$ and $\nu_B(x)$ are continuous functions on topological space (X, τ) . Right intuitionistic fuzzy integral of f with respect to m on A about B from first and second kind is defined respectively as

$$(RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm = < \lambda_1 \sup_{x \in X} [\mu_B(x) \wedge m(A \cap F_{\nu_B(x)})], \lambda_2 \inf_{x \in X} [\nu_B(x) \vee m(A \cap F_{\mu_B(x)})] >,$$

$$(RIFIS_B^{2(\gamma_1, \gamma_2)}) \int_A f dm = < \gamma_1 \sup_{x \in X} [\mu_B(x) \wedge m(A \cap E_{\mu_B(x)})], \gamma_2 \inf_{x \in X} [\nu_B(x) \vee m(A \cap E_{\nu_B(x)})] >,$$

where $(\lambda_1, \lambda_2), (\gamma_1, \gamma_2) \in [0, 1] \times [0, 1]$, $\lambda_1 + \lambda_2 \leq 1$, $\gamma_1 + \gamma_2 \leq 1$.

Left intuitionistic fuzzy integral of f with respect to m on A about B from first and second kind is defined respectively as

$$(LIFIS_B^{1(q_1, q_2)}) \int_A f dm = < q_1 \inf_{x \in X} [\mu_B(x) \vee m(A \cap F_{\nu_B(x)})], q_2 \sup_{x \in X} [\nu_B(x) \wedge m(A \cap F_{\mu_B(x)})] >,$$

$$(LIFIS_B^{2(r_1,r_2)}) \int_A f dm = < r_1 \inf_{x \in X} [\mu_B(x) \vee m(A \cap E_{\mu_B(x)})], r_2 \sup_{x \in X} [\nu_B(x) \wedge m(A \cap E_{\nu_B(x)})] >,$$

where $(q_1, q_2), (r_1, r_2) \in [0, 1] \times [0, 1]$, $q_1 + q_2 \leq 1$, $r_1 + r_2 \leq 1$.

Let the sets

$$F_{\mu_B(x)} = \{x' | x' \in X, f(x') \geq \mu_B(x)\}, x \in X$$

and

$$E_{\nu_B(x)} = \{x' | x' \in X, f(x') \leq \nu_B(x)\}, x \in X$$

be called a $\mu_B(x)$ -R-level set and a $\nu_B(x)$ -L-level set off, respectively.

From [1] we have that $< a, b > \leq < c, d >$ if and only if $a \leq c$ and $b \geq d$ (Using this order we compare defined above integrals.).

$$\min(< a, b >, < c, d >) = < \min(a, c), \max(b, d) >$$

and

$$\max(< a, b >, < c, d >) = < \max(a, c), \min(b, d) > .$$

Proposition 1. Let B and C are IFS. m is a monotone measure. From [1] we have that $B \subseteq C$ if and only if for every $x \in X$

$$\mu_B(x) \leq \mu_C(x) \& \nu_B(x) \geq \nu_C(x)$$

is performed.

Then we have that:

$$(1) (RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm,$$

$$(2) (RIFIS_B^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_C^{2(\gamma_1, \gamma_2)}) \int_A f dm,$$

$$(3) (LIFIS_B^{1(q_1, q_2)}) \int_A f dm \leq (LIFIS_C^{1(q_1, q_2)}) \int_A f dm,$$

$$(4) (LIFIS_B^{2(r_1, r_2)}) \int_A f dm \leq (LIFIS_C^{2(r_1, r_2)}) \int_A f dm.$$

Proof. (1) From $\nu_B(x) \geq \nu_C(x)$ we have that $F_{\nu_B(x)} \subseteq F_{\nu_C(x)}$. Whence $A \cap F_{\nu_B(x)} \subseteq A \cap F_{\nu_C(x)}$. Since m is monotone, $m(A \cap F_{\nu_B(x)}) \leq m(A \cap F_{\nu_C(x)})$. From $\mu_B(x) \leq \mu_C(x)$ follows $\mu_B(x) \wedge m(A \cap F_{\nu_B(x)}) \leq \mu_C(x) \wedge m(A \cap F_{\nu_C(x)})$ for every $x \in X$. Therefore, we have $\sup_{x \in X} [\mu_B(x) \wedge m(A \cap F_{\nu_B(x)})] \leq \sup_{x \in X} [\mu_C(x) \wedge m(A \cap F_{\nu_C(x)})]$.

From $\mu_B(x) \leq \mu_C(x)$ we have that $F_{\mu_C(x)} \subseteq F_{\mu_B(x)}$. Whence $A \cap F_{\mu_C(x)} \subseteq A \cap F_{\mu_B(x)}$. Since m is monotone, $m(A \cap F_{\mu_C(x)}) \leq m(A \cap F_{\mu_B(x)})$. From $\nu_B(x) \geq \nu_C(x)$ follows $\nu_C(x) \vee m(A \cap F_{\mu_C(x)}) \leq \nu_B(x) \vee m(A \cap F_{\mu_B(x)})$.

$F_{\mu_C(x)} \leq \nu_B(x) \vee m(A \cap F_{\mu_B(x)})$ for every $x \in X$. Therefore, we have $\inf_{x \in X} [\nu_C(x) \vee m(A \cap F_{\mu_C(x)})] \leq \inf_{x \in X} [\nu_B(x) \vee m(A \cap F_{\mu_B(x)})]$.

(2) From $\mu_B(x) \leq \mu_C(x)$ we have that $E_{\mu_B(x)} \subseteq E_{\mu_C(x)}$. Whence $A \cap E_{\mu_B(x)} \subseteq A \cap E_{\mu_C(x)}$. Since m is monotone, $m(A \cap E_{\mu_B(x)}) \leq m(A \cap E_{\mu_C(x)})$. From $\mu_B(x) \leq \mu_C(x)$ follows $\mu_B(x) \wedge m(A \cap E_{\mu_B(x)}) \leq \mu_C(x) \wedge m(A \cap E_{\mu_C(x)})$ for every $x \in X$. Therefore, we have $\sup_{x \in X} [\mu_B(x) \wedge m(A \cap E_{\mu_B(x)})] \leq \sup_{x \in X} [\mu_C(x) \wedge m(A \cap E_{\mu_C(x)})]$.

From $\nu_B(x) \geq \nu_C(x)$ we have that $E_{\nu_C(x)} \subseteq E_{\nu_B(x)}$. Whence $A \cap E_{\nu_C(x)} \subseteq A \cap E_{\nu_B(x)}$. Since m is monotone, $m(A \cap E_{\nu_C(x)}) \leq m(A \cap E_{\nu_B(x)})$. From $\nu_B(x) \geq \nu_C(x)$ follows $\nu_C(x) \vee m(A \cap E_{\nu_C(x)}) \leq \nu_B(x) \vee m(A \cap E_{\nu_B(x)})$ for every $x \in X$. Therefore, we have $\inf_{x \in X} [\nu_C(x) \vee m(A \cap E_{\nu_C(x)})] \leq \inf_{x \in X} [\nu_B(x) \vee m(A \cap E_{\nu_B(x)})]$.

(3) From $\nu_B(x) \geq \nu_C(x)$ we have that $F_{\nu_B(x)} \subseteq F_{\nu_C(x)}$. Whence $A \cap F_{\nu_B(x)} \subseteq A \cap F_{\nu_C(x)}$. Since m is monotone, $m(A \cap F_{\nu_B(x)}) \leq m(A \cap F_{\nu_C(x)})$. From $\mu_B(x) \leq \mu_C(x)$ follows $\mu_B(x) \vee m(A \cap F_{\nu_B(x)}) \leq \mu_C(x) \vee m(A \cap F_{\nu_C(x)})$ for every $x \in X$. Therefore, we have $\inf_{x \in X} [\mu_B(x) \vee m(A \cap F_{\nu_B(x)})] \leq \inf_{x \in X} [\mu_C(x) \vee m(A \cap F_{\nu_C(x)})]$.

From $\mu_B(x) \leq \mu_C(x)$ we have that $F_{\mu_C(x)} \subseteq F_{\mu_B(x)}$. Whence $A \cap F_{\mu_C(x)} \subseteq A \cap F_{\mu_B(x)}$. Since m is monotone, $m(A \cap F_{\mu_C(x)}) \leq m(A \cap F_{\mu_B(x)})$. From $\nu_B(x) \geq \nu_C(x)$ follows $\nu_C(x) \wedge m(A \cap F_{\mu_C(x)}) \leq \nu_B(x) \wedge m(A \cap F_{\mu_B(x)})$ for every $x \in X$. Therefore, we have $\sup_{x \in X} [\nu_C(x) \wedge m(A \cap F_{\mu_C(x)})] \leq \sup_{x \in X} [\nu_B(x) \wedge m(A \cap F_{\mu_B(x)})]$.

(4) From $\mu_B(x) \leq \mu_C(x)$ we have that $E_{\mu_B(x)} \subseteq E_{\mu_C(x)}$. Whence $A \cap E_{\mu_B(x)} \subseteq A \cap E_{\mu_C(x)}$. Since m is monotone, $m(A \cap E_{\mu_B(x)}) \leq m(A \cap E_{\mu_C(x)})$. From $\mu_B(x) \leq \mu_C(x)$ follows $\mu_B(x) \vee m(A \cap E_{\mu_B(x)}) \leq \mu_C(x) \vee m(A \cap E_{\mu_C(x)})$ for every $x \in X$. Therefore, we have $\inf_{x \in X} [\mu_B(x) \vee m(A \cap E_{\mu_B(x)})] \leq \inf_{x \in X} [\mu_C(x) \vee m(A \cap E_{\mu_C(x)})]$.

From $\nu_B(x) \geq \nu_C(x)$ we have that $E_{\nu_C(x)} \subseteq E_{\nu_B(x)}$. Whence $A \cap E_{\nu_C(x)} \subseteq A \cap E_{\nu_B(x)}$. Since m is monotone, $m(A \cap E_{\nu_C(x)}) \leq m(A \cap E_{\nu_B(x)})$. From $\nu_B(x) \geq \nu_C(x)$ follows $\nu_C(x) \wedge m(A \cap E_{\nu_C(x)}) \leq \nu_B(x) \wedge m(A \cap E_{\nu_B(x)})$ for every $x \in X$. Therefore, we have $\sup_{x \in X} [\nu_C(x) \wedge m(A \cap E_{\nu_C(x)})] \leq \sup_{x \in X} [\nu_B(x) \wedge m(A \cap E_{\nu_B(x)})]$. \square

Proposition 2. Let B and C are IFS. From [1] we have that

$$B \cap C = \{< x, \min(\mu_B(x), \mu_C(x)), \max(\nu_B(x), \nu_C(x)) > | x \in X\}$$

and

$$B \cup C = \{< x, \max(\mu_B(x), \mu_C(x)), \min(\nu_B(x), \nu_C(x)) > | x \in X\}.$$

Then

$$(1) \quad (RIFIS_{B \cap C}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq \min((RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm, (RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm),$$

$$(2) \quad \max((RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm, (RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm) \leq (RIFIS_{B \cup C}^{1(\lambda_1, \lambda_2)}) \int_A f dm,$$

$$(3) \quad (RIFIS_{B \cap C}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq \min((RIFIS_B^{2(\gamma_1, \gamma_2)}) \int_A f dm, (RIFIS_C^{2(\gamma_1, \gamma_2)}) \int_A f dm),$$

$$(4) \quad \max((RIFIS_B^{2(\gamma_1, \gamma_2)}) \int_A f dm, (RIFIS_C^{2(\gamma_1, \gamma_2)}) \int_A f dm) \leq (RIFIS_{B \cup C}^{2(\gamma_1, \gamma_2)}) \int_A f dm,$$

$$(5) \quad (LIFIS_{B \cap C}^{1(q_1, q_2)}) \int_A f dm \leq \min((LIFIS_B^{1(q_1, q_2)}) \int_A f dm, (LIFIS_C^{1(q_1, q_2)}) \int_A f dm),$$

$$(6) \quad \max((LIFIS_B^{1(q_1, q_2)}) \int_A f dm, (LIFIS_C^{1(q_1, q_2)}) \int_A f dm) \leq (LIFIS_{B \cup C}^{1(q_1, q_2)}) \int_A f dm,$$

$$(7) \quad (LIFIS_{B \cap C}^{2(r_1, r_2)}) \int_A f dm \leq \min((LIFIS_B^{2(r_1, r_2)}) \int_A f dm, (LIFIS_C^{2(r_1, r_2)}) \int_A f dm) \text{ and}$$

$$(8) \quad \max((LIFIS_B^{2(r_1, r_2)}) \int_A f dm, (LIFIS_C^{2(r_1, r_2)}) \int_A f dm) \leq (LIFIS_{B \cup C}^{2(r_1, r_2)}) \int_A f dm.$$

Proof. (1) From $B \cap C \subseteq B$ and $B \cap C \subseteq C$ using proposition 1, we obtain that

$$(RIFIS_{B \cap C}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm \text{ and } (RIFIS_{B \cap C}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq$$

$(RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm$. Therefore we have

$$(RIFIS_{B \cap C}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq \min((RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm, (RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm).$$

Analogously, we can prove (3),(5),(7).

(2) From $B \subseteq B \cup C$ and $C \subseteq B \cup C$ by using proposition 1 we obtain that $(RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{B \cup C}^{1(\lambda_1, \lambda_2)}) \int_A f dm$ and $(RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{B \cup C}^{1(\lambda_1, \lambda_2)}) \int_A f dm$. Where from we have

$$\max((RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm, (RIFIS_C^{1(\lambda_1, \lambda_2)}) \int_A f dm) \leq (RIFIS_{B \cup C}^{1(\lambda_1, \lambda_2)}) \int_A f dm.$$

Analogously, we can prove (4),(6),(8). \square

Proposition 3. Let B is IFS. From [1] we have

$$\overline{B} = \{\langle x, \nu_B(x), \mu_B(x) \rangle \mid x \in X\}.$$

Then

$$(RIFIS_{\overline{B}}^{1(\lambda_1, \lambda_2)}) \int_A f dm = \overline{(LIFIS_B^{1(\lambda_2, \lambda_1)}) \int_A f dm},$$

$$(RIFIS_{\overline{B}}^{2(\gamma_1, \gamma_2)}) \int_A f dm = \overline{(LIFIS_B^{2(\gamma_2, \gamma_1)}) \int_A f dm},$$

where $\overline{\langle a, b \rangle} = \langle b, a \rangle$.

$$\begin{aligned}
& \text{Proof. } (RIFIS_{\overline{B}}^{1(\lambda_1, \lambda_2)}) \int_A f dm = \\
& < \lambda_1 \sup_{x \in X} [\nu_B(x) \wedge m(A \cap F_{\mu_B(x)})], \lambda_2 \inf_{x \in X} [\mu_B(x) \vee m(A \cap F_{\nu_B(x)})] > = \\
& \frac{< \lambda_2 \inf_{x \in X} [\mu_B(x) \vee m(A \cap F_{\nu_B(x)})], \lambda_1 \sup_{x \in X} [\nu_B(x) \wedge m(A \cap F_{\mu_B(x)})] >}{} = \\
& \frac{(LIFIS_B^{1(\lambda_2, \lambda_1)}) \int_A f dm}{(LIFIS_B^{1(\lambda_2, \lambda_1)}) \int_A f dm} \\
& (RIFIS_{\overline{B}}^{2(\gamma_1, \gamma_2)}) \int_A f dm = \\
& < \gamma_1 \sup_{x \in X} [\nu_B(x) \wedge m(A \cap E_{\nu_B(x)})], \gamma_2 \inf_{x \in X} [\mu_B(x) \vee m(A \cap E_{\mu_B(x)})] > = \\
& \frac{< \gamma_2 \inf_{x \in X} [\mu_B(x) \vee m(A \cap E_{\mu_B(x)})], \gamma_1 \sup_{x \in X} [\nu_B(x) \wedge m(A \cap E_{\nu_B(x)})] >}{} = \\
& \frac{(LIFIS_B^{2(\gamma_2, \gamma_1)}) \int_A f dm}{(LIFIS_B^{2(\gamma_2, \gamma_1)}) \int_A f dm} \quad \square
\end{aligned}$$

Proposition 4. From [1] we have that

$$\begin{aligned}
C(B) &= \{\langle x, \sup_{y \in X} [\mu_B(y)], \inf_{y \in X} [\nu_B(y)] \rangle | x \in X\}, \\
I(B) &= \{\langle x, \inf_{y \in X} [\mu_B(y)], \sup_{y \in X} [\nu_B(y)] \rangle | x \in X\}, \\
C_\nu(B) &= \{\langle x, \mu_B(x), \inf_{y \in X} [\nu_B(y)] \rangle | x \in X\}, \\
I_\mu(B) &= \{\langle x, \inf_{y \in X} [\mu_B(y)], \nu_B(x) \rangle | x \in X\}, \\
C_\mu(B) &= \{\langle x, \sup_{y \in X} [\mu_B(y)], \min(1 - \sup_{y \in X} [\mu_B(y)], \nu_B(x)) \rangle | x \in X\}, \\
I_\nu(B) &= \{\langle x, \min(1 - \sup_{y \in X} [\nu_B(y)], \mu_B(x)), \sup_{y \in X} [\nu_B(y)] \rangle | x \in X\}.
\end{aligned}$$

Then the following inequalities are true.

- (1) $(RIFIS_{I(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{I_\mu(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq$
 $(RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{C_\nu(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{C(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm,$
- (2) $(RIFIS_{I(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{I_\nu(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq$
 $(RIFIS_B^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{C_\mu(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm \leq (RIFIS_{C(B)}^{1(\lambda_1, \lambda_2)}) \int_A f dm,$
- (3) $(RIFIS_{I(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_{I_\mu(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq$
 $(RIFIS_B^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_{C_\nu(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_{C(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm,$
- (4) $(RIFIS_{I(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_{I_\nu(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq$
 $(RIFIS_B^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_{C_\mu(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm \leq (RIFIS_{C(B)}^{2(\gamma_1, \gamma_2)}) \int_A f dm,$

$$(5) \quad (LIFIS_{I(B)}^{1(q_1,q_2)}) \int_A f dm \leq (LIFIS_{I_\mu(B)}^{1(q_1,q_2)}) \int_A f dm \leq \\ (LIFIS_B^{1(q_1,q_2)}) \int_A f dm \leq (LIFIS_{C_\nu(B)}^{1(q_1,q_2)}) \int_A f dm \leq (LIFIS_{C(B)}^{1(q_1,q_2)}) \int_A f dm,$$

$$(6) \quad (LIFIS_{I(B)}^{1(q_1,q_2)}) \int_A f dm \leq (LIFIS_{I_\nu(B)}^{1(q_1,q_2)}) \int_A f dm \leq \\ (LIFIS_B^{1(q_1,q_2)}) \int_A f dm \leq (LIFIS_{C_\mu(B)}^{1(q_1,q_2)}) \int_A f dm \leq (LIFIS_{C(B)}^{1(q_1,q_2)}) \int_A f dm,$$

$$(7) \quad (LIFIS_{I(B)}^{2(r_1,r_2)}) \int_A f dm \leq (LIFIS_{I_\mu(B)}^{2(r_1,r_2)}) \int_A f dm \leq \\ (LIFIS_B^{2(r_1,r_2)}) \int_A f dm \leq (LIFIS_{C_\nu(B)}^{2(r_1,r_2)}) \int_A f dm \leq (LIFIS_{C(B)}^{2(r_1,r_2)}) \int_A f dm,$$

$$(8) \quad (LIFIS_{I(B)}^{2(r_1,r_2)}) \int_A f dm \leq (LIFIS_{I_\nu(B)}^{2(r_1,r_2)}) \int_A f dm \leq \\ (LIFIS_B^{2(r_1,r_2)}) \int_A f dm \leq (LIFIS_{C_\mu(B)}^{2(r_1,r_2)}) \int_A f dm \leq (LIFIS_{C(B)}^{2(r_1,r_2)}) \int_A f dm.$$

Proof. It follows from

$$I(B) \subseteq I_\mu(B) \subseteq B \subseteq C_\mu(B) \subseteq C(B), \\ I(B) \subseteq I_\nu(B) \subseteq B \subseteq C_\nu(B) \subseteq C(B)$$

and proposition 1. \square

Definition 2. Let $(q_1, q_2) \in [0, 1] \times [0, 1]$, $q_1 + q_2 \leq 1$ for every $x \in X$ $f(x) \in [0, 1]$. (X, Σ, m) is measure space with σ -algebra Σ and general monotone measure m , $m(X) = 1$, $A \in \Sigma$. Let

$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$$

is IFS, where $\mu_B(x)$ and $\nu_B(x)$ are continuous functions on topological space (X, τ) . We define integral topological operators of closure, C' , C_0 and interior I' , I_0 .

$$C'(B) = \{ \langle x, q_1 K, q_2 L \rangle / x \in X \},$$

where

$$K = \sup_{y \in X} [\mu_B(y) \wedge m(A \cap F_{\mu_B(y)})], \\ L = \inf_{y \in X} [\nu_B(y) \vee m(A \cap E_{\nu_B(y)})].$$

$$I'(B) = \{ \langle x, q_1 k, q_2 l \rangle / x \in X \},$$

where

$$k = \inf_{y \in X} [\mu_B(y) \vee m(A \cap F_{\mu_B(y)})], \\ l = \sup_{y \in X} [\nu_B(y) \wedge m(A \cap E_{\nu_B(y)})].$$

$$I_1(B) = \{ \langle x, q_1 K', q_2 L' \rangle / x \in X \},$$

where

$$K' = \inf_{y \in X} [\mu_B(y) \wedge m(A \cap F_{\mu_B(y)})],$$

$$L' = \sup_{y \in X} [\nu_B(y) \vee m(A \cap E_{\nu_B(y)})].$$

If B is IFS with $\mu_B, \nu_B \in C(X)$, then we define intuitionistic fuzzy set

$$B' = \{\langle x, q_1(\mu_B(x) \wedge m(A \cap F_{\mu_B(x)})), q_2(\nu_B(x) \vee m(A \cap E_{\nu_B(x)})) \rangle / x \in X\},$$

Then we have

$$C'(B) = C(B')$$

and

$$I_1(B) = I(B'),$$

where C and I are topological operators on IFS. See [1].

$$C_0(B) = \{\langle x, K, L_0 \rangle / x \in X\},$$

where

$$K = \sup_{y \in X} [\mu_B(y) \wedge m(A \cap F_{\mu_B(y)})],$$

$$L_0 = \inf_{y \in X} [\nu_B(y) \wedge m(A \cap F_{\nu_B(y)})].$$

$$I_0(B) = \{\langle x, K', L'_0 \rangle / x \in X\},$$

where

$$K' = \inf_{y \in X} [\mu_B(y) \wedge m(A \cap F_{\mu_B(y)})],$$

$$L'_0 = \sup_{y \in X} [\nu_B(y) \wedge m(A \cap F_{\nu_B(y)})].$$

If B is IFS with $\mu_B, \nu_B \in C(X)$, then we define intuitionistic fuzzy set

$$B^1 = \{\langle x, \mu_B(x) \wedge m(A \cap F_{\mu_B(x)}), \nu_B(x) \wedge m(A \cap F_{\nu_B(x)}) \rangle / x \in X\}.$$

Then we have

$$C_0(B) = C(B^1)$$

and

$$I_0(B) = I(B^1),$$

where C and I are topological operators on IFS. See [1].

References

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