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# Trapezoidal/triangular intuitionistic fuzzy numbers versus interval-valued trapezoidal/triangular fuzzy numbers and applications to multicriteria decision making methods

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**Abstract:** We establish relationships between the set of trapezoidal intuitionistic fuzzy numbers and the set of interval-valued trapezoidal fuzzy numbers and, on the other hand, between the set of triangular intuitionistic fuzzy numbers and the set of triangular fuzzy numbers. Based on these main results of the paper, the methods or procedures elaborated for interval-valued trapezoidal or triangular fuzzy numbers as input data can be easy transferred to the case of trapezoidal or triangular intuitionistic fuzzy numbers as input data. We exemplify by transferring an interval-valued trapezoidal multicriteria decision making method in a trapezoidal intuitionistic fuzzy method.
**Keywords:** Trapezoidal/triangular fuzzy number, Trapezoidal/triangular intuitionistic fuzzy number, Interval-valued trapezoidal/triangular fuzzy number.
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## 1 Introduction

Among various generalizations of fuzzy sets such as intuitionistic fuzzy sets and interval-valued fuzzy sets have gained more attention from researchers. On the other hand, the practitioners are interested to elaborate methods and procedures with input and output data in simple form. Between these, trapezoidal and triangular are often used because the computation is simplified and the results are easier interpreted and implemented.

The relationship between interval-valued fuzzy sets and intuitionistic fuzzy sets is well known (see [10]). In the present paper we give a bijection between the set of trapezoidal intuitionistic fuzzy numbers and a subset of interval-valued trapezoidal fuzzy numbers and a bijection between the set of triangular intuitionistic fuzzy numbers and a subset of interval-valued triangular fuzzy numbers. These bijections have good properties with respect to arithmetic operations and parameters associated with trapezoidal/triangular intuitionistic fuzzy numbers and interval-valued trapezoidal/triangular fuzzy numbers. The immediate benefit is that many problems with trapezoidal/triangular intuitionistic fuzzy numbers data can be solved by using an already elaborated method for interval-valued trapezoidal/triangular fuzzy case.

We illustrate the theoretical development by an application related with a multicriteria decision making problem inspired from [3].

## 2 Trapezoidal and triangular intuitionistic fuzzy numbers

In this section we recall some basic notions on trapezoidal and triangular intuitionistic fuzzy numbers.

**Definition 1.** *A trapezoidal fuzzy number ($TFN$) $A = (a_1, a_2, a_3, a_4)$ is a fuzzy set on $\mathbb{R}$ with the membership function given by*

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \textit{if } x \in [a_1, a_2) \\ 1, & \textit{if } x \in [a_2, a_3] \\ \frac{a_4-x}{a_4-a_3}, & \textit{if } x \in (a_3, a_4] \\ 0, & \textit{otherwise,} \end{cases}$$

*where $a_1 \leq a_2 \leq a_3 \leq a_4$.*

If $a_2 = a_3$ in the above definition then $A$ is called a triangular fuzzy number ($\Delta FN$). We denote by $(a_1, a_2, a_3)$ a $\Delta FN$. We denote by $TFN(\mathbb{R})$ and $\Delta FN(\mathbb{R})$ the sets of all $TFNs$ and all $\Delta FNs$, respectively.

We recall (see [1, 2]) that an intuitionistic fuzzy set in $X \neq \emptyset$ is an object $A$ given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle ; x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ satisfy the condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1,$$

for every $x \in X$.

The trapezoidal intuitionistic fuzzy numbers are important to quantify an ill-known information and can be easily employed in applications (see e.g. [8, 9, 13, 14]).

**Definition 2.** *(see e.g. [13]) A trapezoidal intuitionistic fuzzy number (TIFN)*

$$\widetilde{A} = \langle(\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4})\rangle$$

*is an intuitionistic fuzzy set on* $\mathbb{R}$*, with the membership function* $\mu_{\tilde{A}}$ *and the non-membership function* $\nu_{\tilde{A}}$ *defined as*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-\underline{a_1}}{\underline{a_2}-\underline{a_1}}, & \textit{if } x \in [\underline{a_1}, \underline{a_2}) \\ 1, & \textit{if } x \in [\underline{a_2}, \underline{a_3}] \\ \frac{\underline{a_4}-x}{\underline{a_4}-\underline{a_3}}, & \textit{if } x \in (\underline{a_3}, \underline{a_4}] \\ 0, & \textit{otherwise} \end{cases}$$

*and*

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{\overline{a_2}-x}{\overline{a_2}-\overline{a_1}}, & \textit{if } x \in [\overline{a_1}, \overline{a_2}) \\ 0, & \textit{if } x \in [\overline{a_2}, \overline{a_3}] \\ \frac{x-\overline{a_3}}{\overline{a_4}-\overline{a_3}}, & \textit{if } x \in (\overline{a_3}, \overline{a_4}] \\ 1, & \textit{otherwise} \end{cases}$$

*where* $\underline{a_1} \leq \underline{a_2} \leq \underline{a_3} \leq \underline{a_4}, \overline{a_1} \leq \overline{a_2} \leq \overline{a_3} \leq \overline{a_4}, \overline{a_1} \leq \underline{a_1}, \overline{a_2} \leq \underline{a_2}, \underline{a_3} \leq \overline{a_3}$ *and* $\underline{a_4} \leq \overline{a_4}$*.*

The conditions imposed on $\underline{a_i}, \overline{a_i}, i \in \{1,2,3,4\}$ assure that $\widetilde{A}$ is an intuitionistic fuzzy set. If $\overline{a_2} = \overline{a_3}$ (which implies $\underline{a_2} = \underline{a_3}$) then the $TIFN$ $\widetilde{A}$ is called a triangular intuitionistic fuzzy number ($\Delta IFN$). We denote by

$$\widetilde{A} = \langle (\underline{a_1}, \underline{a_2}, \underline{a_3}), (\overline{a_1}, \overline{a_2}, \overline{a_3}) \rangle$$

a $\Delta IFN$. If $\widetilde{A}$ is a $TIFN$ then $\mu_{\tilde{A}}$ and $1-\nu_{\tilde{A}}$, where $(1-\nu_{\tilde{A}})(x) = 1-\nu_{\tilde{A}}(x)$ for every $x \in \mathbb{R}$, are $TFNs$. If $\overline{a_i} = \underline{a_i} = a, i \in \{1,2,3,4\}$ then $\widetilde{A}$ can be identified with the trapezoidal fuzzy number $(a,a,a,a)$, the triangular fuzzy number $(a,a,a)$, or the real number $a$.

The addition and the scalar multiplication on intuitionistic fuzzy numbers (see [4] or [5]) become in the particular case of trapezoidal intuitionistic fuzzy numbers as follows:

$$\begin{aligned} &\widetilde{A} + \widetilde{B} = \langle (\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}) \rangle + \langle (\underline{b_1}, \underline{b_2}, \underline{b_3}, \underline{b_4}), (\overline{b_1}, \overline{b_2}, \overline{b_3}, \overline{b_4}) \rangle \\ &= \langle (\underline{a_1} + \underline{b_1}, \underline{a_2} + \underline{b_2}, \underline{a_3} + \underline{b_3}, \underline{a_4} + \underline{b_4}), (\overline{a_1} + \overline{b_1}, \overline{a_2} + \overline{b_2}, \overline{a_3} + \overline{b_3}, \overline{a_4} + \overline{b_4}) \rangle, \end{aligned}$$

$$\begin{aligned} \lambda \cdot \widetilde{A} &= \lambda \cdot \langle (\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}) \rangle \\ &= \langle (\lambda\underline{a_1}, \lambda\underline{a_2}, \lambda\underline{a_3}, \lambda\underline{a_4}), (\lambda\overline{a_1}, \lambda\overline{a_2}, \lambda\overline{a_3}, \lambda\overline{a_4}) \rangle \end{aligned}$$

if $\lambda \in \mathbb{R}, \lambda \geq 0$ and

$$\begin{aligned} \lambda \cdot \widetilde{A} &= \lambda \cdot \langle (\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}) \rangle \\ &= \langle (\lambda\underline{a_4}, \lambda\underline{a_3}, \lambda\underline{a_2}, \lambda\underline{a_1}), (\lambda\overline{a_4}, \lambda\overline{a_3}, \lambda\overline{a_2}, \lambda\overline{a_1}) \rangle \end{aligned}$$

if $\lambda \in \mathbb{R}, \lambda < 0$.

# 3 Interval-valued trapezoidal/triangular fuzzy numbers

We recall, an interval-valued fuzzy set $\tilde{\tilde{C}} = [\tilde{\tilde{C}}^L, \tilde{\tilde{C}}^U]$ is defined as (see e.g. [10, 12])

$$\tilde{\tilde{C}} = \{(x, [\mu_{\tilde{\tilde{C}}^L}(x), \mu_{\tilde{\tilde{C}}^U}(x)]); x \in X\},$$

where the pair of fuzzy sets $\mu_{\tilde{\tilde{C}}^L}$,

$$\mu_{\tilde{\tilde{C}}^U} : X \to [0,1],$$

called the lower fuzzy set of $\tilde{\tilde{C}}$ and the upper fuzzy set of $\tilde{\tilde{C}}$ satisfies $\tilde{\tilde{C}}^L \subseteq \tilde{\tilde{C}}^U$, that is $0 \leq \mu_{\tilde{\tilde{C}}^L}(x) \leq \mu_{\tilde{\tilde{C}}^U}(x) \leq 1$, for any $x \in X$.

**Definition 3.** *(see, e.g., [15]) An interval-valued trapezoidal fuzzy number ($IVTFN$) $\tilde{\tilde{A}}$ is an interval-valued fuzzy set on $\mathbb{R}$, defined by*

$$\tilde{\tilde{A}} = [\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U],$$

*where* $\tilde{\tilde{A}}^L = (a_1^L, a_2^L, a_3^L, a_4^L)$ *and* $\tilde{\tilde{A}}^U = (a_1^U, a_2^U, a_3^U, a_4^U)$ *are trapezoidal fuzzy numbers such that* $\tilde{\tilde{A}}^L \subseteq \tilde{\tilde{A}}^U$.

If $a_2^L = a_3^L$ and $a_2^U = a_3^U$ in the above definition then $\tilde{\tilde{A}}$ is a interval-valued triangular fuzzy number ($IV\Delta FN$). We denote by $\tilde{\tilde{A}} = \left[\left(a_1^L, a_2^L, a_3^L\right), \left(a_1^U, a_2^U, a_3^U\right)\right]$ a $IV\Delta FN$. If $a_i^L = a_i^U = a$ then $\tilde{\tilde{A}}$ can be identified with the trapezoidal fuzzy number $(a,a,a,a)$, the triangular fuzzy number $(a,a,a)$, or the real number $a$.

We denote by $IVTFN\,(\mathbb{R})$ the sets of all IVTFNs and by $IV\Delta FN\,(\mathbb{R})$ the sets of all ΔIFNs, respectively. It is immediate that the conditions $a_1^U \leq a_1^L$ and $a_4^L \leq a_4^U$ are necessary as $\tilde{\tilde{A}} \in IVTFN\,(\mathbb{R})$.

The addition and the scalar multiplication on interval-valued fuzzy numbers (see e.g. [15]) become in the particular case of interval-valued trapezoidal fuzzy numbers as follows:

$$\begin{aligned}\tilde{\tilde{A}} + \tilde{\tilde{B}} &= \left[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)\right] + \left[(b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U)\right] \\ &= \langle (a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U) \rangle,\end{aligned}$$

$$\begin{aligned}\lambda \cdot \tilde{\tilde{A}} &= \lambda \cdot \left[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)\right] \\ &= \left[(\lambda a_1^L, \lambda a_2^L, \lambda a_3^L, \lambda a_4^L), (\lambda a_1^U, \lambda a_2^U, \lambda a_3^U, \lambda a_4^U)\right]\end{aligned}$$

if $\lambda \in \mathbb{R}, \lambda \geq 0$ and

$$\begin{aligned}\lambda \cdot \tilde{\tilde{A}} &= \lambda \cdot \left[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)\right] \\ &= \left[(\lambda a_4^L, \lambda a_3^L, \lambda a_2^L, \lambda a_1^L), (\lambda a_4^U, \lambda a_3^U, \lambda a_2^U, \lambda a_1^U)\right]\end{aligned}$$

if $\lambda \in \mathbb{R}, \lambda < 0$.

# 4 Relations between trapezoidal (triangular) intuitionistic fuzzy numbers and interval-valued trapezoidal (triangular) fuzzy numbers and properties

The relationship between interval-valued fuzzy sets and intuitionistic fuzzy sets is well-known (see [10]). To prove a similar result in the case of IVTFNs and TIFNs, then in the case of $IV\Delta FNs$ and $\Delta IFNs$ we consider

$$IVTFN_*(\mathbb{R}) = \Big\{ \widetilde{\widetilde{A}} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)] \\ \in IVTFN(\mathbb{R}) : a_2^U \leq a_2^L \text{ and } a_3^L \leq a_3^U \Big\}$$

and

$$IV\Delta FN_*(\mathbb{R}) = \Big\{ \widetilde{\widetilde{A}} = [(a_1^L, a_2^L, a_3^L), (a_1^U, a_2^U, a_3^U)] \\ \in IV\Delta FN(\mathbb{R}) : a_2^L = a_2^U \Big\}.$$

**Proposition 4.** *The mapping* $\Phi : TIFN(\mathbb{R}) \to IVTFN_*(\mathbb{R})$ *defined as*

$$\Phi(\langle(\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4})\rangle) \\ = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)],$$

*where*

$$a_i^L = \underline{a_i}, i \in \{1, 2, 3, 4\}$$

*and*

$$a_i^U = \overline{a_i}, i \in \{1, 2, 3, 4\},$$

*is a bijection.*

*Proof.* It is immediate and $\Phi^{-1}$: $IVTFN_*(\mathbb{R}) \to TIFN(\mathbb{R})$ is defined as

$$\Phi^{-1}[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)] \\ = \langle(\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4})\rangle,$$

where

$$\underline{a_i} = a_i^L, i \in \{1, 2, 3, 4\}$$

and

$$\overline{a_i} = a_i^U, i \in \{1, 2, 3, 4\},$$

is the inverse of $\Phi$. □

As above, we obtain the following result.

**Proposition 5.** *The mapping* $\Omega : \Delta IFN(\mathbb{R}) \to IV\Delta FN_*(\mathbb{R})$ *defined as*

$$\Omega(\langle(\underline{a_1}, \underline{a_2}, \underline{a_3}), (\overline{a_1}, \overline{a_2}, \overline{a_3})\rangle)$$
$$= [\left(a_1^L, a_2^L, a_3^L\right), \left(a_1^U, a_2^U, a_3^U\right)],$$

*where*

$$a_i^L = \underline{a_i}, i \in \{1, 2, 3\}$$

*and*

$$a_i^U = \overline{a_i}, i \in \{1, 2, 3\},$$

*is a bijection.*

Any parameter (real number or real interval) associated with a fuzzy number can be extended in a natural way to an intuitionistic fuzzy number or to an interval-valued fuzzy number (see e.g. [4, 6]) and then particularized to trapezoidal (triangular) intuitionistic fuzzy numbers and interval-valued trapezoidal (triangular) fuzzy numbers, respectively. For example, the expected value $EV$ introduced for fuzzy numbers in [11] becomes

$$EV\left(\widetilde{A}\right) = \frac{1}{8}\left(\sum_{i=1}^{4}\underline{a_i} + \sum_{i=1}^{4}\overline{a_i}\right)$$

for $\widetilde{A} = \langle(\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}), (\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4})\rangle$ and

$$EV\left(\widetilde{\widetilde{A}}\right) = \frac{1}{8}\left(\sum_{i=1}^{4}a_i^L + \sum_{i=1}^{4}a_i^U\right)$$

for $\widetilde{\widetilde{A}} = \left[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)\right]$. It is immediate that

$$EV\left(\widetilde{A} + \widetilde{B}\right) = EV\left(\widetilde{A}\right) + EV\left(\widetilde{B}\right),$$
$$EV\left(\widetilde{\widetilde{A}} + \widetilde{\widetilde{B}}\right) = EV\left(\widetilde{\widetilde{A}}\right) + EV\left(\widetilde{\widetilde{B}}\right)$$

and

$$EV\left(\lambda \cdot \widetilde{A}\right) = \lambda EV\left(\widetilde{A}\right),$$
$$EV\left(\lambda \cdot \widetilde{\widetilde{A}}\right) = \lambda EV\left(\widetilde{\widetilde{A}}\right),$$

for every $\widetilde{A}, \widetilde{B} \in TIFN(\mathbb{R}), \widetilde{\widetilde{A}}, \widetilde{\widetilde{B}} \in IVTFN(\mathbb{R})$ and $\lambda \in \mathbb{R}$. We choose to discuss in detail the case of the expected value because it is considered (see [7]) a very good option in the ranking of fuzzy numbers. We introduce the ranking of trapezoidal (triangular) intuitionistic fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ and of interval-valued trapezoidal (triangular) fuzzy numbers $\widetilde{\widetilde{A}}$ and $\widetilde{\widetilde{B}}$ by

$$\widetilde{A} \leq_{EV} \widetilde{B} \Leftrightarrow EV\left(\widetilde{A}\right) \leq EV\left(\widetilde{B}\right)$$

and

$$\widetilde{\widetilde{A}} \leq_{EV} \widetilde{\widetilde{B}} \Leftrightarrow EV\left(\widetilde{\widetilde{A}}\right) \leq EV\left(\widetilde{\widetilde{B}}\right)$$

The following properties are very easy to be proved.

**Theorem 6.** *(i)* $\Phi\left(\widetilde{A}+\widetilde{B}\right)=\Phi\left(\widetilde{A}\right)+\Phi\left(\widetilde{B}\right)$*, for every* $\widetilde{A},\widetilde{B}\in TIFN\left(\mathbb{R}\right);$

$(ii)$ $\Phi\left(\lambda\cdot\widetilde{A}\right)=\lambda\cdot\Phi\left(\widetilde{A}\right)$*, for every* $\widetilde{A}\in TIFN\left(\mathbb{R}\right)$ *and* $\lambda\in\mathbb{R};$

$(iii)$ $EV\left(\Phi\left(\widetilde{A}\right)\right)=EV\left(\widetilde{A}\right)$*, for every* $\widetilde{A}\in TIFN\left(\mathbb{R}\right);$

$(iv)$ $\widetilde{A}\leq_{EV}\widetilde{B}\Leftrightarrow\Phi\left(\widetilde{A}\right)\leq_{EV}\Phi\left(\widetilde{B}\right);$

$(v)$ $\Omega\left(\widetilde{A}+\widetilde{B}\right)=\Omega\left(\widetilde{A}\right)+\Omega\left(\widetilde{B}\right)$*, for every* $\widetilde{A},\widetilde{B}\in\Delta IFN\left(\mathbb{R}\right);$

$(vi)$ $\Omega\left(\lambda\cdot\widetilde{A}\right)=\lambda\cdot\Omega\left(\widetilde{A}\right)$*, for every* $\widetilde{A}\in\Delta IFN\left(\mathbb{R}\right)$ *and* $\lambda\in\mathbb{R};$

$(vii)$ $EV\left(\Omega\left(\widetilde{A}\right)\right)=EV\left(\widetilde{A}\right)$*, for every* $\widetilde{A}\in\Delta IFN\left(\mathbb{R}\right);$

$(viii)$ $\widetilde{A}\leq_{EV}\widetilde{B}\Leftrightarrow\Omega\left(\widetilde{A}\right)\leq_{EV}\Omega\left(\widetilde{B}\right).$

# 5 An application to multicriteria decision making methods

A standard multicriteria decision making problem assumes the evaluation (and ranking) of $m$ alternatives $A_1,...,A_m$, under $n$ criteria $C_1,...,C_n$. Let us denote by $e_{ij}, i\in\{1,...,m\}$, $j\in\{1,...,n\}$ the performance of alternative $A_i$ with respect to criterion $C_j$. The matrix $E=(e_{ij})_{i\in\{1,...,m\},j\in\{1,...,n\}}$ is called the decision matrix of the problem. In addition, let us consider $W=(w_j)_{j\in\{1,...,n\}}$, where $w_j$ represents the weight of the criterion $C_j$. The solution of the problem is based on $E$ and $W$. If the evaluations $(e_{ij})_{i\in\{1,...,m\},j\in\{1,...,n\}}$ and/or $(w_j)_{j\in\{1,...,n\}}$ are fuzzy numbers then we have a fuzzy multicriteria decision making problem. A method based on the expected value of the result of data aggregation was proposed in [3]. We obtain an intuitionistic fuzzy multicriteria decision making problem or an interval-valued fuzzy multicriteria decision making problem if the input data in $E$ and $W$ are intuitionistic fuzzy sets (in fact intuitionistic fuzzy numbers and often trapezoidal or triangular intuitionistic fuzzy numbers) or interval-valued fuzzy sets (in fact interval-valued fuzzy numbers and often interval-valued trapezoidal or triangular fuzzy numbers). Let us assume that an interval-valued trapezoidal fuzzy multicriteria decision making method was already elaborated such that only the addition, scalar multiplication and the ranking method based on the expected value described in Section 4 were used for evaluating the alternatives. Based on the results in Theorem 6 it is enough to apply this method for $E_\Phi=\left(e_{ij}^{\Phi}\right)_{i\in\{1,...,m\},j\in\{1,...,n\}}$ and $W_\Phi=\left(w_j^{\Phi}\right)_{j\in\{1,...,n\}}$, where $e_{ij}^{\Phi}=\Phi\left(e_{ij}\right)\in IVTFN\left(\mathbb{R}\right), w_j^{\Phi}=\Phi\left(w_j\right)\in IVTFN\left(\mathbb{R}\right)$, if $e_{ij}\in TIFN\left(\mathbb{R}\right), w_j\in TIFN\left(\mathbb{R}\right)$ are the input data of the problem. We can proceed similarly in the triangular case: the input data $e_{ij}\in\Delta IFN\left(\mathbb{R}\right), w_j\in\Delta IFN\left(\mathbb{R}\right), i\in\{1,...,m\}, j\in\{1,...,n\}$ are transformed into interval-valued triangular fuzzy numbers by application $\Omega$ and an existing interval-valued triangular fuzzy multicriteria decision making method can be applied to $e_{ij}^{\Omega}=\Omega\left(e_{ij}\right)\in IV\Delta FN\left(\mathbb{R}\right)$, $w_j^{\Omega}=\Omega\left(w_j\right)\in IV\Delta FN\left(\mathbb{R}\right)$ to obtain an evaluation of alternatives.

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