Notes on Intuitionistic Fuzzy Sets
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The convergence of intuitionistic fuzzy sets

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Abstract: In the present paper, we first introduce a new intuitionistic fuzzy distance. Relationships between three kinds of convergences compared to this distance are studied in this paper. We will give necessary and sufficient conditions to have a convergence equivalence for these four metrics.
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1 Introduction

The concept of intuitionistic fuzzy sets is introduced by Atanassov [1]. This notion is a generalization of the notion of fuzzy sets. By the graph of membership and nonmembership we can analytically give a sense of the distance between two sets of this type, hence we can construct a metric space and arrive to understand some topological properties of this space. As we know, the study of a sequences in such a space has an important role. In [8] the convergence of fuzzy sets is studied. The author has used a metric defined as the Hausdorff metric between the supported endographs, or shortly sendographs, of fuzzy sets. Kaleva in [7] investigates three kinds of convergences on a space of fuzzy sets, and the relationships between these convergences are studied.

The authors in [6] build the concept of intuitionistic fuzzy metric space and intuionistic fuzzy numbers. In [5], Melliani introduced the extension of Hukuhara difference in the intuitionistic fuzzy case. We will use the idea of O. Kaleva as an inspiration in order to study the convergence of intuitionistic fuzzy sets with respect to a new distance defined below.

This paper is organized as follows. In Section 2, we recall some notions and properties concerning the intuitionistic fuzzy sets. The new metric on the set of all intuitionistic fuzzy sets take place in Section 3. Section 4 is specific to comparing the kind of convergence.

2 Preliminaries

Definition 2.1 ([6]). *The set of all intuitionistic fuzzy numbers is given by*

$$IF_1 = IF_1(\mathbb{R}) = \Big\{\langle u, v\rangle : \mathbb{R} \to [0,1]^2,\ 0 \leq u + v \leq 1\Big\}$$

with the following conditions:

1. *For each $\langle u, v\rangle \in IF_1$ is normal, i.e., $\exists x_0, x_1 \in \mathbb{R}$, such that $u(x_0) = 1$ and $v(x_1) = 1$.*
2. *For each $\langle u, v\rangle \in IF_1$ is a convex intuitionistic set, i.e., u is fuzzy convex and v is fuzzy concave.*
3. *For each $\langle u, v\rangle \in IF_1$, u is lower continuous and v is upper continuous.*
4. *$cl\,\{x \in \mathbb{R},\ v(x) < 1\}$ is bounded.*

Definition 2.2 ([6]). *For $\alpha \in [0,1]$, we define the (α, β)-cut of $\langle u, v\rangle \in IF_1$ by*

$$[\langle u, v\rangle]^{\alpha,\beta} = \{x \in X,\ u(x) \geq \alpha \quad and \quad v(x) \leq \beta\},$$

where $\quad (\alpha, \beta) \in L^* = \{(x, y) \in \mathbb{R}^2, \quad 0 \leq x + y \leq 1\}$.

It is clear that the following proposition holds.

Proposition 2.3.

$$[\langle u, v\rangle]^{\alpha,\beta} = [\langle u, v\rangle]^{\alpha} \cap [\langle u, v\rangle]_{\beta},$$

where $[\langle u, v\rangle]^{\alpha} = \{x \in X,\ u(x) \geq \alpha\} \quad$ *and* $\quad [\langle u, v\rangle]_{\beta} = \{x \in X,\ v(x) \leq \beta\}$

Remark 2.4. *If $\alpha + \beta = 1, \quad$ then $\quad [\langle u, v\rangle]^{\alpha} \subset [\langle u, v\rangle]_{\beta}$.*

We get the following proposition.

Proposition 2.5. *For all $\langle u, v\rangle, \langle u', v'\rangle \in IF_1$, $\forall \alpha, \beta \in L^*$, we have*

$$\langle u, v\rangle = \langle u', v'\rangle \iff [\langle u, v\rangle]^{\alpha,\beta} = [\langle u', v'\rangle]^{\alpha,\beta}. \tag{2.1}$$

Proof. If $\beta = 1 - \alpha$, we have $u = u'$. Now, if $\alpha = 0$ we get $[1 - v]^{1-\beta} = [1 - v']^{1-\beta}$ which implies that $v = v'$. □

We define two operations on IF_1 by:

$$\langle u, v\rangle \oplus \langle u', v'\rangle = \langle u \vee v, u' \wedge v'\rangle,\ \forall \langle u, v\rangle, \langle u', v'\rangle \in \mathrm{IF}_1,$$
$$\lambda\langle u, v\rangle = \langle \lambda u, \lambda v\rangle,\ \forall \lambda \in \mathbb{R},\ \forall \langle u, v\rangle \in \mathrm{IF}_1 .$$

According to Zadeh extension, we have:

$$[\langle u, v\rangle \oplus \langle u', v'\rangle]^{\alpha,\beta} = [\langle u, v\rangle]^{\alpha,\beta} + [\langle u', v'\rangle]^{\alpha,\beta} .$$
$$[\lambda\langle u, v\rangle]^{\alpha,\beta} = \lambda\, [\langle u, v\rangle]^{\alpha,\beta} .$$

Definition 2.6. *The intuitionistic fuzzy zero is an intuitionistic fuzzy set defined by:*

$$\widetilde{0}(x) = \begin{cases} (1, 0) & x = 0. \\ (0, 1) & x \neq 0. \end{cases}$$

Definition 2.7. *We define the Hausdorff distance between two compact subsets of a metric space (X, d) by:*

$$d_H(A, B) = \max\left(\sup_{a\in A} \inf_{b\in B} \|a - b\|, \sup_{b\in B} \inf_{a\in A} \|a - b\|\right).$$

Theorem 2.8 ([7]). *If A and B are two compact subsets of a metric space (X, d), then there exists $a \in A$ and $b \in B$ such that*

$$d_H(A, B) = d(a, b).$$

Lemma 2.9 ([3]). *Let C_n be an increasing sequence of compact convex subsets of X. If this sequence admits a convergent subsequence to C compared to d_H, then*

$$d_H\left(C_n, C\right) \to 0$$

and

$$C = \bigcap_{n\geq 1} \overline{\bigcup_{m\geq n} C_m}$$

Theorem 2.10 ([6]). *Let $\mathcal{M} = \{M_\alpha, M^\alpha, \alpha \in [0, 1]\}$ be a family of subsets in $\mathbb{R}$ satisfying the following conditions:*

1. *If $\alpha \leq s$, then $M_s \subset M_\alpha$ and $M^s \subset M^\alpha$, for each $\alpha, s \in [0, 1]$.*
2. *M_α and M_s are nonempty compact convex sets in $\mathbb{R}$ for each $\alpha \in [0, 1]$.*
3. *For any non-decreasing sequence $\alpha_i \to \alpha$ on $[0, 1]$, we have $M_\alpha = \bigcap_i M_{\alpha_i}$ and $M^\alpha = \bigcap_i M^{\alpha_i}$.*

We define u and v by

$$u(x) = \begin{cases} 0, & x \notin M_0. \\ \sup\limits_{\alpha\in[0,1]} M_\alpha, & x \in M_0. \end{cases}$$

$$v(x) = \begin{cases} 1, & x \notin M^0. \\ 1 - \sup\limits_{\alpha\in[0,1]} M_\alpha, & x \in M^0. \end{cases}$$

Then

$$\langle u,v\rangle \in IF_1$$

with $M_\alpha = [\langle u,v\rangle]_\alpha$ *and* $M^\alpha = [\langle u,v\rangle]^\alpha$.

Remark 2.11 ([6]). 1. *The family* $\{[\langle u,v\rangle]_\alpha, [\langle u,v\rangle]^\alpha, \alpha\in[0,1]\}$ *satisfying statements* 1 *to* 3 *of the previous Theorem 2.10.*

2. *For all* $\alpha \in [0,1]$,

$$[\langle u,v\rangle]_\alpha \subset [\langle u,v\rangle]^\alpha.$$

Theorem 2.12 ([6]). *On* IF_1 *we can define the metric*

$$\begin{aligned} d_\infty\Big((u,v),(z,w)\Big) =\ & \frac{1}{4}\sup_{0\langle\alpha\leq 1}\left|\Big[(u,v)\Big]_r^+(\alpha)-\Big[(z,w)\Big]_r^+(\alpha)\right| \\ &+\frac{1}{4}\sup_{0\langle\alpha\leq 1}\left|\Big[(u,v)\Big]_l^+(\alpha)-\Big[(z,w)\Big]_l^+(\alpha)\right| \\ &+\frac{1}{4}\sup_{0\langle\alpha\leq 1}\left|\Big[(u,v)\Big]_r^-(\alpha)-\Big[(z,w)\Big]_r^-(\alpha)\right| \\ &+\frac{1}{4}\sup_{0\langle\alpha\leq 1}\left|\Big[(u,v)\Big]_l^-(\alpha)-\Big[(z,w)\Big]_l^-(\alpha)\right|. \end{aligned}$$

and

$$\begin{aligned} d_p\left(\langle u,v\rangle,\langle u',v'\rangle\right) = \Big(&\frac{1}{4}\int_0^1 |[\langle u,v\rangle]_l^+(\alpha)-[\langle u',v'\rangle]_l^+(\alpha)|d\alpha \\ &+\frac{1}{4}\int_0^1 |[\langle u,v\rangle]_r^+(\alpha)-[\langle u',v'\rangle]_r^+(\alpha)|d\alpha \\ &+\frac{1}{4}\int_0^1 |[\langle u,v\rangle]_l^-(\alpha)-[\langle u',v'\rangle]_l^-(\alpha)|d\alpha \\ &+\frac{1}{4}\int_0^1 |[\langle u,v\rangle]_r^-(\alpha)-[\langle u',v'\rangle]_r^-(\alpha)|d\alpha\Big)^{\frac{1}{p}}. \end{aligned}$$

For $p\in[1,\infty)$, *we have that* (IF_1, d_p) *is a complete metric space.*

We set that

$$\mathcal{S} = \Big\{\langle u,v\rangle \in \mathrm{IF}_1,\ x\mapsto u(x) \text{ is a concave function, and } x\mapsto v(x) \text{ is a convex function}\Big\}$$
$$\mathcal{S}_1 = \Big\{\langle u,v\rangle \in \mathcal{S}, [\langle u,v\rangle]^{1,0} \text{ has one element}\Big\}.$$

3 New metric on IF_1

We define the following mapping:

$$d^1 : \begin{cases} \mathrm{IF}_1\times\mathrm{IF}_1 \to \mathbb{R}^+ \\ (\langle u,v\rangle,\langle u',v'\rangle) \longmapsto \int_0^1\int_0^{1-\alpha} d_H\left([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\right)d\beta d\alpha \\ \qquad\qquad\qquad + \int_0^1\int_0^{1-\beta} d_H\left([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\right)d\alpha d\beta \end{cases}$$

Lemma 3.1. *The mapping d^1 is well defined.*

Proof. Let α_n a be nondecreasing sequance and β_n a decreasing sequence. We have

$$\lim_{n\to\infty}\left[\langle u,v\rangle^{\alpha_n,\beta_n}\right]=\bigcap_n\left[\langle u,v\rangle^{\alpha_n,\beta_n}\right],$$

which implies that $(\alpha,\beta)\to[\langle u,v\rangle]^{\alpha,\beta}$ is a measurable mapping. $\square$

Using the above argument, it is easy to show the following lemma.

Lemma 3.2. *Let $\beta\in[0,1]$. For all $\langle u,v\rangle,\langle u',v'\rangle\in IF_1$,*

$$\alpha\to\int_0^{1-\alpha}d_H\left([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\right)$$

is a continuous mapping.

Theorem 3.3. *(IF_1,d^1) is a metric space.*

Proof. It remains to prove that $d^1(\langle u,v\rangle,\langle u',v'\rangle)=0$ implies that $\langle u,v\rangle=\langle u',v'\rangle$.
By Lemma 3.2 and Remark 4.12,

$$\alpha\to\int_0^{1-\alpha}d_H\left([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\right),$$
$$\beta\to\int_0^{1-\beta}d_H\left([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\right),$$

are continuous and nonnegative functions. Thus

$$d^1(\langle u,v\rangle,\langle u',v'\rangle)=0\to\int_0^{1-\alpha}d_H\Big([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\Big)=\int_0^{1-\beta}d_H\Big([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\Big)=0.$$

If we set $\alpha=\beta=0$, it follows that $d_1(\langle u,v\rangle,\langle u',v'\rangle)=0$, which implies that $u=u'$ and $v=v'$. $\square$

Remark 3.4. *If $\alpha+\beta=1$, then by* [6] *we have*

$$d^1\left(\langle u,v\rangle,\langle u',v'\rangle\right)=d_1\left(\langle u,v\rangle,\langle u',v'\rangle\right).$$

4 Relation for convergence sets

Definition 4.1. *Let $\langle u_n,v_n\rangle$ be a sequence of IF_1. We say that this sequence is levelwise converging to $\langle u,v\rangle$ if $\forall(\alpha,\beta)\in L^*$,*

$$d_H\left([\langle u_n,v_n\rangle]^{\alpha,\beta},[\langle u,v\rangle]^{\alpha,\beta}\right)\to 0.$$

Theorem 4.2. *We have the equivalence between* (1) *and* (2)*, where:*
(1) *The levelwise convergence.*
(2) *The convergence in (IF_1,d^1).*

Proof. Let $(\langle u_n, v_n\rangle)$ be a sequence which converges levelwise to $\langle u, v\rangle$. Since L^* is compact, then we have the convergence in (IF_1, d^1).
Conversely, by [2, Theorem 4.9, p. 94] there is a subsequence $(\langle u_{n_k}, v_{n_k}\rangle)$ which converges to $\langle u, v\rangle$ a.e. to respect d_H, and by [6] we get $d_H\left(\langle u_n, v_n\rangle, \langle u, v\rangle\right) \longrightarrow 0$. □

We define the endograph and sendograph of an element $\langle u, v\rangle$ in IF_1 as follows

$$\operatorname{ende}\left(\langle u, v\rangle\right) = \left\{(x, (\alpha, \beta)) \in \mathbb{R} \times L^*, \quad x \in [\langle u, v\rangle]^{\alpha,\beta}\right\}$$

and

$$\operatorname{sende}\left(\langle u, v\rangle\right) = [\langle u, v\rangle]^{0,1} \times L^* \cap \operatorname{inde}\left(\langle u, v\rangle\right)$$

and we define the following metric

$$H\left(\langle u, v\rangle, \langle u', v'\rangle\right) = d^*\left(\operatorname{sende}\left(\langle u, v\rangle\right), \operatorname{sende}\left(\langle u', v'\rangle\right)\right),$$

where d^* is the Hausdorff distance on $\mathbb{R} \times L^*$.

Theorem 4.3. *The convergence in (IF_1, d^1) implies the convergence in (IF_1, H).*

Proof. Let $\langle u_n, v_n\rangle$ be a sequence of IF_1 that converges to $\langle u, v\rangle$ with respect to the distance d^1. Since d_H is positive then by [2, Theorem 4.9, p. 94] there is a subsequence $\langle u_{n_k}, v_{n_k}\rangle$ that converges a.e to $\langle u, v\rangle$ with respect to d_H. Now, we use the idea of [6, Lemma 2, p. 4], we conclude that $d_H\left(\langle u_{n_k}, v_{n_k}\rangle, \langle u, v\rangle\right) \to 0$. The proof continues in the same way as in [7, Theorem 3.1, p. 4]. □

Remark 4.4. *The converse of the above implication is not true.*

Example 4.5. *Define*

$$u(x) = \begin{cases} 1, & 0 \le x \le 1. \\ 0, & \textit{otherwise.} \end{cases}$$

$$v(x) = \begin{cases} 0, & 0 \le x \le 1. \\ 1, & \textit{otherwise.} \end{cases}$$

$$u_n(x) = \begin{cases} 1 + \frac{x-1}{n}, & 0 \le x \le 1. \\ 0, & \textit{otherwise.} \end{cases}$$

$$u(x) = \begin{cases} \frac{1-x}{n}, & 0 \le x \le 1. \\ 1, & \textit{otherwise.} \end{cases}$$

Clearly, $H\left(\langle u_n, v_n\rangle, \langle u, v\rangle\right) \to 0$. *But* $d^1\left(\langle u_n, v_n\rangle, \langle u, v\rangle\right) = (2 - (\alpha + \beta))\, d_H\left(L^*, [0, 1]\right)$.

Now, we will study the converse of the previous results.

Lemma 4.6. *Let $\langle u_n, v_n\rangle$ be a sequence in $\mathcal{S}$ converging levelwise to $\langle u, v\rangle$. Then*

$$\lim_{n\to\infty} d_H([\langle u_n, v_n\rangle]^{0,1}, [\langle u, v\rangle]^{0,1}) = 0.$$

Proof. Using Proposition 2.5 and the inequality $d_H(A\cap B, C\cap D) \leq d_H(A, C) + d_H(B, D)$, we have

$$d_H([\langle u_n, v_n\rangle]^{0,1}, [\langle u, v\rangle]^{0,1}) \leq d_H\left(u_n^0, u_0\right) + d_H\left((1-v)_n^0, (1-v)_0\right).$$

Now, [7, Lemma 3.1] answers the question. □

Lemma 4.7. *Let $\langle u, v\rangle \in \mathcal{S}$ and $(\alpha', \beta') \in L^*$ be fixed. Then*

$$g(\alpha, \beta) = d_H\left([\langle u, v\rangle]^{\alpha,\beta}, [\langle u, v\rangle]^{\alpha',\beta'}\right)$$

is continuous at (α, β).

Proof. Just using the following inequality

$$g(\alpha, \beta) \leq d_H\left(u^\alpha, [\langle u, v\rangle]^{\alpha',\beta'}\right) + d_H\left((1-v)^{1-\beta}, [\langle u, v\rangle]^{\alpha',\beta'}\right),$$

the proof follows from by [7, Lemma 3.2]. □

It is easy to observe that:

Corollary 4.8. *The function $g(\alpha, \beta) = d_H\left([\langle u, v\rangle]^{\alpha,\beta}, [\langle u, v\rangle]^{\alpha',\beta'}\right)$ is continuous.*

Also by the previous inequality we get the following lemma.

Lemma 4.9. *Let (α_n, β_n) be a sequence of L^* converging to $(\alpha, \beta) \in L^*$. Then under the assumptions of Lemma 4.6:*

$$\lim_{n\to\infty} d_H\left([\langle u, v\rangle]^{\alpha,\beta}, [\langle u_n, v_n\rangle]^{\alpha_n,\beta_n}\right) = 0.$$

Theorem 4.10. *The levelwise convergence implies the convergence in $(\mathcal{S}, d^1)$.*

Proof. Let $\epsilon > 0$ be arbitrary. Since $\langle u_n, v_n\rangle$ converges levelwise to $\langle u, v\rangle$, then

$$d_H(\langle u_n, v_n\rangle, \langle u, v\rangle) \to 0,$$

but there is $N \in \mathbb{N}$ such that

$$n \geq N \Rightarrow d^1(\langle u_n, v_n\rangle, \langle u, v\rangle) \leq 2d_H(\langle u_n, v_n\rangle, \langle u, v\rangle) \to 0,$$

as desired. □

Lemma 4.11. *Let $\langle u, v\rangle \in \mathcal{S}$. The mapping $\alpha \to \int_0^{1-\alpha} d_H\left([\langle u, v\rangle]^{\alpha,\beta}, [\langle u', v'\rangle]^{\alpha,\beta}\right)$ is a continuous function.*

Proof. If $(\alpha,\beta)=(0,1)$. Let $\alpha_n\to 0$ and $\beta_n\to 1$. We have

$$\lim_{n\to\infty} d_H\left([\langle u,v\rangle]^{\alpha_n,\beta_n},[\langle u,v\rangle]^{0,1}\right)=\lim_{n\to\infty} d_H\left(\{u\geq\alpha_n\}\cap\{v\leq\beta_n\},\{u\geq 0\}\cap\{v\leq 1\}\right).$$

By [7], we have

$$\begin{cases}\lim_{n\to\infty} d_H\left(\{u\geq\alpha_n\},\{u\geq 0\}\right)=0,\\ \lim_{n\to\infty} d_H\left(\{v\leq\beta_n\},\{v\leq 1\}\right)=0,\end{cases}$$

which implies that

$$\lim_{n\to\infty} d_H\left([\langle u,v\rangle]^{\alpha_n,\beta_n},[\langle u,v\rangle]^{0,1}\right)=0.$$

Thus, the continuity at $(0,1)$ is proved. Now, we consider the case when $\alpha>0$ and $\beta<1$.

$i)$ Let $\alpha_n\nearrow\alpha$ and $\beta_n\searrow\beta$ with α_n is a nondecreasing sequence and β_n is a decreasing sequence. By Lemma 2.9, we get

$$\begin{cases}d_H\left(\{u\geq\alpha_n\},\bigcap_{n\geq 1}\overline{\bigcup_{k\geq n}\{u\geq\alpha_k\}}\right) &=0,\\ d_H\left(\{v\leq\beta_n\},\bigcap_{n\geq 1}\overline{\bigcup_{k\geq n}\{v\leq\beta_k\}}\right) &=0,\end{cases}$$

which implies that

$$d_H\left(\{u\geq\alpha_n\}\cap\{v\leq\beta_n\},\bigcap_{n\geq 1}\overline{\bigcup_{k\geq n}\{u\geq\alpha_k\}\cap\{v\leq\beta_k\}}\right)=0.$$

Thus, $\alpha_n\nearrow\to\alpha$ and $\beta_n\searrow\beta$ where α_n is a nondecreasing sequence and β_n is a decreasing sequence.

$$d_H\left([\langle u,v\rangle]^{\alpha_n,\beta_n},\bigcap_{n\geq 1}\overline{\bigcup_{k\geq n}[\langle u,v\rangle]^{\alpha,\beta}}\right)=0,$$

which implies the right continuity.

$ii)$ Let $\alpha_n\to\alpha$ and $\beta_n\to\beta$ with α_n is a decreasing sequence and β_n is a nondecreasing sequence. We get

$$\bigcap_{n\geq 1}\overline{\bigcup_{k\geq n}[\langle u,v\rangle]^{\alpha,\beta}}=\bigcap_{n\geq 1}[\langle u,v\rangle]^{\alpha_n,\beta_n}.$$

By [9, Lemma 2.1], we have

$$\bigcap_{n\geq 1}\overline{\bigcup_{k\geq n}[\langle u,v\rangle]^{\alpha_k,\beta_k}}=[\langle u,v\rangle]^{\alpha,\beta},$$

which implies the left continuity. □

Remark 4.12. *If* $\alpha\in[0,1]$*, then* $\beta\to\int_0^{1-\beta} d_H\left([\langle u,v\rangle]^{\alpha,\beta},[\langle u',v'\rangle]^{\alpha,\beta}\right)$ *is a continuous function.*

Theorem 4.13. *Let $\langle u_n, v_n \rangle$ be a sequence in $\mathcal{S}_1$. If $\langle u_n, v_n \rangle$ converges to $\langle u, v \rangle \in \mathcal{S}_1$ in (S_1, H), then it also converges levelwise to $\langle u, v \rangle$.*

Proof. By the inequality used in the previous proof of all lemmas and [7, Theorem 3.2] we can affirm that $\langle u_n, v_n \rangle$ converges to $\langle u, v \rangle \in (\mathcal{S}_1, d^1)$. Thus, $d_H(\langle u_n, v_n \rangle, \langle u, v \rangle) \to 0$, a.e., we conclude by using the idea of [6, Lemma 2, p. 4]. □

Combining the previous theorems, we get the following theorem.

Theorem 4.14. *The levelwise convergence implies the equivalence between the convergence in $(d^1, \mathcal{S}_1)$ and $(H, \mathcal{S}_1)$.*

5 Conclusions

In this paper, we introduced a new distance on the set of all intuitionistic fuzzy numbers. We studied the convergence of intuitionistic fuzzy sets in such metric space and we compare this convergence with three kinds of metrics on IF_1. Finally, we gave necessary and sufficient conditions in order to have the equivalence between this new metric and the others.

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