Notes on Intuitionistic Fuzzy Sets
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Some relationships between new intuitionistic fuzzy modal operators

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Abstract: The concept of Intuitionistic Fuzzy Sets was introduced in 1983 [1] as an extention of fuzzy sets [7]. Intuitionistic fuzzy modal operators firstly are defined in [1] and new modal operators are defined by different reserchers [3, 4, 5, 6]. Some properties of these operators studied by several authors. In this paper, some relationships between some of the new modal operators are examined.

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1 Introduction

Intuitionistic fuzzy sets was introduced in 1983 [1], as an extension of fuzzy sets [7]. Intuitionistic fuzzy modal operators were introduced in [1, 2]. Then several extensions of these operators were defined by different authors. Some algebraic and characteristic properties of these operators were studied in several papers. In this study, we will examine some relationships between $B_{\alpha,\beta}$, $\boxminus_{\alpha,\beta}$, $E^{\omega,\theta}_{\alpha,\beta}$, $L^{\omega}_{\alpha,\beta}$, $K^{\omega}_{\alpha,\beta}$, $T_{\alpha,\beta}$, $S_{\alpha,\beta}$ and $\otimes_{\alpha,\beta,\gamma,\delta}$ intuitionistic fuzzy modal operators.

Definition 1. *[1] An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is called the "degree of membership of x in A", $\nu_A : X \to [0,1])$ and $\nu_A(x)$ is called the "degree of non-membership of x in A", and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \textit{for all } x \in X.$$

The class of intuitionistic fuzzy sets on X is denoted by $IFS(X)$.

Definition 2. *[1] An IFS A is said to be contained in an IFS B (notation $A \sqsubseteq B$) if and only if, for all $x \in X$,*

$$\mu_A(x) \leq \mu_B(x) \textit{ and } \nu_A(x) \geq \nu_B(x).$$

It is clear that $A = B$ if and only if $A \sqsubseteq B$ and $B \sqsubseteq A$.

Definition 3. *[1] Let $A \in IFS(X)$ and let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ then the above set is called the complement of A*

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}.$$

The following operations of two IFSs A and B on X is defined by

$$A \sqcap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$$

$$A \sqcup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$$

Now we will give the definition of some intuitionistic fuzzy modal operators which studied in this paper. In 2013, $B_{\alpha,\beta}, \boxminus_{\alpha,\beta}$ and $E^{\omega,\theta}_{\alpha,\beta}$ modal operotors were defined as follows:

Definition 4. *[4] Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta \in [0,1]$ then*

$$B_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}.$$

Definition 5. *[4] Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta, \omega \in [0,1]$ then*

$$\boxminus_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\beta)\nu_A(x)), \alpha((1-\alpha)\mu_A(x) + \nu_A(x))\rangle : x \in X\}.$$

Definition 6. *[4] Let X be a universal and $A \in IFS(X)$, $\alpha, \beta, \omega, \theta \in [0,1]$ then*

$$E^{\omega,\theta}_{\alpha,\beta}(A) = \{\langle x, \beta((1-(1-\alpha)(1-\theta))\mu_A(x) + (1-\alpha)\theta\nu_A(x) + (1-\alpha)(1-\theta)\omega),$$

$$\alpha((1-\beta)\theta\mu_A(x) + (1-(1-\beta)(1-\theta))\nu_A(x) + (1-\beta)(1-\theta)\omega)\rangle : x \in X\}.$$

Proposition 1. *[4] Let X be a universal and $A \in IFS(X)$, $\alpha, \beta, \omega \in [0,1]$ then*

1. $E^{\omega,\theta}_{1,1}(A) = A$

2. $E_{1,0}^{0,0}(A) = \emptyset$

3. $E_{0,1}^{0,0}(A) = X$

$K_{\alpha,\beta}^{\omega}$ and $L_{\alpha,\beta}^{\omega}$ intuitionistic fuzzy modal operators were defined in 2014 and same year $\otimes_{\alpha,\beta,\gamma,\delta}$ was introduced by Atanassov.

Definition 7. *[6] Let X be a universal and $A \in IFS(X), \alpha, \beta, \omega, \alpha+\beta \in [0,1]$.*

1. $L_{\alpha,\beta}^{\omega}(A) = \{\langle x, \alpha\mu_A(x) + \omega(1-\alpha), \alpha(1-\beta)\nu_A(x) + \alpha\beta(1-\omega)\rangle \, x \in X\}$

2. $K_{\alpha,\beta}^{\omega}(A) = \{\langle x, \alpha(1-\beta)\mu_A(x) + \alpha\beta(1-\omega), \alpha\nu_A(x) + \omega(1-\alpha)\rangle \, x \in X\}$

Proposition 2. *[6] Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta, \alpha+\beta \in [0,1]$, $\alpha \neq 1$.*

1. $L_{\alpha,0}^{\frac{\beta}{1-\alpha}}(A) = \boxtimes_{\alpha,\beta}(A)$

2. $K_{\alpha,0}^{\frac{\beta}{1-\alpha}}(A) = \boxplus_{\alpha,\beta}(A)$

Proposition 3. *[6] Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta, \omega, \alpha+\beta \in [0,1]$.*

1. $L_{\alpha,\beta}^{\omega}(A)^c = K_{\alpha,\beta}^{\omega}(A^c)$

2. $L_{\alpha,\beta}^{\omega}(A^c) = K_{\alpha,\beta}^{\omega}(A)^c$

Definition 8. *[3] Let X be a universal and $A \in IFS(X)$.*

$$\otimes_{\alpha,\beta,\gamma,\delta}(A) = \{\langle x, \alpha\mu_A(x) + \gamma\nu_A(x), \beta\mu_A(x) + \delta\nu_A(x)\rangle\},$$

where $\alpha, \beta, \gamma, \delta \in [0,1]$ and $\alpha+\beta \leq 1, \gamma+\delta \leq 1$.

After the definition of $\otimes_{\alpha,\beta,\gamma,\delta}$ intuitionistic fuzzy modal operator, we saw that some modal operators may not be associated with any operator in the diagram. $T_{\alpha,\beta}$, $S_{\alpha,\beta}$ modal operators introduced in 2015 and they also are not associated with any operator in the diagram.

Definition 9. *[5] Let X be a universal and $A \in IFS(X), \alpha, \beta, \alpha+\beta \in [0,1]$.*

(1) $T_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x) + \alpha), \alpha(\nu_A(x) + (1-\beta)\mu_A(x))\rangle : x \in X\}$,

(2) $S_{\alpha,\beta}(A) = \{\langle x, \alpha(\mu_A(x) + (1-\beta)\nu_A(x)), \beta(\nu_A(x) + (1-\alpha)\mu_A(x) + \alpha)\rangle : x \in X\}$.

Theorem 1. *[5] Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ then*

$$T_{\alpha,\beta}(A)^c = S_{\alpha,\beta}(A^c).$$

Proposition 4. *[5] Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ then*

1. $T_{\beta,\alpha}(A)^c \sqsubseteq T_{\alpha,\beta}(A^c)$

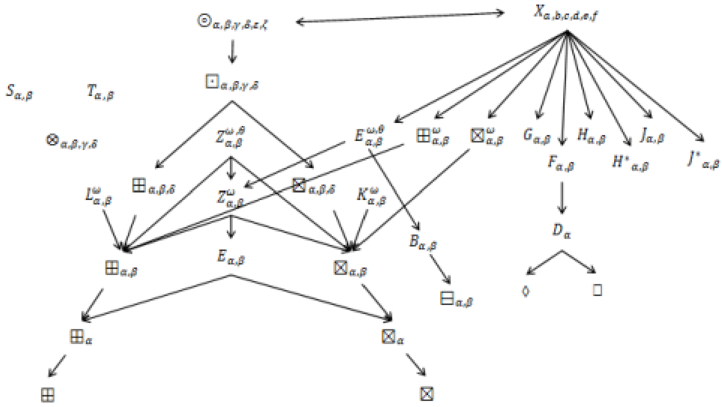
2. $S_{\alpha,\beta}(A^c) \sqsubseteq S_{\beta,\alpha}(A)^c$

Theorem 2. *[5] Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ and $\beta \leq \alpha$ then*

1. $T_{\alpha,\beta}(A) \sqsubseteq T_{\beta,\alpha}(A)$

2. $S_{\beta,\alpha}(A) \sqsubseteq S_{\alpha,\beta}(A)$

Hence, the latest diagram of intuitionistic fuzzy modal operators as following:



2 Main results

In this section, new results on last intuitionistic fuzzy modal operators are introduced. We have obtained some relationships between these modal operators.

Proposition 5. *Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta, \theta \in [0, 1]$ then*

1. $\otimes_{\beta(\theta+\alpha-\alpha\theta),\alpha\theta(1-\beta),\beta\theta(1-\alpha),\alpha(\theta+\beta-\beta\theta)}(A) = E^{0,\theta}_{\alpha,\beta}(A)$
2. $\otimes_{\beta,\alpha(1-\beta),\beta(1-\alpha),\alpha}(A) = B_{\alpha,\beta}(A)$
3. $\otimes_{\beta,\alpha(1-\alpha),\beta(1-\beta),\alpha}(A) = \boxminus_{\alpha,\beta}(A)$

Proof. Let $\alpha, \beta, \theta \in [0, 1]$,

$$\begin{aligned}
\beta(\theta + \alpha - \alpha\theta) + \alpha\theta(1 - \beta) &= \beta\theta + \alpha\,(\beta - \beta\theta + \theta - \beta\theta) \\
&\leq \beta\theta + \beta - \beta\theta + \theta - \beta\theta = \beta(1 - \theta) + \theta \\
&\leq 1 - \theta + \theta \leq 1
\end{aligned}$$

From definition of $\otimes_{\alpha,\beta,\gamma,\delta}$ modal operator and above inequality

$$\otimes_{\beta(\theta+\alpha-\alpha\theta),\alpha\theta(1-\beta),\beta\theta(1-\alpha),\alpha(\theta+\beta-\beta\theta)}(A) = E^{0,\theta}_{\alpha,\beta}(A)$$

is clear. □

Theorem 3. *Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$ then*

$$\otimes_{\alpha,\beta,\beta,\alpha}(B_{\alpha,\alpha}(A)) = B_{\alpha,\alpha}(\otimes_{\alpha,\beta,\beta,\alpha}(A))$$

Proof. Let $\alpha, \beta \in [0, 1]$,

$$\begin{aligned}
&\otimes_{\alpha,\beta,\beta,\alpha}(B_{\alpha,\alpha}(A)) \\
&= \{\langle x, \alpha\,(\alpha\mu_A(x) + \alpha(1-\alpha)\nu_A(x)) + \beta\,(\alpha(1-\alpha)\mu_A(x) + \alpha\nu_A(x))\,, \\
&\beta\,(\alpha\mu_A(x) + \alpha(1-\alpha)\nu_A(x)) + \alpha\,(\alpha(1-\alpha)\mu_A(x) + \alpha\nu_A(x))\rangle : x \in X\}
\end{aligned}$$

$$= \{\langle x, \alpha\left(\alpha\mu_A(x) + \beta\nu_A(x)\right) + \alpha(1-\alpha)\left(\beta\mu_A(x) + \alpha\nu_A(x)\right),$$
$$\alpha(1-\alpha)\left(\alpha\mu_A(x) + \beta\nu_A(x)\right) + \alpha\left(\beta\mu_A(x) + \alpha\nu_A(x)\right)\rangle : x \in X\}$$
$$= B_{\alpha,\alpha}(\otimes_{\alpha,\beta,\beta,\alpha}(A)).$$

Hence the proof. □

Theorem 4. *Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta, \gamma, \delta \in [0,1]$ and $\alpha+\beta \leq 1, \gamma+\delta \leq 1$ then*

1. $\otimes_{\alpha,\beta,\gamma,\delta}\left(L^1_{\alpha,\beta}(A)\right) \sqsubseteq L^1_{\alpha,\beta}(\otimes_{\alpha,\beta,\gamma,\delta}(A))$
2. $K^1_{\alpha,\beta}(\otimes_{\alpha,\beta,\gamma,\delta}(A)) \sqsubseteq \otimes_{\alpha,\beta,\gamma,\delta}\left(K^1_{\alpha,\beta}(A)\right)$

Proof. (1) Let $\alpha, \beta, \gamma, \delta \in [0,1]$,

$$\begin{aligned} \alpha\gamma(1-\beta) &\leq \alpha\gamma \Rightarrow \alpha\gamma(1-\beta)\nu_A(x) \leq \alpha\gamma\nu_A(x) \\ &\Rightarrow \alpha^2\mu_A(x) + \alpha\gamma(1-\beta)\nu_A(x) + \alpha(1-\alpha) \leq \alpha^2\mu_A(x) + \alpha\gamma\nu_A(x) + 1 - \alpha \end{aligned}$$

and

$$\begin{aligned} \alpha\beta(1-\beta) &\leq \alpha\beta \Rightarrow \alpha\beta(1-\beta)\mu_A(x) \leq \alpha\beta\mu_A(x) \\ &\Rightarrow \alpha\beta(1-\beta)\mu_A(x) + \alpha\delta(1-\beta)\nu_A(x) \leq \alpha\beta\mu_A(x) + \alpha\delta(1-\beta)\nu_A(x) \end{aligned}$$

So, we prove that $\otimes_{\alpha,\beta,\gamma,\delta}\left(L^1_{\alpha,\beta}(A)\right) \sqsubseteq L^1_{\alpha,\beta}(\otimes_{\alpha,\beta,\gamma,\delta}(A))$.
(2) It can similarly proved. □

Theorem 5. *Let X be a universal and $A \in IFS(X)$. If $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1, \alpha \leq \frac{1}{2}$ then*

1. $S_{\alpha,\alpha}(\otimes_{\alpha,\beta,\beta,\alpha}(A)) \sqsubseteq \otimes_{\alpha,\beta,\beta,\alpha}(S_{\alpha,\alpha}(A))$
2. $\otimes_{\alpha,\beta,\beta,\alpha}(T_{\alpha,\alpha}(A)) \sqsubseteq T_{\alpha,\alpha}(\otimes_{\alpha,\beta,\beta,\alpha}(A))$

Proof. (1) Let $\alpha, \beta \in [0,1]$,

$$\begin{aligned} 0 &\leq \alpha^2\beta \Rightarrow \alpha(\alpha+\beta(1-\alpha))\mu_A(x) + \alpha(\beta+\alpha(1-\alpha))\nu_A(x) \\ &\leq \alpha(\alpha+\beta(1-\alpha))\mu_A(x) + \alpha(\beta+\alpha(1-\alpha))\nu_A(x) + \alpha^2\beta \end{aligned}$$

and

$$\begin{aligned} \alpha^3 &\leq \alpha^2 \Rightarrow \alpha(\beta+\alpha(1-\alpha))\mu_A(x) + \alpha(\alpha+\beta(1-\alpha))\nu_A(x) + \alpha^3 \\ &\leq \alpha(\beta+\alpha(1-\alpha))\mu_A(x) + \alpha(\alpha+\beta(1-\alpha))\nu_A(x) + \alpha^2 \end{aligned}$$

From above inequalities we have obtain that

$$S_{\alpha,\alpha}(\otimes_{\alpha,\beta,\beta,\alpha}(A)) \sqsubseteq \otimes_{\alpha,\beta,\beta,\alpha}(S_{\alpha,\alpha}(A)).$$

(2) The proof is obvious. □

3 Conclusion

In this study, some ralationships between $\otimes_{\alpha,\beta,\gamma,\delta}$ modal operator and last intuitionistic fuzzy modal operators are examined. Following this, we can study new properties on extended modal operators.

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