# REFUTABILITY OF PHYSICAL THEORIES: A NEW APPROACH ${ }^{1}$ 

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## §1. The problem

The problem concerns the empirical test of physical theories. Finding confirmations of consequences (especially predictions) of the theory is a widely accepted technique among working physicists. This issue is treated by Carnap [1] whose views are known as confirmationism.

In contrast to confirmationism and having in mind the supplanting of classical dynamics and theory of gravity by the relativity theory, Popper developed a view known as falsificationism about refutability as the only reliable procedure for empirical verification of physical theories [2].

The opinion has been established for a long time that there is an asymmetry between the procedures of corraboration and refutation in the sense that the latter is stronger. This is also a subject of discussion within the framework of the general criticism of falsificationism.

Notwitstanding the so far existing discussion on the issue, a special attention should be paid to the Duhem's thesis [3]. Considering a physical theory a collection of statements (hypotheses) allows to relate the case of disagreement with empirical data to one or another individual hypothesis rather than to the theory as a whole. This thesis reveals possibilities for modifying some of the theory's integral constituents in order to eliminate the disagreement with the data.

In symbols, this can be written as
(a) $\left.T_{1} \equiv((A \supset C) \& \neg C) \supset \neg A\right)$,
(b) $T_{2} \equiv((A \& B \supset C) \& \neg C \& B) \supset \neg A$,
(c) $T_{3} \equiv((A \& B \supset C) \& \neg C) \supset(\neg A \vee \neg B)$.

The case of replacement of one or more statements (premises) with their opposite is examined in [4].

## §2. A new approach

The above considerations could be generalized (extended) by representing the propositions as fuzzy ones within the framework of intuitionistic fuzzy logic [5,6] and accepting fuzzy values instead of truth based on complementing each other values of refutation and corraboration. This can be represented as follows.

To each proposition (see, e.g., [7]) we can assign its truth value: truth - denoted by 1, or falsity -0 . In the case of fuzzy logic this truth value is a real number in the interval $[0,1]$

[^0]and may be called "truth degree" of a particular proposition. Here we add one more value - "falsity degree" - which will be in the interval $[0,1]$ as well. Thus two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition $p$ with the following constraint to hold:
$$
\mu(p)+\nu(p) \leq 1
$$

Let this assignment be provided by an evaluation function $V$ defined over a set of propositions $S$ in such a way that:

$$
V(p)=\langle\mu(p), \nu(p)\rangle .
$$

Hence the function $V: S \rightarrow[0,1] \times[0,1]$ gives the truth and falsity degrees of all propositions in $S$.

When the values $V(p)$ and $V(q)$ of the propositions $p$ and $q$ are known, the evaluation function $V$ can be extended also for the operations " $\&$ ", " $\vee$ " through the definition :

$$
\begin{gathered}
V(\neg p)=\langle\nu(p), \mu(p)\rangle, \\
V(p \& q)=\langle\min (\mu(p), \mu(q)), \max (\nu(p), \nu(q))\rangle, \\
V(p \vee q)=\langle\max (\mu(p), \mu(q)), \min (\nu(p), \nu(q))\rangle, \\
V(p \supset q)=\langle\max (\nu(p), \mu(q)), \min (\mu(p), \nu(q))\rangle,
\end{gathered}
$$

It will be convenient to define for the propositions $p, q \in S$ :

$$
\begin{aligned}
\neg V(p) & =V(\neg p), \\
V(p) \wedge V(q) & =V(p \& q), \\
V(p) \sqcup V(q) & =V(p \vee q), \\
V(p) \rightarrow V(q) & =V(p \supset q) .
\end{aligned}
$$

For the needs of the discussion below we shall define the notion of tautology and of intuitionistic fuzzy tautology (IFT) through:

$$
\begin{aligned}
& \text { " } A \text { is a tautology" iff } V(A)=\langle 1,0\rangle \text {, } \\
& \text { " } A \text { is an IFT" iff if } V(A)=\langle a, b\rangle \text {, then } a \geq b .
\end{aligned}
$$

The following assertions are valid.
THEOREM: For every three propositional forms $A, B$ and $C$ :
(a) $((A \supset C) \& \neg C) \supset \neg A)$,
(b) $((A \& B \supset C) \& \neg C \& B) \supset \neg A$,
(c) $((A \& B \supset C) \& \neg C) \supset(\neg A \vee \neg B)$
are IFTs.
Proof: Let $V(A)=<\mu_{A}, \nu_{A}>, V(B)=<\mu_{B}, \nu_{B}>, V(C)=<\mu_{C}, \nu_{C}>$.
(a) $V\left(T_{1}\right)$
$=\left(\left(<\mu_{A}, \nu_{A}>\rightarrow<\mu_{C}, \nu_{C}>\right) \wedge<\nu_{C}, \mu_{C}>\right) \rightarrow<\nu_{A}, \mu_{A}>$
$=\left(<\max \left(\nu_{A}, \mu_{C}\right), \min \left(\mu_{A}, \nu_{C}\right)>\wedge<\nu_{C}, \mu_{C}>\right) \rightarrow<\nu_{A}, \mu_{A}>$
$=\left(<\min \left(\max \left(\nu_{A}, \mu_{C}\right), \nu_{C}\right), \max \left(\min \left(\mu_{A}, \nu_{C}\right), \mu_{C}\right)>\rightarrow<\nu_{A}, \mu_{A}>\right.$
$=\left(<\max \left(\min \left(\mu_{A}, \nu_{C}\right), \mu_{C}, \nu_{A}\right), \min \left(\max \left(\nu_{A}, \mu_{C}\right), \nu_{C}, \mu_{A}\right)>\right.$.

Now, we see that

$$
\begin{gathered}
X \equiv \max \left(\min \left(\mu_{A}, \nu_{C}\right), \mu_{C}, \nu_{A}\right)-\min \left(\max \left(\nu_{A}, \mu_{C}\right), \nu_{C}, \mu_{A}\right) \\
\geq \min \left(\mu_{A}, \nu_{C}\right)-\min \left(\nu_{C}, \mu_{A}\right)=0
\end{gathered}
$$

i.e. $T_{1}$ is an IFT.
(b) $V\left(T_{2}\right)$
$=\left(\left(\left(<\mu_{A}, \nu_{A}>\wedge<\mu_{B}, \nu_{B}>\right) \rightarrow<\mu_{C}, \nu_{C}>\right) \wedge<\nu_{C}, \mu_{C}>\right.$

$$
\left.\wedge<\mu_{B}, \nu_{B}>\right) \rightarrow<\nu_{A}, \mu_{A}>
$$

$$
=\left(<\max \left(\nu_{A}, \nu_{B}, \mu_{C}\right), \min \left(\mu_{A}, \mu_{B}, \nu_{C}>\wedge\right.\right.
$$

$$
\left.<\min \left(\nu_{C}, \mu_{B}\right), \max \left(\mu_{C}, \nu_{B}\right)>\right) \rightarrow<\nu_{A}, \mu_{A}>
$$

$$
=<\min \left(\nu_{C}, \mu_{B}, \max \left(\nu_{A}, \nu_{B}, \mu_{C}\right), \max \left(\mu_{C}, \nu_{B}\right)\right.
$$

$$
\min \left(\mu_{A}, \mu_{B}, \nu_{C}\right)>\rightarrow<\nu_{A}, \mu_{A}>
$$

$$
=<\max \left(\mu_{C}, \nu_{B}, \nu_{A}, \min \left(\mu_{A}, \mu_{B}, \nu_{C}\right)\right)
$$

$$
\min \left(\nu_{C}, \mu_{B}, \mu_{A}, \max \left(\nu_{A}, \nu_{B}, \mu_{C}\right)\right)>
$$

Let

$$
\begin{gathered}
Y \equiv \max \left(\mu_{C}, \nu_{B}, \nu_{A}, \min \left(\mu_{A}, \mu_{B}, \nu_{C}\right)\right)-\min \left(\nu_{C}, \mu_{B}, \mu_{A}, \max \left(\nu_{A}, \nu_{B}, \mu_{C}\right)\right) \\
\left.\geq \min \left(\mu_{A}, \mu_{B}, \nu_{C}\right)\right)-\min \left(\nu_{C}, \mu_{B}, \mu_{A}\right)=0
\end{gathered}
$$

Therefore $T_{2}$ is an IFT.
$(c) V\left(T_{3}\right)=\left(\left(\left(<\mu_{A}, \nu_{A}>\wedge<\mu_{B}, \nu_{B}>\right) \rightarrow<\mu_{C}, \nu_{C}>\right) \wedge<\nu_{C}, \mu_{C}>\right)$
$\rightarrow<\nu_{A}, \mu_{A}>\sqcup<\nu_{B}, \mu_{B}>$
$=\left(\left(<\min \left(\mu_{A}, \mu_{B}\right), \max \left(\nu_{A}, \nu_{B}\right)>\rightarrow<\mu_{C}, \nu_{C}>\right) \wedge<\nu_{C}, \mu_{C}>\right)$
$\rightarrow<\max \left(\nu_{A}, \nu_{B}\right), \min \left(\mu_{A}, \mu_{B}\right)>$
$=\left(<\max \left(\nu_{A}, \nu_{B}, \mu_{C}\right), \min \left(\mu_{A}, \mu_{B}, \nu_{C}\right)>\wedge<\nu_{C}, \mu_{C}>\right)$
$\rightarrow<\max \left(\nu_{A}, \nu_{B}\right), \min \left(\mu_{A}, \mu_{B}\right)>$
$=<\min \left(\max \left(\nu_{A}, \nu_{B}, \mu_{C}\right), \nu_{C}\right), \max \left(\min \left(\mu_{A}, \mu_{B}, \nu_{C}\right), \mu_{C}\right)>$
$\rightarrow<\max \left(\nu_{A}, \nu_{B}\right), \min \left(\mu_{A}, \mu_{B}\right)>$
$=<\max \left(\min \left(\mu_{A}, \mu_{B}, \nu_{C}\right), \mu_{C}, \nu_{A}, \nu_{B}\right)$,
$\min \left(\max \left(\nu_{A}, \nu_{B}, \mu_{C}\right), \nu_{C}, \mu_{A}, \mu_{B}\right)>$.
Now, we see that

$$
\begin{gathered}
Z \equiv \max \left(\min \left(\mu_{A}, \mu_{B}, \nu_{C}\right), \mu_{C}, \nu_{A}, \nu_{B}\right)-\min \left(\max \left(\nu_{A}, \nu_{B}, \mu_{C}\right), \nu_{C}, \mu_{A}, \mu_{B}\right) \\
\geq \min \left(\mu_{A}, \mu_{B}, \nu_{C}\right)-\min \left(\nu_{C}, \mu_{A}, \mu_{B}\right)=0
\end{gathered}
$$

i.e. $T_{3}$ is an IFT.

From the values of the expressions Y and Z we see, that $\mathrm{Y}=\mathrm{Z}$. On the other hand, easily it can be check directly, that $T_{2} \equiv T_{3}$ (i.e., $T_{2} \supset T_{3} \& T_{3} \equiv T_{2}$ ) is an IFT. Moreover, $T_{2} \equiv T_{3}$ is an ordinary tautology, too, i.e.,

$$
V\left(T_{2} \equiv T_{3}\right)=<1,0>
$$

Also, $T_{1} \supset T_{2}$ and $T_{1} \supset T_{3}$ are IFTs, but not always $T_{2} \supset T_{1}$ and $T_{3} \supset T_{1}$ are IFTs, while from the first order's point of view, $T_{1}, T_{2}$ and $T_{3}$ are tautologies. From this, we
can see that the intuitionistic fuzzy logic approach gives a possibility for a more detailed description of the situations than the standard (first-order) logic approach.

## §3. Consequences

Apart of the possibility of a new application of the approach at issue, there are at least two new conclusions to be drawn from it, namely:

- the existence of an additional relationship between confirmation and refutation. This concerns not only the actual proof of theories but more generally it is also, significant for overcoming the onesidedness of both confirmationalism and falsificationism.
- the schematic nature of the general solution of the question and accordingly the need for a specific fixing (definiting) of the truth values in every particular case, basing on a specific analysis.

Let us define for the propositional forms $A$ and $B$, for which $V(A)=<\mu_{A}, \nu_{A}>$ and $V(B)=<\mu_{B}, \nu_{B}>$, that:

$$
\begin{aligned}
& V(A) \leq V(B) \text { iff }\left(\mu_{A} \leq \mu_{B} \& \nu_{A} \geq \nu_{B}\right), \\
& V(A)>V(B) \text { iff }\left(\mu_{A}>\mu_{B} \& \nu_{A}<\nu_{B}\right) .
\end{aligned}
$$

Let us assume that the intuitionistic fuzzy values of $A \mu_{A}$ and $\nu_{A}$ be fixed. Then, from the form of $T_{2}$ we see that the bigger $\mu_{C}$ and the smaller $\nu_{C}$ (i.e. the more increasing the intuitionistic fuzzy truth of $\mu_{B}$ ), the more reliable is $T_{2}$.

Now, let us assume that $\mu_{A}$ be fixed. If $V(A) \leq V(B)$ or if $V(A) \leq V(\neg C)$, then the truth value of $T_{2}$ will not be changed. If $V(A)>V(B)$ or if $V(A)>V(\neg C)$, then the truth value of $T_{2}$ can increase.

Let us assume that $\nu_{A}$ be fixed. If $V(A) \geq V(B)$ or if $V(B) \leq V(\neg C)$, then the truth value of $T_{2}$ will not be changed. If $V(A)<V(B)$ or if $V(B)<V(\neg C)$, then the truth value of $T_{2}$ can increase.

From above we see that if $\mu_{B}$ is fixed, then if $V(A) \geq V(\neg C)$ or if $V(B) \geq V(\neg C)$, then the truth value of $T_{2}$ will not be changed, but if $V(A)<V(\neg C)$ or if $V(B)<V(\neg C)$, then the truth value of $T_{2}$ can increase.

## §4. An example

We shall apply in retrospection the obtained result to a characteristic situation concerning the testing of the Special Theory of Relativity (STR). Its empirical basis comprises three kinds of results which may be rank-ordered according to their significance, as follows: confirmation of predictions, explanation of new facts, and new explanation of well-known facts.

As to disproving or embaracing facts one may distinguish three successive stages, namely:
A. Immediatly after the framing of the STR, represented by the findings of W.Kaufmann's experiments (1906).
B. In the 20 -ies and the 30 -ies when the positive results establishing the absolute motion of the Earth were annonced (D.Miller) [8]
C. Since the middle 60 -ies when the possibility to measure the velocities of cosmic objects relative to the background cosmic radiation was stated, as well as superluminous velocities (tachions) were admitted, etc. [9].

The most dramatic, interesting and instructive seems to be the first stage. Kaufmann's attempt to establish the dependence of the electron mass increase with the velocity did not confirm the consequence of the electron theory and the STR. This knocked out Lorentz who conceived it as a failure of his theory, i.e. he proceeded along the lines of falsificationism.

Another is Einstein's attitude, viz. that "only after a more diverse body of observations becomes available will be possible to decide with confidence whether the systematic deviations are due to a not yet recognized source of errors or to the circumstance that the foundations of the theory of relativity do not correspond to the facts" [10]. This is an evaluation along the lines of our truth function, wherein - even at this stage - the empirical support of the theory - prevails over the alleged disproof.

Somewhat similar is the situation after the report of positive results of the repeated Michelson's experiments. By then the STR is well-founded to such an extent that the physical community did not attach almost any significance to these communications. Acctually the value of V with a prevailing confirmation is adopted.

## References:

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[^0]:    ${ }^{1}$ The present paper has been prepared about a week before the sudden death of Prof. A. Polikarov (9 Oct. 1921 - 16 March 2000), who was one of the greatest Bulgarian philosophers and physicists, Academician in the Bulgarian Academy of Sciences. Prof. Polikarov's relatives and his coauthor in this research decided that despite the two-year delay, this paper should be published.

