Notes on Intuitionistic Fuzzy Sets
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Some notes on the relationships between intuitionistic fuzzy sets and correlation analysis

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Abstract: In the real world applications it is common that relationship between tuples of attributes of dimension higher than two need to be examined. It is well known that correlation analysis is focused on measuring of strength and direction of relationship between a pair of attributes. Algorithms using intercriteria analysis that solve the problem of measuring the strength of relationship between triples, quadruples, etc., were designed previously. The research presented in this paper is motivated by possibilities of using intuitionistic fuzzy equivalence relations to classify the data into the specific classes. The objective of this work is to use the values of correlation coefficients and compute the relationship between more than two attributes. The results are compared with the results obtained by intercriteria analysis.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy relations, InterCriteria Analysis, Correlation analysis.
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1 Introduction

Intuitionistic fuzzy sets were introduced by professor Krassimir Atanassov in 1983 [2] as an extension of the fuzzy sets and they represent one of the tools for working with uncertainty. They could be used in a number of areas of interest. One of these is the use of intercriteria analysis (ICA) where the relation between objects and criteria is expressed by a matrix which contains intuitionistic fuzzy pairs [3, 4, 18]. From this matrix, the consonances between criteria are computed. As shown in articles [3, 8, 19], not only consonances between pairs of criteria but also between triples, foursomes, etc. can be computed. For example, these results could be applied in the industry where the information of multi-criteria relations is essential.

This work approaches the problem with the use of another approach, which combines the properties of IFS and correlation analysis to evaluate the relation between more than two criteria and we compare them with the results of ICA. When we use a statistical approach in data analysis, we usually calculate one of three basic types of correlation coefficients - Pearson, Spearman, or Kendall. We obtain matrices composed of values from the interval $[-1, 1]$. Moreover, these matrices are reflexive and symmetric but not transitive. On the other hand, in algebra, it is well known, that if some matrices represent the relation matrices and they are reflexive, symmetric and transitive, they could be used for classification. These matrices are called equivalence relations. In [14] author wrote about the equivalence relations based on IFS matrices.

In this paper, we discuss the possibilities of how to construct IFS matrices from correlation matrices. Then we create equivalence IFS relations matrices and we compare the tuples of criteria obtained using this process with those which were obtained via intercriteria analysis.

2 Intuitionistic fuzzy sets

In this part of the paper, we introduce the basic preliminaries about Intuitionistic fuzzy sets.

Definition 2.1. *Let X be the universe. An intuitionistic fuzzy set A is a set*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

of the functions $\mu_A : X \to [0, 1], \nu_A : X \to [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Function μ_A is called the membership function and function ν_A is called the non-membership function. $\mathcal{F}$ denotes the family of all intuitionistic fuzzy sets.

Definition 2.2. *Let $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets. Then it holds*

$$\begin{aligned} A = B &\iff (\mu_A = \mu_B) \,\&\, (\nu_A = \nu_B), \\ A \leq B &\iff (\mu_A \leq \mu_B) \,\&\, (\nu_A \geq \nu_B), \\ A \bigwedge B &= ((\mu_A \wedge \mu_B),\ (\nu_A \vee \nu_B)), \\ A \bigvee B &= ((\mu_A \vee \mu_B),\ (\nu_A \wedge \nu_B)). \end{aligned}$$

In addition

$$(0,1) \leq A \leq (1,0),$$

which means, that element $(0,1)$ *is the smallest element and element* $(1,0)$ *is the greatest element of the set* $\mathcal{F}$.

Remark 1. In this text operations $\wedge$ and $\vee$ represent operations $\min$ and $\max$ respectively. In general, any t-norm and t-conorm could be used instead of these operations. t-norms and t-conorms are special operations defined on fuzzy sets, which are used to define various types of intersections and unions on fuzzy sets.

Definition 2.3. *Let* $R = (r_{i,j})_{n\times m}$ *be a matrix. If all elements of the matrix* R *belong to* $\mathcal{F}$, *then* R *is called an intuitionistic fuzzy matrix.*

It is obvious, that the intuitionistic fuzzy matrix R represents the intuitionistic fuzzy relation between two sets, for example X and Y. In the following text, we work with the intuitionistic fuzzy relations defined on the Cartesian product $X \times X$ exclusively.

Definition 2.4. *Let* $X = \{x_1, x_2, \ldots, x_n\}$ *be a finite set. An intuitionistic fuzzy relation* R *on* X *is called*

- *reflexive, iff* $R(x_i, x_i) = (1,0)$ *holds for each* $x_i \in X$,
- *symmetric, iff* $R(x_i, x_j) = R(x_j, x_i)$ *holds for each* $x_i, x_j \in X$,
- *transitive, iff* $\sup\limits_{x_j \in X} \min[R(x_i, x_j), R(x_j, x_k)] \leq R(x_i, x_k)$ *holds for each* $x_i, x_j, k_k \in X$.

Definition 2.5. *Let* X *be a finite set and let* R *be the intuitionistic fuzzy relation on* X. *If an intuitionistic fuzzy relation* R *is reflexive, symmetric and transitive, then* R *is called an intuitionistic fuzzy equivalence relation. If intuitionistic fuzzy relation* R *satisfies the properties of reflexivity and symmetry, then* R *is called an intuitionistic fuzzy tolerance relation.*

Since all operations defined in the next text are used for membership and non-membership parts separately, it is useful to follow the definition:

Definition 2.6. *The intuitionistic fuzzy relation matrix* R *could be written using two matrices* $R = [R_\mu, R_\nu]$. *Then the first matrix* R_μ *contains the membership degrees of elements of Cartesian product* $X \times X$ *and the second matrix* R_ν *contains the non-membership degrees of elements of Cartesian product* $X \times X$.

Remark 2. From the previous definitions it follows, that if fuzzy relation R is an intuitionistic fuzzy equivalence relation then matrix R_μ is symmetric with diagonal elements equal to one and matrix R_ν is symmetric with diagonal elements equal to zero. Moreover, both matrices are transitive.

Definition 2.7. *Let X be a finite set and let $R = [R_\mu, R_\nu]$, $S = [S_\mu, S_\nu]$ and $T = [T_\mu, T_\nu]$ be the intuitionistic fuzzy relations on X. Let $\circ$ represent the* $\max - \min$ *composition and let $\diamond$ represent the* $\min - \max$ *composition of two fuzzy relations (see for example [3]). Then the* $\max - \min$ *composition (denoted by $\star$) of intuitionistic fuzzy relations R and S is defined in the following way*

$$T = R \star S = [R_\mu \circ S_\mu, R_\nu \diamond S_\nu].$$

Theorem 1. Let X be a finite set, let R be the intuitionistic fuzzy tolerance relation on X and let $\star$ represent the $\max - \min$ composition of two intuitionistic fuzzy relations. Denote $R^2 = R \star R$ and $R^k = R^{k-1} \star R$. If it holds

$$R^k = R^{k-1},$$

then the intuitionistic fuzzy relation R^{k-1} is the intuitionistic fuzzy equivalence relation.

Remark 3. Let R be the intuitionistic fuzzy tolerance relation on X. From the previous Theorem, it follows

- If $R^2 = R$, then relation R satisfies the property of transitivity and therefore R is the fuzzy equivalence relation.
- If $R^2 \neq R$, then relation R does not satisfy the property of transitivity. But if we need to generate the intuitionistic fuzzy equivalence relation from the relation R, we should use the $\max - \min$ composition of this relation with itself. After a finite number of steps, we obtain the intuitionistic fuzzy equivalence relation. In addition, it was proved that the number of steps is always smaller or at most equal to the dimension n of the matrix R.

Definition 2.8. *(see [2]) Let X be a finite set and let $(\alpha, \beta) \in \mathcal{F}$. Then (α, β)-cut of the intuitionistic fuzzy set A is given by the following formula*

$$A^{(\alpha,\beta)} = \{x \in X, \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}.$$

Definition 2.9. *(see [9]) Let X be a finite set and let R be the intuitionistic fuzzy equivalence relation on X. Let a be any element of X. Then the IFS defined by*

$$aR = \{\langle x, (a\mu_R)(x), (a\nu_R)(x)\rangle | x \in X\}$$

where

$$(a\mu_R)(x) = \mu_R(a, x) \text{ and } (a\nu_R)(x) = \nu_R(a, x)$$

for each $x \in X$, is called an intuitionistic fuzzy equivalence class of a with respect to R.

Theorem 2. (see [9]) Let X be a finite set and let R be the intuitionistic fuzzy equivalence relation on X. Let a be any element of X. Then for any $(\alpha, \beta) \in \mathcal{F}$ the IFS defined by

$$R^{(\alpha,\beta)}(a) = [a]$$

is the equivalence class of a with respect to the intuitionistic fuzzy equivalence relation $R^{(\alpha,\beta)}$.

Theorem 3. (see [9]) Let X be a finite set and let R be the intuitionistic fuzzy equivalence relation on X. Let a, b be any element of X. $[a] = [b]$ denotes the equivalence classes of a and b with respect to the intuitionistic fuzzy equivalence relation $R^{(\alpha,\beta)}$. Then

$$[a] = [b] \text{ iff } (a, b) \in R^{(\alpha,\beta)}.$$

3 InterCriteria Analysis

InterCriteria Analysis (ICA) has been developed in Bulgaria with the aim of supporting decision-making in multiobject multicriteria problems, using the paradigms of intuitionistic fuzzy sets and index matrices. In ICA the term "correlation" is not being used, but changed to "positive/negative consonance" or "dissonance".

As input data, the method requires a two-dimensional table with the measurements or evaluations of m objects against n criteria and returns an $n \times n$ table with intuitionistic fuzzy pairs, defining the degrees of consonance between each pair of criteria, hence the name "intercriteria". The algorithm is completely dependent on the input data (measurements), and so far works well with complete datasets, without any missing values. The essence of the method is in the exhaustive pairwise comparison of the values of the measurements of all objects in the set against pairs of criteria, with all possible pairs being traversed, while counters being maintained for the percentage of the cases when the relations between the pairs of evaluations have been 'greater than', 'less than' or 'equal'. The method has been proposed and described in detail in 2014, [3] and extensively researched in the next years (e.g. [1,4,8,16–19,21]).

To be able to compare some specific results, in this paper, we work with the same data as presented in the paper *Intercriteria analysis: From pairs to triples* ([8]). These data are about the exhibited competitiveness of the EU Member States in the Year 2016–2017, derived from the Global Competitiveness Report of the World Economic Forum. Data from these reports have been among the most analysed with the apparatus of InterCriteria Analysis (see e.g. [5–7]), which presents a good basis for comparison. The objects here are the 28 EU Member States, and the criteria are the 12 main indicators in the methodology of the GCRs, namely, 1. Institutions, 2. Infrastructure, 3. Macroeconomic environment, 4. Health and primary education, 5. Higher education and training, 6. Goods market efficiency, 7. Labor market efficiency, 8. Financial market development, 9. Technological readiness, 10. Market size, 11. Business sophistication, and 12. Innovation.

In the paper [8] as a result of applying the ICA method to the mentioned data, authors obtained two matrices of size 12×12, one containing the membership parts, and the other one containing the non-membership parts of the intuitionistic fuzzy pairs. In the next step the distance of obtained values from the point $(0, 1)$ was computed and the pairs of the criteria were sorted in ascending order with respect to these distances. To compute the triples of criteria with the highest consonance, the algorithm proposed in the mentioned paper was used. A similar algorithm can be used for the determination of quadruples, quintuples, etc. of criteria.

4 Correlation analysis, coefficients and matrices

In descriptive and predictive data analysis the relationships between a pair of attributes (criteria) can be measured in various ways - the most common of which are covariance and correlation. Since the covariance of the attribute pair A and B illustrates how the distances from the mean values of the two attributes affect each other, it is computed as [13]:

$$cov(A, B) = \frac{\sum_{i=1}^{n}(A_i - \overline{A})(B_i - \overline{B})}{n}$$

where A_i and B_i denote i-th measurement of attributes A and B, $\overline{A}$ and $\overline{B}$ denote the mean value of the considered attributes and n is the number of data instances over which are the attributes of interest measured.

In this way, the covariance value between attributes is defined as $cov(A, B) \in [-\infty, \infty]$. Since this coefficient is measured in the units of input attributes, the covariance clearly defines the direction of the relationship between attributes only. This direction can be positive (values of both attributes grow or descend simultaneously) or negative (the value of one attribute grows and the other descends or vice versa). Therefore, the covariance coefficient lacks a clear description of the strength of the relationship between two attributes, which is added via the concept of correlation.

Correlation measures the predictive potential between two attributes of interest via correlation coefficient $corr(A, B) \in [-1, 1]$. Compared to covariance, the correlation coefficient naturally defines the direction and strength of the relationship between the attributes [10,15]. In the $[-1, 1]$ interval, the following three points are [20]:

- Complete positive correlation, where $corr(A, B) = 1$. In this case, the values of both attributes are directly proportional.
- No correlation, where $corr(A, B) = 0$, which is the worst situation from the point of view of correlation analysis. In such a case, there is no relationship between the values of the two attributes.
- Complete negative correlation (anticorrelation), where $corr(A, B) = -1$. This presents the opposite situation to a complete positive correlation, therefore the values of attributes are directly disproportional.

Naturally, all of these three situations are very uncommon, leading to the need to identify correlation levels in the considered interval. In [12] authors summarize these levels (cor_{lvl}) over $|corr(A_1, A_2)|$ as:

$$cor_{lvl} = \begin{cases} weak, & \text{in the case } |corr(A, B)| \leq 0.3, \\ moderate, & \text{in the case } 0.3 < |corr(A, B)| \leq 0.7, \\ strong, & \text{in the case } 0.7 < |corr(A, B)| \leq 1. \end{cases}$$

There are generally three correlation coefficients - Pearson correlation coefficient (r), Spearman correlation coefficient (ρ) and Kendall correlation coefficient (τ).

Pearson Correlation Coefficient

The simplest and the most commonly used type of correlation coefficient is the Pearson correlation coefficient, which measures the amount of linear relationship between a pair of attributes. Pearson correlation coefficient for attributes A and B is computed as follows [15]:

$$r(A,B) = \frac{\sum_{i=1}^{n}(A_i - \overline{A})(B_i - \overline{B})}{\sqrt{\sum_{i=1}^{n}(A_i - \overline{A})^2}\sqrt{\sum_{i=1}^{n}(B_i - \overline{B})^2}}$$

where A_i and B_i are i-th measurements of attributes A and B, and $\overline{A}$ and $\overline{B}$ denote the mean value of these attributes and n is number of entities in the dataset. It is natural, that this coefficient efficiently measures the strength (and direction) of the relationship in normally distributed data with no outliers.

Spearman (Rank) Correlation Coefficient

Spearman (rank) correlation coefficient is a non-linear alternative to measuring predictive potential between a pair of attributes. Since the Spearman method is one of the non-parametric, rank-based measures, each value of attributes in a studied pair is assigned a ranking. This correlation coefficient is then computed as [10]:

$$\rho(A,B) = 1 - \frac{6\sum(rank(A_i) - rank(B_i))^2}{n(n^2-1)}$$

where $rank(A_i)$ and $rank(B_i)$ are rankings for the i-th value of the considered attributes and n is number of measurements in the studied dataset. Even though, this coefficient works well even without normal distribution (needed in Pearson correlation coefficient), the Spearman correlation coefficient is not effective in the cases of repeated values - and therefore rankings.

Kendall (Rank) Correlation Coefficient

Second, the less frequently used, ranking model for correlation value measurement is the Kendall correlation coefficient measured between two attributes (A, B), which is computed as [20]:

$$\tau(A,B) = \frac{n_c - n_d}{\frac{n(n-1)}{2}}$$

where n_c is the number of concordant pairs of rankings for attributes A and B, n_d is the number of discordant pairs of such rankings and n is the number of data instances in the dataset. The concept of concordance of a combination of ranking pairs can be described as the monotonicity of rankings in such a combination. Discordance is the opposite situation to concordance.

Since conventional data sets contain more then two attributes and correlation coefficients measure the relationship between pairs of attributes, we need a model for summation of correlations in the data set. Most commonly used method of such summation is correlation matrix visualized through correlation heatmap (see Figure 1) [12].

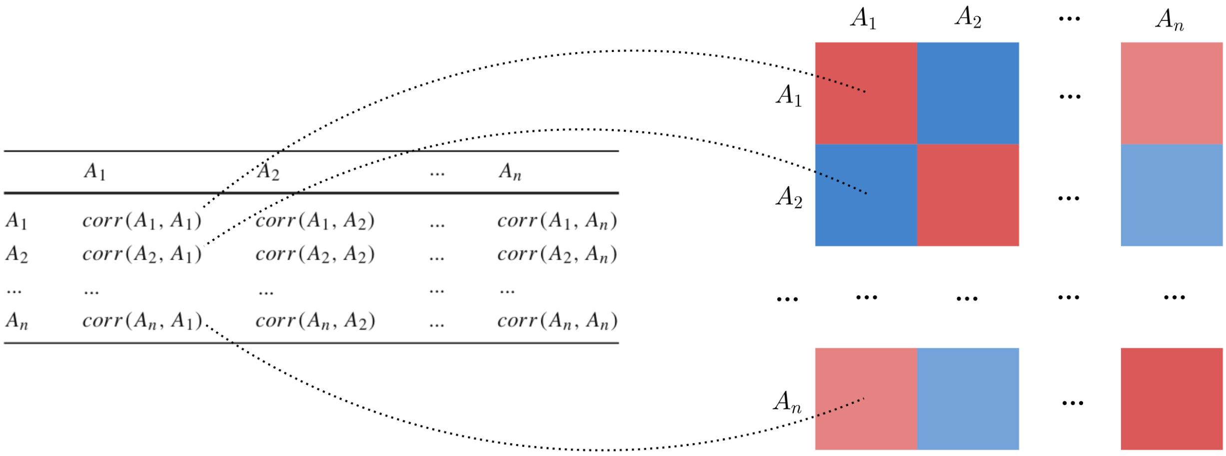


Figure 1. Illustrative example of the correlation matrix and its correlation heatmap visualization

5 From correlation matrices to IFS matrices

After calculating some correlation coefficients, we obtain the matrix that has the values from the interval $[-1, 1]$. Moreover, this matrix is reflexive and symmetric but not transitive. Now we transform the obtained matrix into the intuitionistic fuzzy matrix in a such way, that it fulfils the properties of intuitionistic fuzzy tolerance relation.

If there are two criteria which are in strong positive consonance, then the correlation coefficient that is assigned to them is very close to the number 1. In IFS theory these two criteria belong to the same set with a high membership degree. On the opposite side, if two criteria are in strong negative consonance, then the correlation coefficient that is assigned to them is very close to the number -1. We interpret this relation in such a way, that these two criteria belong to the same set with a high non-membership degree. Moreover, the correlation coefficient of one criterion with itself is equal to one and the correlation coefficients for two criteria are the same in both orders of criteria, i. e. it is symmetric.

We use various approaches to create intuitionistic fuzzy relation matrices. We made the calculations for all the mentioned correlation coefficients (Pearson, Spearman, Kendall). Since the results for them are comparable, in the next text we describe only those results, which were obtained by using Spearman data.

5.1 Simple creation of IFS matrices

Our first idea was to simply turn the positive values of the correlation coefficient into the membership matrix R_μ, absolute values of negative values of the correlation coefficient into the non-membership matrix R_ν and all other elements we put equal to zero. Using this approach we got the matrix, which satisfied all requirements of intuitionistic fuzzy tolerance relation. All diagonal elements of the matrix R_μ were equal to one and all diagonal elements of the matrix R_ν were equal to zero. Both matrices were symmetric. Therefore we composed the matrices R_μ and R_ν with themselves.

Let's reiterate that in the paper [14] there were made similar $\max - \min$ compositions on intuitionistic fuzzy tolerance matrices. The result of that paper was, that in used examples, the number of compositions of matrix R_μ with itself and the number of compositions of matrix R_ν with itself was the same. After the $\max - \min$ composition of these matrices, we get the following results:

- The number of compositions of matrix R_μ was different from the number of compositions of matrix R_ν. Specifically for matrix R_μ 7 compositions were used and for matrix R_ν 3 compositions were used.
- After the compositions of matrix R_ν with itself, all values of the final matrix were equal to zero, which means that we lost information for the non-membership part, i. e. for negative values of the correlation coefficient. Since information about negative correlation is also important, we decided to use another approach. The reason for obtaining such a zero matrix was that it contained too many zero elements.

From the obtained results it is obvious that we need to use another approach.

5.2 Creation of IFS matrices by using Yager IFS generator – first approach

There are several different intuitionistic fuzzy generators (shortly IFS generators). Let us first give a general definition.

Definition 5.1. *(see [11]) A function* $\varphi : [0,1] \to [0,1]$ *is called intuitionistic fuzzy generator, if the inequality*

$$\varphi(x) \leq 1 - x$$

holds for each $x \in [0,1]$.

We decide to use the Yager IFS generator defined by the following formula:

$$\varphi_\omega(x) = (1 - x^\omega)^{1/\omega} \quad \text{and} \quad \omega \in (0,1].$$

Since the inverse function of the function φ has the same prescription as the original function φ, we took the same matrices R_μ and R_ν as used in subsection 5.1 and we applied Yager IFS generator following way:

- We took matrix R_μ and for each matrix element which was equal to zero, we computed a new value using the Yager IFS generator on the corresponding element of the matrix R_ν. We did not change the non-zero elements of matrix R_μ.
- Similarly we took matrix R_ν and for the matrix element which was equal to zero, we computed a new value using the Yager IFS generator on the corresponding element of the matrix R_μ. We did not change the non-zero elements of matrix R_ν.

We used different values of Yager parameter ω. Then we looked at the created matrices and concluded the following:

- When we use the value of ω parameter close to one, the obtained new matrix values are in many cases greater than the original values of matrices. This means, that in the next classification, we compare incorrect data. Therefore we did not compute the $\max - \min$ composition of the matrices.
- When we use the value of ω parameter close to zero, the obtained matrix values are also quite low. For example, when we used $\omega = 0.1$, we got new values of matrices with the level $10^{-11} - 10^{-15}$. After using these values and computing $\max - \min$ composition of both matrices, we were not able to draw a dendrogram graph and also made a classification.
- It is possible to find such a value of ωparameter, that we get such values of matrices that all the original values are greater than values computed using the Yager IFS generator. This value of the ω parameter always depends on the smallest non-zero element of original matrices, therefore we decided to use another approach, that employs the Yager IFS generator.

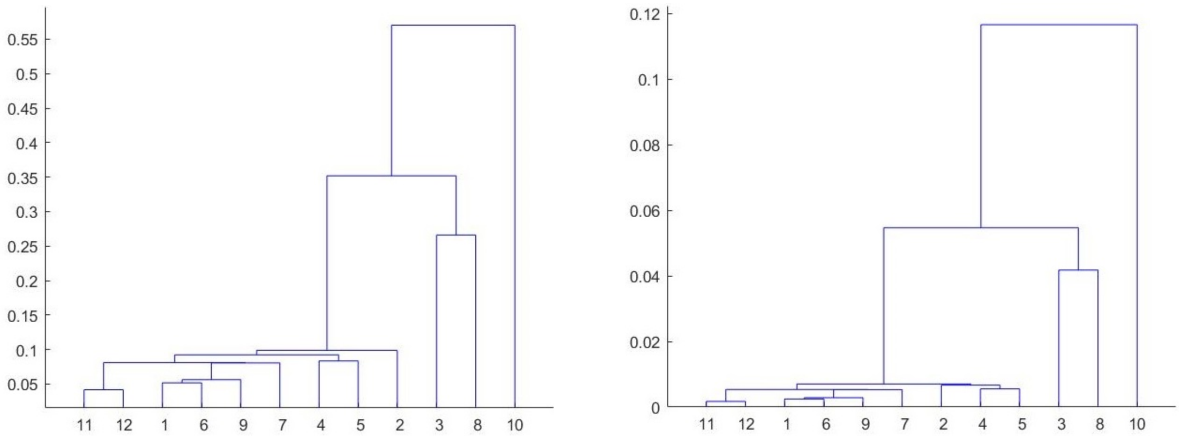
5.3 Creation of IFS matrices by using Yager IFS generator – second approach

As mentioned in the previous section, using the Yager IFS generator on original matrices R_μ and R_ν could cause the situation, that original values, which represent some correlation between criteria, could be smaller than values computed using Yager IFS generator. This causes the comparison of incorrect values in classification. Therefore, we decide to strengthen original values of matrices R_μ and R_ν using transformation from interval $(0, 1]$ to interval $(0.5, 1]$. Specifically, we create new matrices $\tilde{R}_\mu$ and $\tilde{R}_\nu$ of the same size as the original matrices. We took the values of the original matrices. If they were equal to zero, we just rewrite them to new matrices. If the values of the original matrices were greater than zero, we use the following formula:

$$\tilde{r}_{i,j} = 0.5r_{i,j} + 0.5 \ .$$

When we used the Yager IFS generator with an arbitrary value of ω parameter to create new matrices, the values computed using the Yager IFS generator were always smaller than the original values.

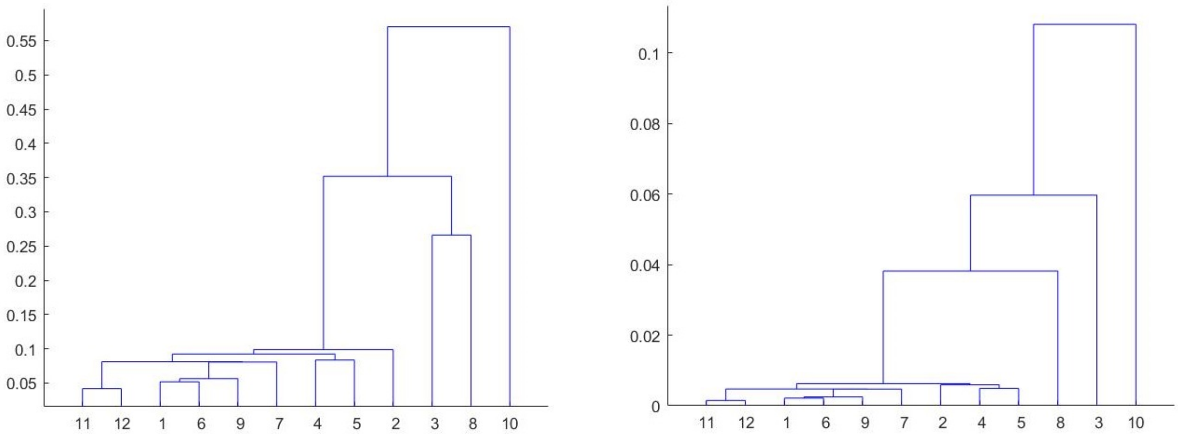
An interesting observation is that for some values of ω parameter, we got the same structure of dendrograms for membership and non-membership matrix. In our example, it holds for $\omega \in [0.5976137595, 1]$. In these cases, the value of ω parameter causes just the different lengths of dendrogram branches, but the assignment of criteria to individual branches of the dendrograms remain the same (see Figure 2). The results of this approach are referenced as results obtained using *Method 1*.



(a) Dendrogram for membership matrix (b) Dendrogram for non-membership matrix

Figure 2. Dendrograms for parameter $\omega = 0.6$

When we took the values of $\omega \in [0, 0.5976137595)$ the assignment of criteria to individual branches of the dendrograms was different for membership and for non-membership dendrogram. The results for different values of ω parameter from the mentioned interval were always the same (see Figure 3). The results of this approach are referred to as results obtained using *Method 2*.



(a) Dendrogram for membership matrix (b) Dendrogram for non-membership matrix

Figure 3. Dendrograms for parameter $\omega = 0.59$

Another interesting observation is that the values of the ω parameter affected only the values of the non-membership dendrogram. The dendrogram for membership values did not change after using different values of the ω parameter. This could be caused by the values of the original matrix R_μ that contains a small number of non-zero elements.

We have not yet been able to determine the general dependence between the values of the matrices $\tilde{R}_\mu, \tilde{R}_\nu$ and the value of the ω parameter. The solution of this relationship is left as an open problem for the future. In this text, we focus further on the comparison of results for our specific example.

Based on the obtained matrices and dendrograms, we can say which couples, triples, etc. of criteria are in strongest correlation. The comparison of the obtained couples and triples using the different methods is listed in Table 1.

Table 1. Comparison of obtained results

Method	**Couples**	**Triples**
Method1	{11, 12}, {1, 6}, {1, 9}, {6, 9}, {1, 11}, {1, 12} {6, 11}, {6, 12}, {9, 11}, {9, 12}	{1, 6, 9}, {1, 11, 12}, {6, 11, 12}, {9, 11, 12}
Method2	{11, 12}, {1, 6}, {1, 9}, {6, 9}, {1, 11}, {1, 12} {6, 11}, {6, 12}, {9, 11}, {9, 12}	{1, 6, 9}, {1, 11, 12}, {6, 11, 12}, {9, 11, 12}
Method ICA	{11, 12}, {1, 9}, {1, 6}, {9, 11}, {6, 7}	{9, 11, 12}, {1, 11, 12}, {1, 6, 9}, {1, 9, 11}
Spearman	{11, 12}, {1, 6}, {1, 9}, {9, 11}, {6, 7}	—

The values of couples and triples are mentioned in such order, as the criteria are in strongest correlation/consonance. We wanted to add into the table first five couples and four triples with the strongest correlation/consonance, but when the obtained values were the same for more than five couples/ four triples, than we add all of them into the table.

The first row of the table represents the elements which were obtained from the membership matrix R_μ. The couples reach following values $\mu(11,12) = 0.970471$, $\mu(1,6) = 0.963342$, $\mu(1,9) = \mu(6,9) = 0.960154$, $\mu(1,11) = \mu(1,12) = \mu(6,11) = \mu(6,12) = \mu(9,11) = \mu(9,12) = 0.945242$. The triples reach following values $\mu(1,6,9) = 0.963342$, $\mu(1,11,12) = \mu(6,11,12) = \mu(9,11,12) = 0.945242$.

The second row of the table represents the elements which were obtained from the non-membership matrix R_ν, when the ω parameter was equal to 0.59. In this case, we consider the smallest values of ν between copules. The couples reach following values $\nu(11,12) = 0.001055$, $\nu(1,6) = 0.001526$, $\nu(1,9) = \nu(6,9) = 0.001760$, $\nu(1,11) = \nu(1,12) = \nu(6,11) = \nu(6,12) = \nu(9,11) = \nu(9,12) = 0.002871$. The triples reach following values $\nu(1,6,9) = 0.001760$, $\nu(1,11,12) = \nu(6,11,12) = \nu(9,11,12) = 0.002871$.

The third row of the table represents the elements which were obtained using an algorithm, which was described in the paper [8]. The obtained values represent the distance between the IFS pair belonging to the considered couple of criteria and the greatest element of the IFS, element $(1,0)$. Therefore, as the lower value of distance is achieved, the consonance between characters grows. The couples reach following values $d(11,12) = 0.1491$, $d(1,9) = 0.1792$, $d(1,6) = 0.1883$, $d(9,11) = 0.2028$, $d(6,7) = 0.2094$. The triples reach following values $d(9,11,12) = 0.5758$, $d(1,11,12) = 0.5976$, $d(1,6,9) = 0.5985$, $d(1,9,11) = 0.6207$.

The fourth row of the table represents the elements which were obtained using the Spearman correlation coefficient on considered data. The couples reach following values of the Spearman correlation coefficient $\rho(11,12) = 0.940942$, $\rho(1,6) = 0.926685$, $\rho(1,9) = 0.920308$, $\rho(9,11) = 0.893920$, $\rho(6,7) = 0.890485$.

6 Conclusions

In data analysis, the question of determining the correlation between more than two criteria (attributes/objects/variables) is not sufficiently resolved. In this paper, we present some new ideas about the possibility of using a composition of IFS matrices to generate the triples, four-tuples, etc. of criteria from the values of correlation coefficients. We describe three different approaches to the transformation of correlation coefficients into the IFS matrices and we discussed which approach is correct to use. We compare the obtained results with some previous works. We could conclude, that these results are comparable. Therefore this approach seems to be useful. We use the model where most of the data exhibited mutual positive correlation between criteria. In future work, we are planning to use other data, with more criteria in negative correlation. We also compare the relation between the use of different types of IFS generators and the obtained results.

Mentioned results have another benefit from the view of the composition of the IFS matrices. In this paper In this paper we present the example of the matrices, where the number of $\max - \min$ compositions of the matrices R_μ and R_ν were not equal. Moreover the result of the composition of the matrix R_ν with itself results in a zero matrix. It is important from that point, that we lost information about non-membership part of the problem being solved.

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