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Determining intuitionistic fuzzy estimates for decision making in medical tasks

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Abstract: In the present paper, a new way for determining intuitionistic fuzzy estimates for a decision making process in medicine based on objective observations is proposed.
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1 Basic definitions and preliminaries

Intuitionistic fuzzy sets were proposed by K. Atanassov in 1983 [1] as an extension of the fuzzy sets proposed by L. Zadeh in [15]. Here we recall some basic definitions and properties:

Definition 1 (cf. [2])**.** Let $A \subset X$ and $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ are mappings such that for any $x \in X$ the inequality

$$\mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

holds. The set $\tilde{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\}$ is called intuitionistic fuzzy set (or Atanassov set) over X.

The mappings μ_A and ν_A are called membership and non-membership function, respectively.

It is usually accepted that the membership and non-membership functions are generated by expert evaluation. However, this is not always possible in practice.

In the present work an algorithm determining the membership and non-membership and hesitancy values of a given pattern to one of the two possible classes based on the parameters sensitivity (SEN) and specificity (SPE), positive predictive value (PPV) and negative predictive value (NPV) preliminary calculated over concrete medical data is proposed.

Let us be given 2 classes classes ω_p - class positives (the class of sick patients) and ω_n- class negatives (class of the "healthy" persons). In the terms of medical statistics, the accuracy of recognition is assessed by the parameters sensitivity, specificity, positive and negative predictive values which are calculated as follows:

$$\mathrm{SEN} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}}; \mathrm{SPE} = \frac{\mathrm{TN}}{\mathrm{TN}+\mathrm{FP}}; \mathrm{PPV} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}}; \mathrm{NPV} = \frac{\mathrm{TN}}{\mathrm{TN}+\mathrm{FN}},$$

where:

- TN (true negatives) — the number of healthy patients, determined as healthy;
- TP (true positives) — the number of sick patients, determined as sick;
- FP (false positives) -– the number of healthy patients, determined as sick;
- FN (false negatives) -– the number of sick patients, determined as healthy.

It is evident that the sensitivity determines the per cent of classified by the concrete criterion sick from all sick in the particular sample, i.e. how many of the sick are correctly classified. Specificity determines the per cent of classified by the concrete criterion healthy to all healthy in the particular sample, i.e. how many of the healthy were correctly identified. Positive predictive value is the ratio of correctly identified sick to all classified as such. The negative predictive value determines the per cent of real sick from all identified as such. This permits us to make the following definitions

Definition 2. Degree of membership to class "sick" by a given criterion i (feature) in the sense of IFS we will call the sensitivity with which it classifies the sick:

$$\mu_p^i = \frac{\mathrm{TP}^i}{\mathrm{TP}^i + \mathrm{FN}^i}$$

Definition 3. Degree of non-membership to class "sick" by a given criterion i (feature) in the sense of IFS we will call the (adjusted) negative predictive value, by which it recognizes the sick:

$$\nu_p^i = \frac{\mathrm{TN}^i}{\mathrm{TN}^i + \mathrm{FN}^i}(1 - \mu_p)$$

Definition 4. Degree of membership to class "healthy" by a given criterion i (feature) in the sense of IFS we will call the call the specificity with which it recognizes the sick:

$$\mu_n^i = \frac{\mathrm{TN}^i}{\mathrm{TN}^i + \mathrm{FP}^i}$$

Definition 5. Degree of non-membership to class "healthy" by a given criterion i (feature) in the sense of IFS we will call the (adjusted) positive predictive value, by which it recognizes the sick:

$$\nu_n^i = \frac{\text{TP}^i}{\text{TP}^i + \text{FP}^i}(1 - \mu_n)$$

The hesitancy degrees for each of the two classes by the i-th criterion are determined according to the formulas:

$$\pi_p^i = 1 - \mu_p^i - \nu_p^i$$
$$\pi_n^i = 1 - \mu_n^i - \nu_n^i$$

1.1 Proposed algorithm:

1. Over concrete data we determine SEN, SPE, PPV and NPV for each of the given criteria
2. We calculate the degrees of membership, non-membership and hesitancy to the two classes - μ_p^i, ν_p^i, π_p^i, μ_n^i, ν_n^i, π_n^i.
3. We calculate:
$$\mu_p = \frac{1}{n}\sum_{i=1}^{n}\mu_p^i; \nu_p = \frac{1}{n}\sum_{i=1}^{n}\nu_p^i$$
$$\mu_n = \frac{1}{n}\sum_{i=1}^{n}\mu_n^i; \nu_n = \frac{1}{n}\sum_{i=1}^{n}\nu_n^i,$$
where n is the number of criteria in the particular problem.
4. The classification rule to one of the two classes

- If $\mu_p > \mu_n$ and $\nu_p < \nu_n + \frac{\pi_n}{2}$, the pattern (patient) is assigned to class "sick".
- If $\mu_n > \mu_p$ and $\nu_n < \nu_p + \frac{\pi_p}{2}$, the pattern (patient) is assigned to class "healthy".

2 Example

2.1 Material and method

A retrospective study of 114 patients implanted ventricular shunt due to infantile hydrocephalus in Neurosurgery clinic at University Hospital "Alexandrovska" and later at University Multiprofile Hospital for Active Treatment "St. Ivan Rilski", Medical university – Sofia for a period of 20 years (1984-2003) has been conducted. The study excluded all patients with planned revisions. From the remaining 48% had underwent 3 or more revision. Despite the change of the applied surgical techniques, as well as the new models of valve systems, the patients with large number of revisions continue to be a challenge for the surgeon. The risk factors for shunt complications are:

- There are no commonly accepted data whether there exists a dependency between the implantation age and the frequency and type of complications for cerebrospinal fluid draining shunt systems at child's age. However, many authors point out the age as a predictor of complications [4, 6, 8, 12, 14]

Table 1: **Distribution of patients by age of shunt implantation**

Patient age at implantation	< 3 months	3 to 6 months	6 to 12 months	> year
Number of patients	23	32	23	36

- The interval from the valve implantation to the first revision [9–11, 14] as the researchers state maximal risk in the first few months after the surgery

Table 2: **Distribution based on the period between the main surgery and the first revision**

Period to the first revision	Number of patients
0 – 3 months	29
3 – 6 months	13
6 – 12 months	12
1 – 3 years	23
3 – 5 years	13
Over 5 years	24

- shunt system type (VAA Ventriculoatrial anastomosis and VPA Ventriculo-peritoneal anastomosis) – There is no consensus about the existence of relation between the type of shunt system and the frequency of complications for cerebrospinal fluid drainage systems [5, 13]

  The sick people, included in the retrospective study is divided in two groups according to the type of the initial shunt operation:

  - Arterial (VAA) – 42;
  - Peritoneal (VPA) – 72.

- kind of the implanted valve – the implanted valves are Pudenz, Cordis, Orbis Sigma, Hakim Precision, Omni shunt and in rare cases — Russian valve (by origin, no brand specified), Radionix, Electa, Accu-flo, Uni shunt, Akkura, NMT. The use of one or another valve model is dictated not only by surgeon's preference but often depends on socio-economic and other factors. For the purpose of the current research the valve systems are divided in two groups – with antisiphon mechanism (this are the new generation valves – Orbis Sigma, Cordis, Hakim Precision) and without antisiphon mechanism. According to many authors [3–5, 7] no significant differences in the frequency and type of complication is found between the different valve models.In the study the patients are divided according to the type of implanted valve system as follows:

  - with antisiphon mechanism – 70
  - without antisiphon mechanism – 44

- Type of the complication leading to the first revision. The patient distribution in our investigation is shown in the following table

Table 3: **Patient distribution according to the type of complication leading to the first revision**

Type of complication	Number of patients
Ventricular catheter malfunction	27
Cardiac catheter malfunction	25
Peritoneal catheter malfunction	22
Infection	13
Valve mechanism malfunction	13
Hyperdrainage	14

These results make the task of determining predictors of large number of complications for cerebrospinal shun systems especially topical.

Every patient is represented by a vector of n in number features (in our case $n = 5$), i.e.

$$X = (x_1, x_2, ..., x_n),$$

where: x_1 – age of the patient at shunt implantation; x_2 – interval of time from implantation to the first revision; x_3 – shunt system type; x_4 – type of complication leading to the first revision; x_5 – type of the implanted valve. We divide the patients in two classes. The first class, which we will call "healthy" is assigned every patient who has had less than 3 revisions for the time of existence of the shunt. The second class, which we call "sick" consists of all patients who have at least 3 revisions.

2.2 Results

Criteria for classification of the patients to one of the two classes:

- interval from implantation to I revision (in months) – interval < 6 months is considered as threshold. Patients undergoing revision in this interval are risky for future revisions
- age at shunting implantation – if the patients are implanted before the age of 6 months it is expected that they will have a larger number of revisions;
- shunt system type – VAA is arguably associated with greater risk of complications;
- type of the implanted valve – the new generation of valves with antisiphon mechanism are presumed to reduce the probability of revision;

- type of the complication leading to the first revision – potentially the most hazardous for the number of future complications are considered the ventricular catheter malfunction, the cardiac catheter malfunction and peritoneal catheter malfunction.

The values for the degrees of membership μ, non-membership ν and hesitancy π for each of the features, as well as the final degrees of membership and non-membership are given in the following table:

Table 4: **Features used in finding the intuitionistic fuzzy values**

Feature	μ_p	ν_p	π_p	μ_n	ν_n	π_n
Interval to first revision	0.61	0.2613	0.1287	0.6	0.216	0.184
Age at shunt implantation	0.51	0.2597	0.2303	0.42	0.232	0.348
Type of the shunt system	0.5	0.34	0.16	0.61	0.1677	0.2223
Implanted valve type	0.48	0.338	0.182	0.6	0.172	0.228
Type of Complication	0.69	0.1953	0.1147	0.38	0.2852	0.3348

3 Discussion

The proposed algorithm provides an objective a measurable way for the determination of the degrees of membership, non-membership and hesitancy in the decision making tasks in the medical practice. From the illustrated example it is seen that determining for the classification of the pattern (patient) is the combination of features, for which the patient is classified as sick and those that assign her/him to the class ”healthy”. The more a feature is expressed, the greater impact it would have on the total value for the respective class. In the concrete example it turns out that this are the time interval to the first revision, the age at the shunt implantation, and the type of complication which led to the first revision. According to the obtained results the type of the shunt system and the kind of implanted valve are not indicative for the number of following surgical interventions. This coincides with similar findings in the scientific findings in the literature.

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