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The Hauber's law with intuitionistic fuzzy implications

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Abstract: 25 years ago, in [7], it was proved that the Hauber's law is an intuitionistic fuzzy tautology. In this case, the used implication was the standard intuitionistic fuzzy one. In the present paper, we check which intuitionistic fuzzy implications, defined during these 25 years, satisfy the Hauber's law as a tautology and which if them – as an intuitionistic fuzzy tautology.
Keywords: Hauber's law, Intuitionistic fuzzy implication, Intuitionistic fuzzy tautology, Tautology.
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1 Introduction

The concept of an Intuitionistic Fuzzy pair (IFP) was introduced in [9] as follows: the ordered pair $\langle a, b\rangle$ is an IFP if and only if $a, b, a + b \in [0, 1]$.

The IFP is called an *Intuitionistic Fuzzy Tautology (IFT)* if and only if $a \geq b$ and it is a *tautology* if and only if $a = 1, b = 0$.

For both IFPs the operations conjunction and disjunction have the forms

$$\langle a, b\rangle \wedge \langle c, d\rangle = \langle \min(a, c), \max(b, d)\rangle,$$
$$\langle a, b\rangle \vee \langle c, d\rangle = \langle \max(a, c), \min(b, d)\rangle.$$

In [11], the Hauber's Law is formulated by

$$((A \to B) \wedge (C \to D) \wedge (A \vee C) \wedge \neg(B \wedge D)) \to ((B \to A) \wedge (D \to C)) \qquad (*)$$

and it is proved that it is a standard tautology. In [7] it was proved that it is an IFT when $\to$ is the standard Intuitionistic Fuzzy Implication (IFI) $\to_4$ defined by

$$\langle a, b\rangle \to_4 \langle c, d\rangle = \langle \max(b, c), \min(a, d)\rangle$$

and $\neg$ is the standard Intuitionistic FuzzyNegation (IFN) $\neg_1$ defined by

$$\neg_1\langle a, b\rangle = \langle b, a\rangle.$$

In [8] this fact is discussed in **Theorem 1.5.30**, but there the following **Open Problem** is formulated: *For which other intuitionistic fuzzy implications, negations, disjunctions and conjunctions the Hauber's Law is an IFT? Are there operations of these four types, for which the Law is a standard tautology?*

In the present paper, we give answers to these questions.

2 Main results

First, we must mention that in [1–3, 10], the list of all existing by the moment IFIs and IFNs are given, respectively.

Theorem 1. *The implications $\to_i$ and the standard negation ($\neg_1$) satisfy (*) as an IFTs, where* $i = 1, 4, 7, 17, 21, 29, 30, 33, \dots, 36, 45, 61, 71, 72, 75, 76, 77, 79, \dots, 82, 88, 89, 90, 100, \dots, 105, 109, \dots, 116, 118, 124, 125, 126, 128, \dots, 133, 166, 167, 168, 176, 177, 178, 186, 188, 192, 194, 195, 196, 204, 206.$

Proof. Let the formulas A, B, C, D be given and let everywhere:

$$V(A) = \langle a, b\rangle,$$
$$V(B) = \langle c, d\rangle,$$
$$V(C) = \langle e, f\rangle,$$
$$V(D) = \langle g, h\rangle.$$

Below, we will check the validity of the Theorem for the case, when the implication is $\rightarrow_{33}$ and the negation is $\neg_1$, i.e.,

$$V(A \rightarrow_{33}) = \langle a, b\rangle \rightarrow_{33} \langle c, d\rangle = \langle 1 - \min(a, d), \min(a, d)\rangle$$

and

$$V(\neg_1 A) = \neg_1 \langle a, b\rangle = \langle b, a\rangle$$

(see [8]).

We calculate sequentially:

$$\begin{aligned}
&V(((A \rightarrow_{33} B) \wedge (C \rightarrow_{33} D) \wedge (A \vee C) \wedge \neg_1(B \wedge D)) \rightarrow_{33} ((B \rightarrow_{33} A) \wedge (D \rightarrow_{33} C)))\\
&= (((\langle a, b\rangle \rightarrow_{33} \langle c, d\rangle) \wedge (\langle e, f\rangle \rightarrow_{33} \langle g, h\rangle) \wedge (\langle a, b\rangle \vee \langle e, f\rangle)\\
&\quad \wedge \neg_1(\langle c, d\rangle \wedge \langle g, h\rangle)) \rightarrow_{33} (((\langle c, d\rangle \rightarrow_{33} \langle a, b\rangle) \wedge (\langle g, h\rangle \rightarrow \langle e, f\rangle))\\
&= (\langle 1 - \min(a, d), \min(a, d)\rangle \wedge \langle 1 - \min(e, h), \min(e, h)\rangle \wedge \langle \max(a, e), \min(b, f)\rangle\\
&\quad \wedge \neg\langle \min(c, g), \max(d, h)\rangle) \rightarrow_{33} (\langle 1 - \min(b, c), \min(b, c)\rangle \wedge \langle 1 - \min(f, g), \min(f, g)\rangle)\\
&= (\langle \max(1 - a, 1 - d), \min(a, d)\rangle \wedge \langle \max(1 - e, 1 - h), \min(e, h)\rangle \wedge \langle \max(a, e), \min(b, f)\rangle\\
&\quad \wedge \langle \max(d, h), \min(c, g)\rangle) \rightarrow (\langle \max(1-b, 1-c), \min(b, c)\rangle \wedge \langle \max(1-f, 1-g), \min(f, g)\rangle)\\
&= \langle \min(\max(1 - a, 1 - d), \max(1 - e, 1 - h), \max(a, e), \max(d, h)),\\
&\quad \max(\min(a, d), \min(e, h), \min(b, f), \min(c, g))\rangle\\
&\quad \rightarrow_{33} \langle \min(\max(1 - b, 1 - c), \max(1 - f, 1 - g)), \max(\min(b, c), \min(f, g)\rangle)\\
&= \langle 1 - \min(\min(\max(1 - a, 1 - d), \max(1 - e, 1 - h), \max(a, e), \max(d, h)),\\
&\quad \max(\min(b, c), \min(f, g))), \min(\min(\max(1 - a, 1 - d), \max(1 - e, 1 - h),\\
&\quad \max(a, e), \max(d, h)), \max(\min(b, c), \min(f, g)))\rangle).
\end{aligned}$$

Let

$$\begin{aligned}
X \equiv 1 - 2\min(&\min(\max(1 - a, 1 - d), \max(1 - e, 1 - h), \max(a, e), \max(d, h)),\\
&\max(\min(b, c), \min(f, g))).
\end{aligned}$$

If $a \geq d \geq e \geq h$, then[1]

$$X = 1 - 2\min(\min(1 - d, 1 - h, a, d), \max(\min(b, c), \min(f, g)))$$

(from $1 - d \leq 1 - h$)

$$= 1 - 2\min(\min(1 - d, d), \max(\min(b, c), \min(f, g)))$$

(from $1 - a \geq b, 1 - d \geq c, 1 - e \geq f, 1 - h \geq g$)

$$\geq 1 - 2\min(\min(1 - d, d), \max(\min(1 - a, 1 - d), \min(1 - e, 1 - h)))$$

[1] In this case, we will give the detailed proof, while in all next cases – only the important steps of their proofs.

(from $1-a \leq 1-d, 1-e \leq 1-h$)

$$= 1 - 2\min(\min(1-d, d), \max(1-a, 1-e))$$

(from $1-a \leq 1-e$)

$$= 1 - 2\min(1-d, d, 1-e)$$

(from $1-d \leq 1-e$)

$$= 1 - 2\min(1-d, d) \geq 0,$$

because $\min(d, 1-d) \leq 0.5$.

If $a \geq d \geq h \geq e$, then

$$\begin{aligned}
X &= 1 - 2\min(\min(1-d, 1-e, a, d), \max(\min(b, c), \min(f, g))) \\
&= 1 - 2\min(\min(1-d, d), \max(\min(b, c), \min(f, g))) \\
&\geq 1 - 2\min(\min(1-d, d), \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\
&= 1 - 2\min(\min(1-d, d), \max(1-a, 1-h)) \\
&= 1 - 2\min(1-d, d, 1-h) \\
&= 1 - 2\min(1-d, d) \geq 0.
\end{aligned}$$

If $a \geq e \geq d \geq h$, then

$$\begin{aligned}
X &= 1 - 2\min(\min(1-d, 1-h, a, d), \max(\min(b, c), \min(f, g))) \\
&\geq 1 - 2\min(\min(1-d, d), \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\
&= 1 - 2\min(1-d, d, \max(1-a, 1-e)) \\
&= 1 - 2\min(1-d, d, 1-e) \\
&= 1 - 2\min(d, 1-e) \\
&\geq 1 - 2\min(e, 1-e) \geq 0.
\end{aligned}$$

If $a \geq e \geq h \geq d$, then

$$\begin{aligned}
X &= 1 - 2\min(\min(1-d, 1-h, a, h), \max(\min(b, c), \min(f, g))) \\
&\geq 1 - 2\min(\min(1-h, h), \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\
&= 1 - 2\min(1-h, h, \max(1-a, 1-e)) \\
&= 1 - 2\min(1-h, h, 1-e)) \\
&\geq 1 - 2\min(1-h, h) \geq 0.
\end{aligned}$$

If $a \geq h \geq d \geq e$, then

$$\begin{aligned}
X &\geq 1 - 2\min(\min(1-d, 1-e, a, h), \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\
&= 1 - 2\min(1-d, h, \max(1-a, 1-h)) \\
&= 1 - 2\min(1-d, h, 1-h) \\
&= 1 - 2\min(h, 1-h) \geq 0.
\end{aligned}$$

If $a \geq h \geq e \geq d$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-e, e, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-e, e, \max(1-a, 1-h)) \\ &= 1 - 2\min(1-e, e, 1-h) \geq 0. \end{aligned}$$

If $d \geq a \geq e \geq h$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-h, a, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-a, a, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-a, a, 1-e) \geq 0. \end{aligned}$$

If $d \geq a \geq h \geq e$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-e, a, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &\geq 1 - 2\min(1-a, a, \max(1-d, 1-h)) \\ &= 1 - 2\min(1-a, a, 1-h) \geq 0. \end{aligned}$$

If $d \geq e \geq a \geq h$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-h, e, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-a, e, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-a, e, 1-e) \geq 0. \end{aligned}$$

If $d \geq e \geq h \geq a$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-h, e, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-h, e, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-h, e, 1-e) \geq 0. \end{aligned}$$

If $d \geq h \geq a \geq e$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-e, a, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-a, a, \max(1-a, 1-h)) \\ &= 1 - 2\min(1-a, a, 1-a) \geq 0. \end{aligned}$$

If $d \geq h \geq e \geq a$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-e, e, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-e, e, \max(1-d), 1-h)) \\ &= 1 - 2\min(1-e, e, 1-h) \geq 0. \end{aligned}$$

If $e \geq a \geq d \geq h$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-h, e, d, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-d, d, \max(1-a, 1-e)) \\ &= 1 - 2\min(1-d, d, 1-a) \geq 0. \end{aligned}$$

If $e \geq a \geq h \geq d$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-h, e, d, \max(1-a, 1-e)) \\ &= 1 - 2\min(1-d, d, 1-a) \geq 0. \end{aligned}$$

If $e \geq d \geq a \geq h$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-h, e, d, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-a, d, 1-d) \geq 0. \end{aligned}$$

If $e \geq d \geq h \geq a$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-h, e, d, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-a, d, 1-d) \geq 0. \end{aligned}$$

If $e \geq h \geq a \geq d$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-h, e, h, \max(1-a, 1-e)) \\ &= 1 - 2\min(1-h, h, 1-e) \geq 0. \end{aligned}$$

If $e \geq h \geq d \geq a$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-h, e, h, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-h, h, 1-d) \geq 0. \end{aligned}$$

If $h \geq a \geq e \geq d$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-e, a, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-d, a, \max(1-a, 1-h)) \\ &= 1 - 2\min(1-d, a, 1-a) \\ &= 1 - 2\min(a, 1-a) \geq 0. \end{aligned}$$

If $h \geq a \geq d \geq e$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-e, a, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-d, a, \max(1-a, 1-h)) \\ &= 1 - 2\min(1-d, a, 1-a) \\ &= 1 - 2\min(a, 1-a) \geq 0. \end{aligned}$$

If $h \geq d \geq a \geq e$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-e, a, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-a, a, \max(1-d, 1-h)) \\ &= 1 - 2\min(1-a, a, 1-d) \geq 0. \end{aligned}$$

If $h \geq d \geq e \geq a$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-e, e, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-e, e, \max(1-d, 1-h)) \\ &= 1 - 2\min(1-e, e, 1-d) \geq 0. \end{aligned}$$

If $h \geq e \geq a \geq d$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-d, 1-e, e, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-e, e, \max(1-a, 1-h)) \\ &= 1 - 2\min(1-e, e, 1-a) \geq 0. \end{aligned}$$

If $h \geq e \geq d \geq a$, then

$$\begin{aligned} X &\geq 1 - 2\min(1-a, 1-e, e, h, \max(\min(1-a, 1-d), \min(1-e, 1-h))) \\ &= 1 - 2\min(1-e, e, \max(1-d, 1-e)) \\ &= 1 - 2\min(1-e, e, 1-d) \geq 0. \end{aligned}$$

Therefore, the Hauber's Law is an IFT for implication $\rightarrow_{33}$ and negation $\neg_1$.

Now, we will show that this Law is not valid, e.g., for implication $\rightarrow_{139}$ and the same (standard) negation. Implication $\rightarrow_{139}$ has the form

$$V(A \rightarrow_{139}) = \langle a, b\rangle \rightarrow_{139} \langle c, d\rangle = \langle \frac{b+c}{2}, \frac{a+d}{2}\rangle.$$

In this case, we obtain:

$$\begin{aligned} &V(((A \rightarrow_{139} B) \wedge (C \rightarrow_{139} D) \wedge (A \vee C) \wedge \neg_1(B \wedge D)) \rightarrow_{139} ((B \rightarrow_{139} A) \wedge (D \rightarrow_{139} C))) \\ &= (((\langle a, b\rangle \rightarrow_{139} \langle c, d\rangle) \wedge (\langle e, f\rangle \rightarrow_{139} \langle g, h\rangle) \wedge (\langle a, b\rangle \vee \langle e, f\rangle) \\ &\quad \wedge \neg(\langle c, d\rangle \wedge \langle g, h\rangle)) \rightarrow_{139} ((\langle c, d\rangle \rightarrow_{139} \langle a, b\rangle) \wedge (\langle g, h\rangle \rightarrow_{139} \langle e, f\rangle)) \\ &= \left(\left\langle \frac{b+c}{2}, \frac{a+d}{2}\right\rangle \wedge \left\langle \frac{f+g}{2}, \frac{e+h}{2}\right\rangle \wedge \langle \max(a, e), \min(b, f)\rangle \wedge \langle \max(d, h), \min(c, g)\rangle\right) \\ &\quad \rightarrow_{139} \left(\left\langle \frac{a+d}{2}, \frac{b+c}{2}\right\rangle \wedge \left\langle \frac{f+h}{2}, \frac{e+g}{2}\right\rangle\right) \\ &= \left\langle \min\left(\frac{b+c}{2}, \frac{f+g}{2}, \max(a, e), \max(d, h)\right), \max\left(\frac{a+d}{2}, \frac{e+h}{2}, \min(b, f), \min(c, g)\right)\right\rangle \\ &\quad \rightarrow_{139} \left\langle \min\left(\frac{a+d}{2}, \frac{f+h}{2}\right), \max\left(\frac{b+c}{2}, \frac{e+g}{2}\right)\right\rangle \\ &= \left\langle \frac{1}{2}\left(\max\left(\frac{a+d}{2}, \frac{e+h}{2}, \min(b, f), \min(c, g)\right) + \min\left(\frac{a+d}{2}, \frac{f+h}{2}\right)\right), \right. \\ &\quad \left. \frac{1}{2}\left(\min\left(\frac{b+c}{2}, \frac{f+g}{2}, \max(a, e), \max(d, h)\right) + \max\left(\frac{b+c}{2}, \frac{e+g}{2}\right)\right)\right\rangle. \end{aligned}$$

Let

$$X \equiv \max\left(\frac{a+d}{2}, \frac{e+h}{2}, \min(b,f), \min(c,g)\right) + \min\left(\frac{a+d}{2}, \frac{f+h}{2}\right)$$
$$- \min\left(\frac{b+c}{2}, \frac{f+g}{2}, \max(a,e), \max(d,h)\right) - \max\left(\frac{b+c}{2}, \frac{e+g}{2}\right).$$

When $a = d = 0, b = c = 1$, we obtain

$$X = \max\left(\frac{e+h}{2}, f, g\right) - \min\left(\frac{f+g}{2}, e, h\right) - 1 \leq 0,$$

i.e., the expression is not an IFT.

Therefore, this Law is not valid for implication $\rightarrow_{139}$ and negation $\neg_1$.

In the same manner we can check the validity of all other cases in Theorem 1. □

Again using the same approach, we can prove also the following three theorems.

Theorem 2. The implications $\rightarrow_i$ and the generated by them negations satisfy (*) as IFTs, where $i = 1, 3, 4, 7, 11, 14, 17, 18, 20, \ldots, 23, 27, 28, 29, 35, 61, 71, 72, 74, 76, 77, 79, 80, 81, 100, 109, \ldots, 113, 117, 118, 124, 125, 126, 128, 132, 133, 166, 167, 170, 176, 177, 180, 186, 187, 190, 191, 192, 195, 198, 199, 201, 203, 204, 206$.

Theorem 3. The implications $\rightarrow_i$ and the standard negation ($\neg_1$) satisfy (*) as a tautology, where $i = 77, 88, 206$.

Theorem 4. The implications $\rightarrow_i$ and the generated by them negations satisfy (*) as an IFTs, where $i = 3, 11, 14, 20, 23, 74, 77, 191, 199, 201, 203, 206$.

3 Conclusion

In the present paper, the Open problem, formulated in [8], is solved for the existing at the moment intuitionistic fuzzy implications and intuitionistic fuzzy negations and for the standard conjunction and disjunction, but in [4–6] three (in general case different) intuitionistic fuzzy conjunctions and disjunctions are generated from each one of the intuitionistic fuzzy implications. So, the discussed **Open problem** can be formulated in an essentially more extended form: *For which other (non-standard) intuitionistic fuzzy conjunctions and disjunctions with the related to them IFIs and IFNs the Hauber's Law is an intuitionistic fuzzy tautology and when it is a standard tautology?*

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