Quicksort Algorithm Using Generalized Nets

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Abstract: The generalized net (GN) is used to create a model of Quicksort algorithm. The model can be used to understand and analyze sorting algorithms and to take into account the specificity of the chosen algorithm. The use of GN as a modeling tool is especially suitable because of the parallel implementation of the modeling algorithm. This paper initiates a study and comparative analysis of various sorting algorithms presented with GN tools.

Keywords: Generalized nets, Modelling, Sorting algorithms, Quicksort algorithm. **AMS Classification:** 68Q85, 68P10.

1 Introduction

Quicksort is an algorithm for sorting arrays of data, with average complexity of $O(n \log n)$. It is perfect for parallel execution and the ideal number of working threads would be equal to the number of elements in the array.

The nature of the algorithm itself lays in choosing an element of the array to become a pivot and then braking the rest of the array into two new ones. In one of the new arrays we will put all the elements smaller or equal to the pivot and in the other – we put all the rest. These steps are then repeated, until we are left with arrays of size one, since arrays of size one are by definition sorted. All of these arrays of size one are then merged together relatively to their pivots. The parallelism of the process comes from the fact that each of the splits and subsequently each of the merges can be done by a separate processor.

2 The Apparatus of Generalized Nets

Generalized Nets GNs, (see [1-3]) are extensions of Petri nets and Petri net modifications. GNs nets constitute a discrete tool for universal description of adaptable, flexible, structured and reusable models of complex systems with many different and interacting components, not necessarily of the homogeneous structure and origin, usually involved in parallel, simultaneous activities. Generalized nets represent a significant extension and generalization of the concept of Petri nets, as well as of other Petri nets extensions and modifications. A generalized net consists

of: static and dynamic components, temporal components, and memory components.

The static component consists of objects called transitions, which have input and output places. Two transitions can share a place, but every place can be an input of at most one transition and can be an output of at most one transition. The dynamic component consists of tokens, which act as information carriers and can occupy a single place at every moment of the GN execution. The tokens pass through the transition from one input to another output place; such an ordered pair of places is called transition arc. The tokens' movement is governed by conditions (predicates), contained in the predicate matrix of the transition.

The information carried by a token is contained in its characteristics, which can be viewed as an associative array of characteristic names and values. The values of the token characteristics change in time according to specific rules, called characteristic functions. Every place possesses at most one characteristic function, which assigns new characteristics to the incoming tokens. Apart from movement in the net and change of the characteristics, tokens can also split and merge in the places.

3 A GN-model

The GN-model (Figure 1) contains 3 transitions and 8 places. Initially, there is one α -token that is located in place L_1 with initial characteristic: "*indexed array of known size*".



Figure 1. GN model of Quicksort algorithm

Transition Z_1 checks the number of elements in the array and chooses one of them to become the

pivot.

$$Z_{1} = \langle \{L_{1}, L_{4}, L_{5}, L_{6}\}, \{L_{2}, L_{3}, L_{7}\}, r_{1} \rangle$$

$$r_{1} = \frac{L_{2} \qquad L_{3} \qquad L_{7}}{L_{1} \qquad W_{1,2} \qquad W_{1,3} \qquad W_{1,7}}$$

$$L_{4} \qquad W_{4,2} \qquad W_{4,3} \qquad W_{4,7}$$

$$L_{5} \qquad W_{5,2} \qquad W_{5,3} \qquad W_{5,7}$$

$$L_{6} \qquad false \qquad false \qquad false$$

where

 $W_{1,2} = W_{1,3} = W_{4,2} = W_{4,3} = W_{5,2} = W_{5,3}$ = The size of the array is greater than one. $W_{1,7} = W_{4,7} = W_{5,7}$ = The size of the array is less than or equal to one.

After passing through this transition, the α -token is divided in two: at position L_3 we get a new β -token with characteristic - "pivot" and at position L_2 we get the remaining array. At L_7 are kept the elements of size one.

The transition Z_2 splits the array into two new tokens and adds the characteristics - "parent name and left or right son". In the left array we enter all the values that are smaller or equal to the pivot and in the right one we enter the values greater than the pivot.

$$Z_{2} = \langle \{L_{2}, L_{3}\}, \{L_{4}, L_{5}, L_{6}\}, r_{2} \rangle$$

$$r_{2} = \frac{L_{4}}{L_{2}} \frac{L_{5}}{W_{2,4}} \frac{L_{5}}{W_{2,5}} \frac{L_{6}}{false}$$

$$L_{3} false false true$$

where

 $W_{2,4}$ = The value is less than or equal to the pivot.

 $W_{2,5} = \neg W_{2,4}.$

The β -tokens go to position L_6 .

Transition Z_3 creates a new array and fills it with all the elements of size one by positioning the left and right sons of each pivot accordingly at its sides.

$$Z_3 = \langle \{L_7\}, \{L_8\}, r_3 \rangle$$
$$r_3 = \frac{|L_8|}{|L_7|} \frac{|L_8|}{|true|}.$$

4 Conclusion

The present paper describes generalized net model of the Quicksort algorithm. The opportunity of using GNs as a tool for modelling such process is analyzed as well. The model has the purpose to help the users to understand and analyze the specifics of this sorting algorithm. This GN model gives us the opportunity to use the implementation of various analyses and statistics in order to generate ideas for enhancing this algorithm. Extra model parameters and additional tokens' characteristics can be introduced, while considering factors influencing of the process/

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