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A note on intuitionistic fuzzy almost P-spaces

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Abstract: In this paper, the concepts of intuitionistic fuzzy almost P-spaces, intuitionistic fuzzy weak P-spaces and intuitionistic fuzzy almost GP-spaces are introduced and studied. We will discuss several characterizations of those spaces.
Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy P-spaces, Intuitionistic fuzzy almost P-spaces, Intuitionistic fuzzy almost GP-spaces.
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1 Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [14]. The theory of fuzzy topological spaces was introduced and developed by C. L. Chang [4]. The idea of intuitionistic fuzzy set was first published by Atannasov [1, 2, 3]. Çoker [5] defined the intuitionistic fuzzy topological spaces. The concepts of fuzzy almost P-spaces in fuzzy setting was introduced by the authors in [11, 12, 13]. The concept of intuitionistic fuzzy P-space was introduced and studied by the authors in [9, 10]. In this paper, the concepts of intuitionistic fuzzy weak P-spaces, intuitionistic fuzzy almost P-spaces, intuitionistic fuzzy P-spaces and intuitionistic fuzzy almost GP-spaces are introduced and studied.

2 Preliminaries

Definition 2.1. [1] Let X be a non-empty fixed set and I be the closed interval [0, 1]. An intuitionistic fuzzy set (IFS) A is an object of following form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\},$$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely) $\mu_A(x)$

and the degree of non-membership (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X\}$. Then

(i) $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$;

(ii) $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$;

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X\}$;

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X\}$.

Definition 2.3. [1] $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$.

Definition 2.4. [5] An intuitionistic fuzzy topology (IFT for short) in Çoker sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(T_1) $0_\sim, 1_\sim \in \tau$

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(T_3) $\bigcup_{i \in I} G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in I\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and each IFS in τ is known as a intuitionistic fuzzy open set (IFOS for short) in X.

Definition 2.5. [5] The complement $\bar{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.6. [5] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$, be an IFS in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$IFcl(A) = \cap \{K \mid K \text{ is and IFCS in } X \text{ and } A \subseteq K\}$$

and

$$IFint(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$$

Proposition 2.7. [5] Let (X, τ) be an IFTS and A be IFSs in X. Then the following properties hold:

(i) $1_\sim - IFcl(A) = IFint(1_\sim - A)$,

(ii) $1_\sim - IFint(A) = IFcl(1_\sim - A)$.

Definition 2.8. [7] An IFS A in an IFTS X is called an intuitionistic fuzzy preopen set (IFPOS for short) if $A \subseteq int(cl(A))$. The complement $\bar{A}$ of an IFPOS in X is called an intuitionistic fuzzy preclosed set (IFPCS for short) in X .

Definition 2.9. [7] Let $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ be an IFS in IFTS X. Then, $IFpreint(A) = \cup \{G \mid G$ is an IFPOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy preinterior of A; $IFprecl(A) = \cap \{K \mid K$ is and IFPCS in X and $A \subseteq K\}$ is an intuitionistic fuzzy preclosure of A.

Definition 2.10. [6] An intuitionistic fuzzy set A is called an intuitionistic fuzzy regular open set iff $A = IFint(IFcl((A))$; an intuitionistic fuzzy set K is called an intuitionistic fuzzy regular closed set iff $K = IFcl(IFint(K))$.

Definition 2.11. [10] An IFS A in an IFTS X is called an intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X, τ) such that $A \subseteq B \subseteq 1_{\sim}$

Definition 2.12. [10] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy nowhere dense set if there exists no intuitionistic fuzzy open set U in (X, τ) such that $U \subseteq IFcl\ (A)$. That is, $IFint\ IFcl\ (A) = 0_{\sim}$.

Theorem 2.13. [10] If A is an intuitionistic fuzzy nowhere dense set in IFTS (X, τ), then $1_{\sim} - A$ is an intuitionistic fuzzy dense set in (X, τ).

Definition 2.14. [10] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy G_δ set in (X, τ) if $A = \bigcap_{i=1}^{\infty} A_i$, where $A_i \in \tau$, for each i.

Definition 2.15. [10] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy F_σ set in (X, τ) if $A = \bigcap_{i=1}^{\infty} A_i$, where $A_i \in \tau$, for each i.

Definition 2.16. [10] Let (X, τ) be an IFTS. Then (X, τ) is called an intuitionistic fuzzy fuzzy Volterra space if $IFcl\ (\bigcap_{i=1}^{n} A_i) = 1_{\sim}$, where A_i are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, τ).

Definition 2.17. [10] Let (X, τ) be an IFTS. Then (X, τ) is called an intuitionistic fuzzy fuzzy weakly Volterra space if $IFcl\ (\bigcap_{i=1}^{\infty} A_i) \neq 0_{\sim}$ where A_i are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, τ).

Definition 2.18. [10] An IFTS (X, τ) is an intuitionistic fuzzy almost resolvable space if $\bigcap_{i=1}^{\infty} A_i = 1_{\sim}$, where the IFS, A_i in (X, τ) are such that $IFint\ (A_i) = 0_{\sim}$. Otherwise, (X, τ) is called an intuitionistic fuzzy almost resolvable.

Definition 2.19. [10] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy first category set if $A = \bigcup_{i=1}^{\infty} A_i$, where A_i are intuitionistic fuzzy nowhere dense sets in (X, τ).

Definition 2.20. [10] An IFTS (X, τ) is called an intuitionistic fuzzy first category space set if $1_{\sim} = \bigcup_{i=1}^{\infty} A_i$, where A_i are intuitionistic fuzzy nowhere dense sets in (X, τ). (X, τ) is called an intuitionistic fuzzy second category space if it is not an intuitionistic fuzzy first category space.

Definition 2.21. [10] An IFTS (X, τ) is called an intuitionistic fuzzy fuzzy P-space if countable intersection of intuitionistic fuzzy open sets in (X, τ) is intuitionistic fuzzy open in (X, τ).

Definition 2.22. [10] An IFTS (X, τ) is called an intuitionistic fuzzy submaximal space if for each intuitionistic fuzzy set A in (X, τ) such that $IFcl\ (A)=1_{\sim}$, then $A \in \tau$.

Definition 2.23. [10] An IFTS (X, τ) is called an intuitionistic fuzzy D-Baire space if every intuitionistic fuzzy first category set in (X, τ) is an intuitionistic fuzzy nowhere dense in (X, τ).

Definition 2.24. [9] Let (X, τ) be an IFTS. Then (X, τ) is called an intuitionistic fuzzy fuzzy weakly Volterra space if $IFcl\ (\bigcap_{i=1}^{\infty} A_i) \neq 0_{\sim}$, where A_i are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, τ).

3 Intuitionistic fuzzy almost P-spaces

Definition 3.1. An IFTS (X, τ) is called an intuitionistic fuzzy almost P-space if for every non-zero intuitionistic fuzzy G_δ set A in (X, τ), $IFint\ (A) \neq 0_\sim$ in (X, τ).

It is clear that in intuitionistic fuzzy topological spaces, we have the following implication:

Intuitionistic fuzzy P-space ⇒ Intuitionistic fuzzy almost P-space.

Proposition 3.2. If the IFTS (X, τ) is an intuitionistic fuzzy P-space, then $IFint\ (\bigcap_{i=1}^{\infty} A_i) = \bigcap_{i=1}^{\infty} A_i$, where the IFSs A_i are intuitionistic fuzzy open sets in (X, τ).

Proof. Let A_i be non-zero intuitionistic fuzzy open sets in an intuitionistic fuzzy P-space (X, τ). Then $A = \bigcap_{i=1}^{\infty} A_i$, is an intuitionistic fuzzy G_δ set in (X, τ). Since (X, τ) is an intuitionistic fuzzy P-space, the intuitionistic fuzzy G_δ set A is intuitionistic fuzzy open in (X, τ). Hence, we have $\mathrm{IF}int(A) = A$. This implies that $IFint(\bigcap_{i=1}^{\infty} A_i) = \bigcap_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} IFint(A_i)$, and hence $IFint$ $(\bigcap_{i=1}^{\infty} A_i) = \bigcap_{i=1}^{\infty} A_i$, where A_i are non-zero intuitionistic fuzzy open sets in (X, τ). □

Proposition 3.3. If A_i are intuitionistic fuzzy regular closed sets in an intuitionistic fuzzy P-space (X, τ), then $\mathrm{IF}cl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$.

Proof. Let A_i be intuitionistic fuzzy regular closed sets in an intuitionistic fuzzy P-space (X, τ). Then A_i are intuitionistic fuzzy closed sets in (X, τ), which implies that $(1_\sim - A_i)$ are intuitionistic fuzzy open sets in (X, τ). Then $\bigcap_{i=1}^{\infty}(1_\sim - A_i)$ is a non-zero intuitionistic fuzzy G_δ set in (X, τ). Hence $\mathrm{IF}int(\bigcap_{i=1}^{\infty}(1_\sim - A_i)) = \bigcap_{i=1}^{\infty}(1_\sim - A_i)$.
Therefore $1_\sim - \mathrm{IF}cl(\bigcup_{i=1}^{\infty} A_i) = 1_\sim - \bigcup_{i=1}^{\infty} A_i$.

Hence we have $\mathrm{IF}\ cl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$. □

Proposition 3.4. If each intuitionistic fuzzy G_δ set is an intuitionistic fuzzy dense set in an intuitionistic fuzzy submaximal space (X, τ), then (X, τ) is an intuitionistic fuzzy P-space.

Proof. Let A be an intuitionistic fuzzy G_δ set is in intuitionistic fuzzy submaximal space (X, τ). By hypothesis, A is an intuitionistic fuzzy dense set in (X, τ). Then, A is an intuitionistic fuzzy open set in (X, τ). Hence (X, τ) is an intuitionistic fuzzy P-space. □

Proposition 3.5. If $\mathrm{IF}cl(int(\mathrm{A})) = 1_\sim$, for each intuitionistic fuzzy G_δ set A in an intuitionistic fuzzy submaximal space (X, τ), then (X, τ) is an intuitionistic fuzzy P-space.

Proof. Let A be an intuitionistic fuzzy F_σ set in an intuitionistic fuzzy submaximal space (X, τ). Then $1_\sim - \mathrm{A}$ is an intuitionistic fuzzy G_δ set in (X, τ). Then $1_\sim$-$IFcl(IFint(1_\sim - \mathrm{A}))=1_\sim$.

This implies that $1_\sim - (1_\sim - \mathrm{IF}int(IFcl(A)))=0_\sim$. That is, $\mathrm{IF}int(\mathrm{IF}cl(\mathrm{A})) = 0_\sim$ and hence A is an intuitionistic fuzzy nowhere dense set in (X, τ). Hence (X, τ) is an intuitionistic fuzzy P-space. □

Definition 3.6. An intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy weak P-space if the countable intersection of intuitionistic fuzzy regular open sets in (X, τ) is intuitionistic fuzzy regular open sets in (X,τ). That is, $\bigcap_{i=1}^{\infty} A_i$ is an intuitionistic fuzzy regular open in (X,τ), where A_i are intuitionistic fuzzy regular open sets in (X,τ).

It is clear that in intuitionistic fuzzy topological spaces, we have the following implication:

Intuitionistic fuzzy P-space ⇒ Intuitionistic fuzzy weak P-space.

Proposition 3.7. An intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy weak P-space iff $\bigcup_{i=1}^{\infty} A_i$, where A_i are intuitionistic fuzzy regular closed sets in (X,τ) is an intuitionistic fuzzy regular closed in (X,τ).

Proof. Let (X,τ) be an intuitionistic fuzzy weak P-space. Then $IFint(IFcl(\bigcap_{i=1}^{\infty} A_i))$ = $\bigcap_{i=1}^{\infty} A_i$, where A_i are intuitionistic fuzzy regular open sets in (X,τ). Now

$$1_\sim - IFint(IFcl(\bigcap_{i=1}^{\infty} A_i))=1_\sim-\bigcap_{i=1}^{\infty} A_i, \text{ implies that}$$

$IFcl(IFint(\bigcup_{i=1}^{\infty}(1_\sim - A_i))) = \bigcup_{i=1}^{\infty}(1_\sim - A_i)$. Since $(1_\sim - A_i)$ is an intuitionistic fuzzy regular closed set in (X,τ). Hence $\bigcup_{i=1}^{\infty}(1_\sim - A_i)$ is an intuitionistic fuzzy regular closed in (X,τ).

Conversely, suppose that $IFcl(IFint(\bigcup_{i=1}^{\infty}(1_\sim - A_i))) = \bigcup_{i=1}^{\infty}(1_\sim - A_i)$, where $(1_\sim - A_i)$ are intuitionistic fuzzy regular closed set in (X,τ). Then

$$1_\sim - IFcl(IFint(\bigcup_{i=1}^{\infty}(1_\sim - A_i))) = 1_\sim - \bigcup_{i=1}^{\infty}(1_\sim - A_i),$$

which implies that

$$IFint(IFcl(\bigcap_{i=1}^{\infty}(1_\sim - (1_\sim - A_i))))= \bigcap_{i=1}^{\infty} A_i.$$

Hence (X,τ) is an intuitionistic fuzzy weak P-space. □

Proposition 3.8. If an intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy weak P-space, then $IFcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} IFcl(A_i)$, where A_i are non-zero intuitionistic fuzzy open sets in (X,τ).

Proof. Proof is similar to the Proposition 3.3. □

Definition 3.9. An intuitionistic fuzzy topological space (X,τ) is called an intuitionistic fuzzy almost Lindelöf space if every intuitionistic fuzzy open cover $(A_\alpha)_{\alpha\in\Lambda}$ of (X,τ) there exists a countable subcover $(A_n)_{n\in\mathbb{N}}$ such that $\bigcup_{n\in\mathbb{N}} IFcl(A_n) = 1_\sim$.

Definition 3.10. An intuitionistic fuzzy topological space (X,τ) is said to be intuitionistic fuzzy weakly Lindelöf space if every intuitionistic fuzzy open cover $(A_\alpha)_{\alpha\in\Lambda}$ of (X,τ) there exists a countable subcover $(A_n)_{n\in\mathbb{N}}$ such that $IFcl(\bigcup_{n\in\mathbb{N}} A_n) = 1_\sim$.

Obviously every intuitionistic fuzzy almost Lindelöf space is an intuitionistic fuzzy weakly Lindelöf space.

Proposition 3.11. If the intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy weak P-space, then every intuitionistic fuzzy weakly Lindelöf space is an intuitionistic fuzzy almost Lindelöf space.

Proof. Immediate from the definitions. □

Proposition 3.12. If an intuitionistic fuzzy topological space(X,τ) is an intuitionistic fuzzy P-space, then (X,τ) is an intuitionistic fuzzy weak P-space

Proof. Let A_i be intuitionistic fuzzy regular closed sets in (X,τ). Since (X,τ) is an intuitionistic fuzzy P-space, we have $IFcl(\bigcup_{i=1}^{\infty} A_i)= \bigcup_{i=1}^{\infty} A_i$. Now

$IFcl(IFint(\bigcup_{i=1}^{\infty} A_i))\subseteq IFcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$. Since $\mathrm{IF}cl(IFint(A_i)) = A_i$, then

$$\bigcup_{i=1}^{\infty} \mathrm{IF}cl(\mathrm{IF}int(A_i)) = \bigcup_{i=1}^{\infty} A_i,$$

which implies that $\bigcup_{i=1}^{\infty} A_i \subseteq \mathrm{IF}cl(\mathrm{IF}int(\bigcup_{i=1}^{\infty} A_i))$. Hence $\mathrm{IF}cl(\mathrm{IF}int(\bigcup_{i=1}^{\infty} A_i)) = \bigcup_{i=1}^{\infty} A_i$. By the Proposition 3.7, (X,τ) is an intuitionistic fuzzy weak P-space. □

Proposition 3.13. If A is an intuitionistic fuzzy nowhere dense and intuitionistic fuzzy G_δ set in an intuitionistic fuzzy topological space (X,τ), then (X,τ) is not an intuitionistic fuzzy almost P-space.

Proof. The proof is obvious. □

Definition 3.14. An intuitionistic fuzzy topological space (X,τ) is called an intuitionistic fuzzy almost GP-space if $IFint(A) \neq 0_\sim$, for each non-zero intuitionistic fuzzy dense and intuitionistic fuzzy G_δ set A in (X,τ). That is, (X,τ) is an intuitionistic fuzzy almost GP-space if every non-zero intuitionistic fuzzy G_δ set A in (X,τ) with $IFcl(A)=1_\sim$, $IFint(A) \neq 0_\sim$.

Proposition 3.15. If A is an intuitionistic fuzzy dense and intuitionistic fuzzy G_δ set in an intuitionistic fuzzy almost GP-space (X,τ), then $1_\sim - \mathrm{A}$ is not an intuitionistic fuzzy $dense$ set in (X,τ).

Proof. The proof is obvious. □

Proposition 3.16. If A is An intuitionistic fuzzy F_σ set in an intuitionistic fuzzy almost GP-space (X,τ) such that $\mathrm{IF}int(\mathrm{A}) = 0_\sim$, then A is not an intuitionistic fuzzy dense set in (X,τ).

Proof. The proof is obvious. □

Proposition 3.17. If $IFint(\bigcap_{i=1}^{\infty} A_i) \neq 0_\sim$, where A_i are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in an intuitionistic fuzzy topological space (X,τ), then (X,τ) is an intuitionistic fuzzy almost GP-space.

Proof. Let A_i (i=1,2,3,…) be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in an intuitionistic fuzzy topological space(X,τ)such that $\mathrm{IF}int(\bigcap_{i=1}^{\infty} A_i) \neq 0_\sim$. Then $IFint(\bigcap_{i=1}^{\infty} A_i) \subseteq \bigcap_{i=1}^{\infty} \mathrm{IF}int(A_i)$ implies that $0_\sim \neq \bigcap_{i=1}^{\infty} \mathrm{IF}int(A_i)$. Hence for the intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets A_i in (X,τ), we have $\mathrm{IF}int(A_i) \neq 0_\sim$.Therefore (X,τ) is an intuitionistic fuzzy almost GP-space. □

Proposition 3.18. If IF$cl(\bigcup_{i=1}^{\infty} A_i) \neq 1_{\sim}$, where A_i are intuitionistic fuzzy F_σ sets with IF$int(A_i$=$0_{\sim}$ in an intuitionistic fuzzy topological space (X,τ), then (X,τ) is an intuitionistic fuzzy almost GP-space.

Proof. The proof is similar to Proposition 3.17. □

Definition 3.19. An intuitionistic fuzzy topological space (X,τ) is called resolvable space if there exists an intuitionistic fuzzy dense set in (X,τ) such that $IFcl(1_{\sim} - \text{A}) = 1_{\sim}$.. Otherwise (X,τ) is called an intuitionistic fuzzy irresolvable space.

Proposition 3.20. If intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy irresolvable space, then (X,τ) is an intuitionistic fuzzy almost GP-space.

Proof. Let A be an intuitionistic fuzzy dense and intuitionistic fuzzy G_δ set in an intuitionistic fuzzy irresolvable space (X,τ). Since (X,τ) is an intuitionistic fuzzy irresolvable space, for the intuitionistic fuzzy dense set A in (X,τ), we have $IFcl(1_{\sim} - \text{A}) \neq 1_{\sim}$. But from $1_{\sim}$ – IF$int(\text{A}) =$ IF$cl(1_{\sim} - \text{A}) \neq 1_{\sim}$, we have IF$int(\text{A}) \neq 0_{\sim}$ and hence (X,τ) is an intuitionistic fuzzy almost GP-space. □

Proposition 3.21. If the intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy P-space, then (X,τ) is an intuitionistic fuzzy almost GP-space.

Proof. Obvious. □

Proposition 3.22. If the intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy almost P-space, then (X,τ) is an intuitionistic fuzzy almost GP-space.

Proof. Obvious. □

Proposition 3.23. If each intuitionistic fuzzy G_δ set is an intuitionistic fuzzy dense set in an intuitionistic fuzzy almost GP-space (X,τ), then (X,τ) is an intuitionistic fuzzy almost P-space.

Proof. Let A be a non-zero intuitionistic fuzzy G_δ set in an inuitionistic fuzzy almost GP-space (X,τ). Then A is a non-zero intuitionistic fuzzy dense and intuitionistic fuzzy G_δ set in an intuitionistic fuzzy almost GP-space (X,τ). Hence we have IF$int(\text{A}) \neq 0_{\sim}$ in (X,τ). Therefore, (X,τ) is an intuitionistic fuzzy almost P-space. □

It is clear that in intuitionistic fuzzy topological spaces, we have the following implications:

Intuitionistic fuzzy P-spaces ⇒ Intuitionistic fuzzy almost P-spaces
⇒ Intuitionistic fuzzy almost GP-spaces.

Proposition 3.24. If the intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy submaximal space, then (X,τ) is an intuitionistic fuzzy almost GP-space.

Proof. Let A be a non-zero intuitionistic fuzzy dense and intuitionistic fuzzy G_δ set in an intuitionistic fuzzy submaximal space (X,τ). Since (X,τ) is an intuitionistic fuzzy submaximal space, A is intuitionistic fuzzy open in (X,τ) and thus we have IF$int(\text{A}) \neq 0_{\sim}$. Hence (X,τ) is an intuitionistic fuzzy almost GP-space. □

Proposition 3.25. If the intuitionistic fuzzy topological space (X, τ) is an intuitionistic fuzzy almost GP-space, then (X, τ) is an intuitionistic fuzzy weakly Volterra spaces

Proof. Let A_i $(i = 1, 2, 3, \dots)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in an intuitionistic fuzzy almost GP-space (X, τ). Then, we have $IFint(A_i) \neq 0_\sim$. But from $\left(1_\sim - \mathrm{IF}int(A)\right) = IFcl(1_\sim - \mathrm{A}) \neq 1_\sim$, we have $\mathrm{IF}cl(1_\sim - \bigcap_{i=1}^{n} A_i) \neq 1_\sim$. That is, $IFint(\bigcap_{i=1}^{n} A_i) \neq 0_\sim$ and thus we have $IFcl(\bigcap_{i=1}^{n} A_i) \neq 0_\sim$. Hence (X, τ) is an intuitionistic fuzzy weakly Volterra space. □

Definition 3.26. Let (X, τ) be an intuitionistic fuzzy topological space. Then (X, τ) is called an intuitionistic fuzzy almost resolvable space if $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i are intuitionistic fuzzy open and $IFint(A_i) = 0_\sim$. Otherwise, (X, τ) is called an intuitionistic fuzzy almost irresolvable space.

Proposition 3.27. If the intuitionistic fuzzy topological space (X, τ) is an intuitionistic fuzzy almost irresolvable space, then (X, τ) is an intuitionistic fuzzy weakly Volterra space.

Proof. Immediate from the definitions. □

Definition 3.28. An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy strongly irresolvable space if $IFcl\left(\mathrm{IF}int(A)\right) = 1_\sim$ for each intuitionistic fuzzy dense set A in (X, τ).

Proposition 3.29. If the intuitionistic fuzzy topological space (X, τ) is an intuitionistic fuzzy strongly irresolvable space, then (X, τ) is an intuitionistic fuzzy almost GP-space.

Proof. Let A be a non-zero intuitionistic fuzzy dense and intuitionistic fuzzy G_δ set in an intuitionistic fuzzy strongly irresolvable space (X, τ). Since (X, τ) is an intuitionistic fuzzy strongly irresolvable space, we have $IFcl(IFint(A)) = 1_\sim$ and thus we have $IFint(A) \neq 0_\sim$. Hence (X, τ) is an intuitionistic fuzzy almost GP-space. □

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