Notes on Intuitionistic Fuzzy Sets
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Level operators over primary interval-valued intuitionistic fuzzy *M* group

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Abstract: The concept of interval-valued intuitionistic fuzzy M group is extended by introducing primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group using this concept primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group is defined and using level operators and their properties are established.

Keywords: Intuitionistic fuzzy set, Interval-valued intuitionistic fuzzy set, Primary interval-valued intuitionistic fuzzy *M* group, Primary interval-valued intuitionistic fuzzy anti *M* group.
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1 Introduction

The concept of fuzzy sets was initiated by L. A. Zadeh [8] then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [7] gave the idea of fuzzy subgroup. H. J. Zimmermann [10] gave the idea of fuzzy set theory. The concept of IFS and IVIFS was introduced by K. T. Atanassov [1,2]. The author W. R. Zhang [9] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. K. Chakrabarthy, R. Biwas and S. Nanda [4] investigated a note on union and intersection of intuitionistic fuzzy sets. G. Prasannavengeteswari, K. Gunasekaran and S. Nandakumar [5] introduced the definition of Primary Bipolar Intuitionistic *M* Fuzzy Group and anti *M* Fuzzy Group. A. Balasubramanian, K. L. Muruganantha Prasad, K. Arjunan [3] introduced the definition of Bipolar Interval Valued Fuzzy Subgroups of a Group. G. Prasannavengeteswari, K. Gunasekaran and S. Nandakumar [6] introduced the definition of primary interval-valued intuitionistic fuzzy *M* group and fuzzy anti *M* Group. In this study Level Operators over Primary interval-valued Intuitionistic Fuzzy *M* Group and Fuzzy anti *M* Group and some properties of the same are proved.

2 Preliminaries

Definition 1. An interval-valued intuitionistic fuzzy set (IVIFS) A over the set E is an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle | x \in E\}$, where $M_A(x) \subset [0,1]$ and $N_A(x) \subset [0,1]$ are intervals and $s\text{up}M_A(x) + \text{sup}N_A(x) \leq 1$, for every $x \in E$, Thus we can write IVIFS A as $A = \{\langle x, [inf M_A(x), \text{sup}M_A(x)], [inf N_A(x), \text{sup}N_A(x)]\rangle | x \in E\}$. For simplicity, we write the intervals

$$[\text{inf}M_A(x), \text{sup}M_A(x)] = [\mu_A^-(x), \mu_A^+(x)]$$

and

$$[\text{inf}N_A(x), \text{sup}N_A(x)] = [\nu_A^-(x), \nu_A^+(x)],$$

where $\mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)$ are functions from E into [0, 1] and $(\forall x \in E)$ $(\mu_A^-(x) \leq \mu_A^+(x), \nu_A^-(x) \leq \nu_A^+(x), \mu_A^+(x) + \nu_A^+(x) \leq 1)$ are called the degree of positive membership, degree of negative membership, degree of positive non-membership, and the degree of negative non-membership, respectively. Note that we denote here $\mu_A^-(x) = \text{inf}M_A(x), \mu_A^+(x) = \text{sup}M_A(x), \nu_A^-(x) = \text{inf}N_A(x), \nu_A^+(x) = \text{sup}N_A(x)$.

Definition 2. Let *G* be an *M* group and *A* be an interval-valued intuitionistic fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic fuzzy *M* group of *G*. If for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) \leq \mu_A^+(x^p)$ and $\nu_A^+(mxy) \geq \nu_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) \leq \mu_A^+(y^q)$ and $\nu_A^+(mxy) \geq \nu_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \geq \mu_A^-(x^p)$ and $\nu_A^-(mxy) \leq \nu_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \geq \mu_A^-(y^q)$ and $\nu_A^-(mxy) \leq \nu_A^-(y^q)$, for some $q \in Z_+$.

Example 1.

$$\mu_A^+(x) = \begin{cases} 0.7 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \nu_A^+(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.6 & if\ x = 1 \\ 0.5 & if x = -1 \\ 0.3 & if\ x = i, -i \end{cases} \qquad \nu_A^-(x) = \begin{cases} 0.1 & if\ x = 1 \\ 0.2 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

Definition 3. Let G be an M group and A be an interval-valued intuitionistic anti fuzzy subgroup of G, then A is called a primary interval-valued intuitionistic fuzzy anti M group of G. If for all $x, y \in$ G and $m \in M$, then either $\mu_A^+(mxy) \geq \mu_A^+(x^p)$ and $\nu_A^+(mxy) \leq \nu_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) \geq \mu_A^+(y^q)$ and $\nu_A^+(mxy) \leq \nu_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \leq \mu_A^-(x^p)$ and $\nu_A^-(mxy) \geq \nu(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \leq \mu_A^-(y^q)$ and $\nu_A^-(mxy) \geq \nu_A^-(y^q)$, for some $q \in Z_+$.

Example 2.

$$\mu_A^+(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.7 & if\ x = i, -i \end{cases} \qquad \nu_A^+(x) = \begin{cases} 0.5 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.2 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.3 & if\ x = 1 \\ 0.5 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \nu_A^-(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.2 & if x = -1 \\ 0.1 & if\ x = i, -i \end{cases}$$

Definition 4. Let A be an IVIFS over a set E, then the level operator $!A$ and $?A$ are defined as

$$!A = \left\{\langle x, \left[\min\left\{\frac{1}{2}, \mu_A^+(x)\right\}, \max\left\{\frac{1}{2}, \nu_A^+(x)\right\}\right], \left[\max\left\{\frac{1}{2}, \mu_A^-(x)\right\}, \min\left\{\frac{1}{2}, \nu_A^-(x)\right\}\right]\rangle \,|\, x \in E\right\}$$

$$?A = \left\{\langle x, \left[\max\left\{\frac{1}{2}, \mu_A^+(x)\right\}, \min\left\{\frac{1}{2}, \nu_A^+(x)\right\}\right], \left[\min\left\{\frac{1}{2}, \mu_A^-(x)\right\}, \max\left\{\frac{1}{2}, \nu_A^-(x)\right\}\right]\rangle \,|\, x \in E\right\}$$

3 Some operations on primary interval-valued intuitionistic fuzzy *M* group and primary interval-valued intuitionistic fuzzy anti *M* group

Theorem 1. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then* $!A$ *is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\begin{aligned} \mu_{!A}^+(mxy) &= \min\left(\frac{1}{2}, \mu_A^+(mxy)\right) = \min\left(\frac{1}{2}, supM_A(mxy)\right) \\ &\leq \min\left(\frac{1}{2}, supM_A(x^p)\right) = \min\left(\frac{1}{2}, \mu_A^+(x^p)\right) \\ &= \mu_{!A}^+(x^p) \end{aligned}$$

Therefore, $\mu_{!A}^+(mxy) \leq \mu_{!A}^+(x^p)$, for some $p \in Z_+$.

Consider $\nu^{+}_{!\mathrm{A}}(mxy)) = \max\left(\frac{1}{2}\,, \nu^{+}_{A}(mxy)\right) = \max\left(\frac{1}{2}\,, supN_A(mxy)\right)$

$\geq \max\left(\frac{1}{2}\,, supN_A(x^p)\right) = \max\left(\frac{1}{2}\,, \nu^{+}_{A}(x^p)\right)$

$= \nu^{+}_{!\mathrm{A}}(x^p)$

Therefore, $\nu^{+}_{!\mathrm{A}}(mxy) \geq \nu^{+}_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $\mu^{-}_{!\mathrm{A}}(mxy) = \max\left(\frac{1}{2}\,, \mu^{-}_{A}(mxy)\right) = \max\left(\frac{1}{2}\,, infM_A(mxy)\right)$

$\geq \max\left(\frac{1}{2}\,, infM_A(x^p)\right) = \max\left(\frac{1}{2}\,, \mu^{-}_{A}(x^p)\right)$

$= \mu^{-}_{!\mathrm{A}}(x^p)$

Therefore, $\mu^{-}_{!\mathrm{A}}\,(mxy) \geq \mu^{-}_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $\nu^{-}_{!\mathrm{A}}(mxy)) = \min\left(\frac{1}{2}\,, \nu^{-}_{A}(mxy)\right) = \min\left(\frac{1}{2}\,, infN_A(mxy)\right)$

$\leq \min\left(\frac{1}{2}\,, infN_A(x^p)\right) = \min\left(\frac{1}{2}\,, \nu^{-}_{A}(x^p)\right)$

$= \nu^{-}_{!\mathrm{A}}(x^p)$

Therefore, $\nu^{-}_{!\mathrm{A}}\,(mxy) \leq \nu^{-}_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Therefore, $!\,A$ is a primary interval-valued intuitionistic fuzzy M group of G. □

Theorem 2. *If A and B are primary interval-valued intuitionistic fuzzy M groups of G, then $!(A \cap B) = \,!\,A \cap \,!\,B$ is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $x, y \in B$ and $m \in M$.

Consider $\mu^{+}_{!(A\cap B)}(mxy) = \min\left(\frac{1}{2}\,, \mu^{+}_{A\cap B}(mxy)\right) = \min\left(\frac{1}{2}\,, \min(\mu^{+}_{A}(mxy), \mu^{+}_{B}(mxy))\right)$

$= \min\left(\frac{1}{2}\,, \min(supM_A(mxy), supM_B(mxy))\right)$

$\leq \min\left(\frac{1}{2}\,, \min(supM_A(x^p), supM_B(x^p))\right)$

$= \min\left(\frac{1}{2}\,, \min(\mu^{+}_{A}(x^p), \mu^{+}_{B}(x^p))\right)$

$= \min\left(\min\left(\frac{1}{2}\,, \mu^{+}_{A}(x^p)\right), \min\left(\frac{1}{2}\,, \mu^{+}_{B}(x^p)\right)\right)$

$= \min\left(\mu^{+}_{!\mathrm{A}}(x^p), \mu^{+}_{!\mathrm{B}}(x^p)\right)$

$= \mu^{+}_{!\mathrm{A}\cap!\mathrm{B}}(x^p)$

Therefore, $\mu^{+}_{!(A\cap B)}\,(mxy) \leq \mu^{+}_{!\mathrm{A}\cap!\mathrm{B}}(x^p)$, for some $p \in Z_+$.

Consider $\nu^{+}_{!(A\cap B)}(mxy) = \max\left(\frac{1}{2}\,, \nu^{+}_{A\cap B}(mxy)\right) = \max\left(\frac{1}{2}\,, \max(\nu^{+}_{A}(mxy), \nu^{+}_{B}(mxy))\right)$

$= \max\left(\frac{1}{2}\,, \max(supN_A(mxy), supN_B(mxy))\right)$

$\geq \max\left(\frac{1}{2}\,, \max(supN_A(x^p), supN_B(x^p))\right)$

$= \max\left(\frac{1}{2}\,, \max(\nu^{+}_{A}(x^p), \upsilon^{+}_{B}(x^p))\right)$

$= \max\left(\max\left(\frac{1}{2}\,, \nu^{+}_{A}(x^p)\right), \max\left(\frac{1}{2}\,, \nu^{+}_{B}(x^p)\right)\right)$

$= \max\left(\nu^{+}_{!\mathrm{A}}(x^p), \nu^{+}_{!\mathrm{B}}(x^p)\right)$

$= \nu^{+}_{!\mathrm{A}\cap!\mathrm{B}}(x^p)$

Therefore, $\nu^{+}_{!(A\cap B)}\ (mxy) \geq \nu^{+}_{!\mathrm{A}\cap !\mathrm{B}}(x^p)$, for some $p \in Z_+$.

$$\begin{aligned}
\text{Consider } \mu^{-}_{!(A\cap B)}(mxy) &= \max\left(\frac{1}{2}\ , \mu^{-}_{A\cap B}(mxy)\right) = \max\left(\frac{1}{2}\ , \max(\mu^{-}_{A}(mxy), \mu^{-}_{B}(mxy))\right) \\
&= \max\left(\frac{1}{2}\ , \max(\inf M_A(mxy), \inf M_B(mxy))\right) \\
&\geq \max\left(\frac{1}{2}\ , \max(\inf M_A(x^p), \inf M_B(x^p))\right) \\
&= \max\left(\frac{1}{2}\ , \max(\mu^{-}_{A}(x^p), \mu^{-}_{B}(x^p))\right) \\
&= \max\left(\max\left(\frac{1}{2}\ , \mu^{-}_{A}(x^p)\right), \max\left(\frac{1}{2}\ , \mu^{-}_{B}(x^p)\right)\right) \\
&= \max\left(\mu^{-}_{!\mathrm{A}}(x^p), \mu^{-}_{!\mathrm{B}}(x^p)\right) \\
&= \mu^{-}_{!\mathrm{A}\cap !\mathrm{B}}(x^p)
\end{aligned}$$

Therefore, $\mu^{-}_{!(A\cap B)}\ (mxy) \geq \mu^{-}_{!\mathrm{A}\cap !\mathrm{B}}(x^p)$, for some $p \in Z_+$.

$$\begin{aligned}
\text{Consider } \nu^{-}_{!(A\cap B)}(mxy) &= \min\left(\frac{1}{2}\ , \nu^{-}_{A\cap B}(mxy)\right) = \min\left(\frac{1}{2}\ , \min(\nu^{-}_{A}(mxy), \nu^{-}_{B}(mxy))\right) \\
&= \min\left(\frac{1}{2}\ , \min(inf N_A(mxy), inf N_B(mxy))\right) \\
&\leq \min\left(\frac{1}{2}\ , \min(inf N_A(x^p), inf N_B(x^p))\right) \\
&= \min\left(\frac{1}{2}\ , \min(\nu^{-}_{A}(x^p), \nu^{-}_{B}(x^p))\right) \\
&= \min\left(\min\left(\frac{1}{2}\ , \nu^{-}_{A}(x^p)\right), \min\left(\frac{1}{2}\ , \nu^{-}_{B}(x^p)\right)\right) \\
&= \min\left(\nu^{-}_{!\mathrm{A}}(x^p), \nu^{-}_{!\mathrm{B}}(x^p)\right) \\
&= \nu^{-}_{!\mathrm{A}\cap !\mathrm{B}}(x^p)
\end{aligned}$$

Therefore, $\nu^{-}_{!(A\cap B)}\ (mxy) \leq \nu^{-}_{!\mathrm{A}\cap !\mathrm{B}}(x^p)$, for some $p \in Z_+$.

Therefore, $!\,(A\cap B) = !\,A \cap !\,B$ is a primary interval-valued intuitionistic fuzzy M group of G. □

Theorem 3. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then $?\,A$ is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

$$\begin{aligned}
\text{Consider } \mu^{+}_{?\mathrm{A}}(mxy) &= \max\left(\frac{1}{2}\ , \mu^{+}_{A}(mxy)\right) = \max\left(\frac{1}{2}\ , supM_A(mxy)\right) \\
&\leq \max\left(\frac{1}{2}\ , supM_A(x^p)\right) = \max\left(\frac{1}{2}\ , \mu^{+}_{A}(x^p)\right) \\
&= \mu^{+}_{?\mathrm{A}}(x^p)
\end{aligned}$$

Therefore, $\mu^{+}_{?\mathrm{A}}\ (mxy) \leq \mu^{+}_{?\mathrm{A}}(x^p)$, for some $p \in Z_+$.

$$\begin{aligned}
\text{Consider } \nu^{+}_{?\mathrm{A}}(mxy)) &= \min\left(\frac{1}{2}\ , \nu^{+}_{A}(mxy)\right) = \min\left(\frac{1}{2}\ , supN_A(mxy)\right) \\
&\geq \min\left(\frac{1}{2}\ , supN_A(x^p)\right) \\
&= \min\left(\frac{1}{2}\ , \nu^{+}_{A}(x^p)\right) \\
&= \nu^{+}_{?\mathrm{A}}(x^p)
\end{aligned}$$

Therefore, $\nu^{+}_{?\mathrm{A}}\ (mxy) \geq \nu^{+}_{?\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $$\mu^{-}_{?\mathrm{A}}(mxy) = \min\left(\frac{1}{2}, \mu^{-}_{A}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, inf M_A(mxy)\right)$$
$$\geq \min\left(\frac{1}{2}, inf M_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu^{-}_{A}(x^p)\right)$$
$$= \mu^{-}_{?\mathrm{A}}(x^p)$$

Therefore, $\mu^{-}_{?\mathrm{A}}(mxy) \geq \mu^{-}_{?\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $$\nu^{-}_{?\mathrm{A}}(mxy)) = \max\left(\frac{1}{2}, \nu^{-}_{A}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, inf N_A(mxy)\right)$$
$$\leq \max\left(\frac{1}{2}, inf N_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu^{-}_{A}(x^p)\right)$$
$$= \nu^{-}_{?\mathrm{A}}(x^p)$$

Therefore, $\nu^{-}_{?\mathrm{A}}(mxy) \leq \nu^{-}_{?\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Therefore, $?A$ is a primary interval-valued intuitionistic fuzzy M group of G. ☐

Theorem 4. *If A and B are primary interval-valued intuitionistic fuzzy M groups of G, then $?(A \cap B) =?A \cap ?B$ is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $x, y \in B$ and $m \in M$.

Consider $$\mu^{+}_{?(A\cap B)}(mxy) = \max\left(\frac{1}{2}, \mu^{+}_{A\cap B}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \min(\mu^{+}_{A}(mxy), \mu^{+}_{B}(mxy))\right)$$
$$= \max\left(\frac{1}{2}, \min(supM_A(mxy), supM_B(mxy))\right)$$
$$\leq \max\left(\frac{1}{2}, \min(supM_A(x^p), supM_B(x^p))\right)$$
$$= \max\left(\frac{1}{2}, \min(\mu^{+}_{A}(x^p), \mu^{+}_{B}(x^p))\right)$$
$$= \min\left(\max\left(\frac{1}{2}, \mu^{+}_{A}(x^p)\right), \max\left(\frac{1}{2}, \mu^{+}_{B}(x^p)\right)\right)$$
$$= \min\left(\mu^{+}_{?\mathrm{A}}(x^p), \mu^{+}_{?\mathrm{B}}(x^p)\right)$$
$$= \mu^{+}_{?A\cap ?B}(x^p)$$

Therefore, $\mu^{+}_{?(A\cap B)}(mxy) \leq \mu^{+}_{?\mathrm{A}\cap ?\mathrm{B}}(x^p)$, for some $p \in Z_+$.

Consider $$\nu^{+}_{?(A\cap B)}(mxy) = \min\left(\frac{1}{2}, \nu^{+}_{A\cap B}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \max(\nu^{+}_{A}(mxy), \nu^{+}_{B}(mxy))\right)$$
$$= \min\left(\frac{1}{2}, \max(supN_A(mxy), supN_B(mxy))\right)$$
$$\geq \min\left(\frac{1}{2}, \max(supN_A(x^p), supN_B(x^p))\right)$$
$$= \min\left(\frac{1}{2}, \max(\nu^{+}_{A}(x^p), \nu^{+}_{B}(x^p))\right)$$

$$= \max\left(\min\left(\frac{1}{2}, \nu_A^+(x^p)\right), \min\left(\frac{1}{2}, \nu_B^+(x^p)\right)\right)$$
$$= \max\left(\nu_{?\mathrm{A}}^+(x^p), \nu_{?\mathrm{B}}^+(x^p)\right)$$
$$= \nu_{?\mathrm{A}\cap?\mathrm{B}}^+(x^p)$$

Therefore, $\nu_{?(A\cap B)}^+(mxy) \geq \nu_{?\mathrm{A}\cap?\mathrm{B}}^+(x^p)$, for some $p \in Z_+$.

Consider $\mu_{?(A\cap B)}^-(mxy) = \min\left(\frac{1}{2}, \mu_{A\cap B}^-(mxy)\right)$

$$= \min\left(\frac{1}{2}, \max(\mu_A^-(mxy), \mu_B^-(mxy))\right)$$
$$= \min\left(\frac{1}{2}, \max(inf M_A(mxy), inf M_B(mxy))\right)$$
$$\geq \min\left(\frac{1}{2}, \max(inf M_A(x^p), inf M_B(x^p))\right)$$
$$= \min\left(\frac{1}{2}, \max(\mu_A^-(x^p), \mu_B^-(x^p))\right)$$
$$= \max\left(\min\left(\frac{1}{2}, \mu_A^-(x^p)\right), \min\left(\frac{1}{2}, \mu_B^-(x^p)\right)\right)$$
$$= \max\left(\mu_{?\mathrm{A}}^-(x^p), \mu_{?\mathrm{B}}^-(x^p)\right)$$
$$= \mu_{?\mathrm{A}\cap?\mathrm{B}}^-(x^p)$$

Therefore, $\mu_{?(A\cap B)}^-(mxy) \geq \mu_{?\mathrm{A}\cap?\mathrm{B}}^-(x^p)$, for some $p \in Z_+$.

Consider $\nu_{?(A\cap B)}^-(mxy) = \max\left(\frac{1}{2}, \nu_{A\cap B}^-(mxy)\right)$

$$= \max\left(\frac{1}{2}, \min(\nu_A^-(mxy), \nu_B^-(mxy))\right)$$
$$= \max\left(\frac{1}{2}, \min(inf N_A(mxy), inf N_B(mxy))\right)$$
$$\leq \max\left(\frac{1}{2}, \min(inf N_A(x^p), inf N_B(x^p))\right)$$
$$= \max\left(\frac{1}{2}, \min(\nu_A^-(x^p), \nu_B^-(x^p))\right)$$
$$= \min\left(\max\left(\frac{1}{2}, \nu_A^-(x^p)\right), \max\left(\frac{1}{2}, \nu_B^-(x^p)\right)\right)$$
$$= \min\left(\nu_{?\mathrm{A}}^-(x^p), \nu_{?\mathrm{B}}^-(x^p)\right)$$
$$= \nu_{?\mathrm{A}\cap?\mathrm{B}}^-(x^p).$$

Therefore, $\nu_{?(A\cap B)}^-(mxy) \leq \nu_{?\mathrm{A}\cap?\mathrm{B}}^-(x^p)$, for some $p \in Z_+$. Therefore, $?(A\cap B) = ?A \cap ?B$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*. □

Theorem 5. If *A* is a primary interval-valued intuitionistic fuzzy *M* group of *G*, then $\overline{?\overline{A}} = !A$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*.

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $\mu_{\overline{?\overline{\mathrm{A}}}}^+(mxy) = \nu_{?\overline{\mathrm{A}}}^+(mxy)$

$$= \min\left(\frac{1}{2}, \nu_{\overline{\mathrm{A}}}^+(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^+(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \sup M_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, \sup M_A(x^p)\right)$$

$$= \min\left(\frac{1}{2}, \mu^{+}_{\overline{\mathrm{A}}}(x^p)\right)$$
$$= \mu^{+}_{!\overline{\mathrm{A}}}(x^p)$$

Therefore, $\mu^{+}_{\overline{?\overline{\mathrm{A}}}}(mxy) \leq \mu^{+}_{!\overline{\mathrm{A}}}(x^p)$, for some $p \in Z_{+}$.

Consider $\qquad \nu^{+}_{\overline{?\overline{\mathrm{A}}}}(mxy) = \mu^{+}_{?\overline{\mathrm{A}}}(mxy)$

$$= \max\left(\frac{1}{2}, \mu^{+}_{\overline{\mathrm{A}}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \nu^{+}_{\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \sup N_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, \sup N_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu^{+}_{\mathrm{A}}(x^p)\right)$$
$$= \nu^{+}_{!\mathrm{A}}(x^p)$$

Therefore, $\nu^{+}_{\overline{?\overline{\mathrm{A}}}}(mxy) \geq \nu^{+}_{!\mathrm{A}}(x^p)$, for some $p \in Z_{+}$.

Consider $\qquad \mu^{-}_{\overline{?\overline{\mathrm{A}}}}(mxy) = \nu^{-}_{?\overline{\mathrm{A}}}(mxy)$

$$= \max\left(\frac{1}{2}, \nu^{-}_{\overline{\mathrm{A}}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \mu^{-}_{\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \inf M_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, \inf M_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \mu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \mu^{-}_{!\mathrm{A}}(x^p)$$

Therefore, $\mu^{-}_{\overline{?\overline{\mathrm{A}}}}(mxy) \geq \mu^{-}_{!\mathrm{A}}(x^p)$, for some $p \in Z_{+}$

Consider $\qquad \nu^{-}_{\overline{?\overline{\mathrm{A}}}}(mxy) = \mu^{-}_{?\overline{\mathrm{A}}}(mxy)$

$$= \min\left(\frac{1}{2}, \mu^{-}_{\overline{\mathrm{A}}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \nu^{-}_{\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, inf\, N_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, \inf N_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \nu^{-}_{!\mathrm{A}}(x^p)$$

Therefore, $\nu^{-}_{\overline{?\overline{\mathrm{A}}}}(mxy) \leq \nu^{-}_{!\mathrm{A}}(x^p)$, for some $p \in Z_{+}$. Therefore, $\overline{?\overline{\mathrm{A}}} = !A$ is a primary interval-valued intuitionistic fuzzy M group of G. □

Theorem 6. If A is a primary interval-valued intuitionistic fuzzy M group of G, then $!(?A) = ?(!A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $\quad \mu^{+}_{!(?A)}(mxy) = \min\left(\frac{1}{2}, \mu^{+}_{?A}(mxy)\right)$

$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \mu^{+}_{A}(mxy)\right)\right)$$

$$= \min\left(\frac{1}{2}\,, \max\left(\frac{1}{2}\,, supM_A(mxy)\right)\right)$$
$$\leq \min\left(\frac{1}{2}\,, \max\left(\frac{1}{2}\,, supM_A(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}\,, \max\left(\frac{1}{2}\,, \mu_A^+(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, \mu_A^+(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}\,, \mu_{!A}^+(x^p)\right)$$
$$= \mu_{?(!A)}^+\,(x^p)$$

Therefore, $\mu_{!(?A)}^+\,(mxy) \leq \mu_{?(!A)}^+\,(x^p)$, for some $p \in Z_+$.

Consider $$\nu_{!(?A)}^+(mxy) = \max\left(\frac{1}{2}\,, \nu_{?A}^+(mxy)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, \nu_A^+(mxy)\right)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, supN_A(mxy)\right)\right)$$
$$\geq \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, supN_A(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, \nu_A^+(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}\,, \max\left(\frac{1}{2}\,, \nu_A^+(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}\,, \nu_{!A}^+(x^p)\right)$$
$$= \nu_{?(!A)}^+\,(x^p)$$

Therefore, $\nu_{!(?A)}^+\,(mxy) \geq \nu_{?(!A)}^+\,(x^p)$, for some $p \in Z_+$.

Consider $$\mu_{!(?A)}^-(mxy) = \max\left(\frac{1}{2}\,, \mu_{?A}^-(mxy)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, \mu_A^-(mxy)\right)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, infM_A(mxy)\right)\right)$$
$$\geq \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, infM_A(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}\,, \min\left(\frac{1}{2}\,, \mu_A^-(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}\,, \max\left(\frac{1}{2}\,, \mu_A^-(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}\,, \mu_{!A}^-(x^p)\right)$$
$$= \mu_{?(!A)}^-\,(x^p)$$

Therefore, $\mu_{!(?A)}^-\,(mxy) \geq \mu_{?(!A)}^-\,(x^p)$, for some $p \in Z_+$.

Consider $\nu^-_{!(?A)}(mxy) = \min\left(\frac{1}{2}, \nu^-_{?A}(mxy)\right)$

$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \nu^-_A(mxy)\right)\right)$$
$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, inf N_A(mxy)\right)\right)$$
$$\leq \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, inf N_A(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \nu^-_A(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \nu^-_A(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}, \nu^-_{!A}(x^p)\right)$$
$$= \nu^-_{?(!A)}(x^p)$$

Therefore, $\nu^-_{!(?A)}(mxy) \leq \nu^-_{?(!A)}(x^p)$, for some $p \in Z_+$. Therefore, $!(?A) = ?(!A)$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*. □

Theorem 7. If A is a primary interval-valued intuitionistic fuzzy *M* group of *G*, then $!(\square A) = \square(!A)$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*.

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $\mu^+_{!(\square \mathrm{A})}(mxy) = \min\left(\frac{1}{2}, \mu^+_{\square \mathrm{A}}(mxy)\right)$

$$= \min\left(\frac{1}{2}, \mu^+_{\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, supM_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, supM_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu^+_{\mathrm{A}}(x^p)\right)$$
$$= \mu^+_{!\mathrm{A}}(x^p)$$
$$= \mu^+_{\square(!\mathrm{A})}(x^p)$$

Therefore, $\mu^+_{!(\square \mathrm{A})}(mxy) \leq \mu^+_{\square(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider $\nu^+_{!(\square \mathrm{A})}(mxy) = \max\left(\frac{1}{2}, \nu^+_{\square \mathrm{A}}(mxy)\right)$

$$= \max\left(\frac{1}{2}, 1 - \mu^+_{\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - supM_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, 1 - supM_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu^+_{\mathrm{A}}(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu^+_{\mathrm{A}}(x^p)\right)$$
$$= \nu^+_{!\mathrm{A}}(x^p)$$

$$= 1 - \mu^{+}_{!\mathrm{A}}(x^p)$$
$$= 1 - \mu^{+}_{\square(!\mathrm{A})}(x^p)$$
$$= \nu^{+}_{\square(!\mathrm{A})}(x^p)$$

Therefore, $\nu^{+}_{!(\square\mathrm{A})}(mxy) \geq \nu^{+}_{\square(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider $\mu^{-}_{!(\square\mathrm{A})}(mxy) = \max\left(\frac{1}{2}, \mu^{-}_{\square\mathrm{A}}(mxy)\right)$

$$= \max\left(\frac{1}{2}, \mu^{-}_{\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \inf M_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, \inf M_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \mu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \mu^{-}_{!\mathrm{A}}(x^p)$$
$$= \mu^{-}_{\square(!\mathrm{A})}(x^p)$$

Therefore, $\mu^{-}_{!(\square\mathrm{A})}(mxy) \geq \mu^{-}_{\square(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider $\nu^{-}_{!(\square\mathrm{A})}(mxy) = \min\left(\frac{1}{2}, \nu^{-}_{\square\mathrm{A}}(mxy)\right)$

$$= \min\left(\frac{1}{2}, 1 - \mu^{-}_{\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \inf M_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, 1 - \inf M_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \mu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \nu^{-}_{!\mathrm{A}}(x^p)$$
$$= 1 - \mu^{-}_{!\mathrm{A}}(x^p)$$
$$= 1 - \mu^{-}_{\square(!\mathrm{A})}(x^p)$$
$$= \nu^{-}_{\square(!\mathrm{A})}(x^p)$$

Therefore, $\nu^{-}_{!(\square\mathrm{A})}(mxy) \leq \nu^{-}_{\square(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Therefore, $!(\square A) = \square(!A)$ is a primary interval-valued intuitionistic fuzzy M group of G. □

Theorem 8. *If A is a primary interval-valued intuitionistic fuzzy M group, then* $?(\square A) = \square(?A)$ *is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $\mu^{+}_{?(\square\mathrm{A})}(mxy) = \max\left(\frac{1}{2}, \mu^{+}_{\square\mathrm{A}}(mxy)\right)$

$$= \max\left(\frac{1}{2}, \mu^{+}_{\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \sup M_A(mxy)\right)$$
$$\leq \max\left(\frac{1}{2}, \sup M_A(x^p)\right)$$

$$= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^{+}(x^p)$$
$$= \mu_{\square(?\mathrm{A})}^{+}(x^p)$$

Therefore, $\mu_{?(\square\mathrm{A})}^{+}(mxy) \leq \mu_{\square(?\mathrm{A})}^{+}(x^p)$, for some $p \in Z_{+}$.

Consider $$\nu_{?(\square\mathrm{A})}^{+}(mxy) = \min\left(\frac{1}{2}, \nu_{\square\mathrm{A}}^{+}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{+}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - supM_A(mxy)\right)$$
$$\geq \min\left(\frac{1}{2}, 1 - supM_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^{+}(x^p)$$
$$= 1 - \mu_{?\mathrm{A}}^{+}(x^p)$$
$$= 1 - \mu_{\square(?\mathrm{A})}^{+}(x^p)$$
$$= \nu_{\square(?\mathrm{A})}^{+}(x^p)$$

Therefore, $\nu_{?(\square\mathrm{A})}^{+}(mxy) \geq \nu_{\square(?\mathrm{A})}^{+}(x^p)$, for some $p \in Z_{+}$.

Consider $$\mu_{?(\square\mathrm{A})}^{-}(mxy) = \min\left(\frac{1}{2}, \mu_{\square\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, infM_A(mxy)\right)$$
$$\geq \min\left(\frac{1}{2}, infM_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^{-}(x^p)$$
$$= \mu_{\square(?\mathrm{A})}^{-}(x^p)$$

Therefore, $\mu_{?(\square\mathrm{A})}^{-}(mxy) \geq \mu_{\square(?\mathrm{A})}^{-}(x^p)$, for some $p \in Z_{+}$.

Consider $$\nu_{?(\square\mathrm{A})}^{-}(mxy) = \max\left(\frac{1}{2}, \nu_{\square\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - infM_A(mxy)\right)$$
$$\leq \max\left(\frac{1}{2}, 1 - infM_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^{-}(x^p)$$
$$= 1 - \mu_{?\mathrm{A}}^{-}(x^p)$$

$$= 1 - \mu^-_{\square(?\mathrm{A})}(x^p)$$
$$= \nu^-_{\square(?\mathrm{A})}(x^p)$$

Therefore, $\nu^-_{?(\square A)}(mxy) \le \nu^-_{\square(?A)}(x^p)$, for some $p \in Z_+$.

Therefore, $?(\square A) = \square(?A)$ is a primary interval-valued intuitionistic fuzzy M group of G. $\square$

Theorem 9. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then $!(\lozenge A) = \lozenge(!A)$ is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\begin{aligned}
\mu^+_{!(\lozenge\mathrm{A})}(mxy) &= \min\left(\frac{1}{2}, \mu^+_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \min\left(\frac{1}{2}, 1 - \nu^+_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \min\left(\frac{1}{2}, 1 - \nu^+_{\mathrm{A}}(mxy)\right) \\
&= \min\left(\frac{1}{2}, 1 - supN_A(mxy)\right) \\
&\le \min\left(\frac{1}{2}, 1 - supN_A(x^p)\right) \\
&= \min\left(\frac{1}{2}, 1 - \nu^+_{\mathrm{A}}(x^p)\right) \\
&= \min\left(\frac{1}{2}, \mu^+_{\mathrm{A}}(x^p)\right) \\
&= \mu^+_{!\mathrm{A}}(x^p) \\
&= 1 - \nu^+_{!\mathrm{A}}(x^p) \\
&= 1 - \nu^+_{\lozenge(!\mathrm{A})}(x^p) \\
&= \mu^+_{\lozenge(!\mathrm{A})}(x^p)
\end{aligned}$$

Therefore, $\mu^+_{!(\lozenge\mathrm{A})}(mxy) \le \mu^+_{\lozenge(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}
\nu^+_{!(\lozenge\mathrm{A})}(mxy) &= \max\left(\frac{1}{2}, \nu^+_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, \nu^+_{\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, supN_A(mxy)\right) \\
&\ge \max\left(\frac{1}{2}, supN_A(x^p)\right) \\
&= \max\left(\frac{1}{2}, \nu^+_{\mathrm{A}}(x^p)\right) \\
&= \nu^+_{!\mathrm{A}}(x^p) \\
&= \nu^+_{\lozenge(!\mathrm{A})}(x^p),
\end{aligned}$$

Therefore, $\nu^+_{!(\lozenge\mathrm{A})}(mxy) \ge \nu^+_{\lozenge(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}
\mu^-_{!(\lozenge\mathrm{A})}(mxy) &= \max\left(\frac{1}{2}, \mu^-_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, 1 - \nu^-_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, 1 - \nu^-_{\mathrm{A}}(mxy)\right)
\end{aligned}$$

$$
\begin{aligned}
&= \max\left(\frac{1}{2}, 1 - inf N_A(mxy)\right)\\
&\geq \max\left(\frac{1}{2}, 1 - inf N_A(x^p)\right)\\
&= \max\left(\frac{1}{2}, 1 - \nu_{\mathrm{A}}^{-}(x^p)\right)\\
&= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(x^p)\right)\\
&= \mu_{!\mathrm{A}}^{-}(x^p)\\
&= 1 - \nu_{!\mathrm{A}}^{-}(x^p)\\
&= 1 - \nu_{\lozenge(!\mathrm{A})}^{-}(x^p)\\
&= \mu_{\lozenge(!\mathrm{A})}^{-}(x^p)
\end{aligned}
$$

Therefore, $\mu_{!(\lozenge\mathrm{A})}^{-}(mxy) \geq \mu_{\lozenge(!\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$.

Consider
$$
\begin{aligned}
\nu_{!(\lozenge\mathrm{A})}^{-}(mxy) &= \min\left(\frac{1}{2}, \nu_{\lozenge\mathrm{A}}^{-}(mxy)\right)\\
&= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(mxy)\right)\\
&= \min\left(\frac{1}{2}, inf N_A(mxy)\right)\\
&= \min\left(\frac{1}{2}, inf N_A(x^p)\right)\\
&\leq \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(x^p)\right)\\
&= \nu_{!\mathrm{A}}^{-}(x^p)\\
&= \nu_{\lozenge(!\mathrm{A})}^{-}(x^p).
\end{aligned}
$$

Therefore, $\nu_{!(\lozenge\mathrm{A})}^{-}(mxy) \leq \nu_{\lozenge(!\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$.

Therefore, $!\,(\lozenge\ \mathrm{A}) = \lozenge\ (!\,\mathrm{A})$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*. ☐

Theorem 10. *If A is a primary interval-valued intuitionistic fuzzy M group of G, then* $?\,(\lozenge\ A) = \lozenge\ (?\,A)$ *is a primary interval-valued intuitionistic fuzzy M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$
\begin{aligned}
\mu_{?(\lozenge\mathrm{A})}^{+}(mxy) &= \max\left(\frac{1}{2}, \mu_{\lozenge\mathrm{A}}^{+}(mxy)\right)\\
&= \max\left(\frac{1}{2}, 1 - \nu_{\lozenge\mathrm{A}}^{+}(mxy)\right)\\
&= \max\left(\frac{1}{2}, 1 - \nu_{\mathrm{A}}^{+}(mxy)\right)\\
&= \max\left(\frac{1}{2}, 1 - sup N_A(mxy)\right)\\
&\leq \max\left(\frac{1}{2}, 1 - sup N_A(x^p)\right)\\
&= \max\left(\frac{1}{2}, 1 - \nu_{\mathrm{A}}^{+}(x^p)\right)\\
&= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{+}(x^p)\right)\\
&= \mu_{?\mathrm{A}}^{+}(x^p)\\
&= 1 - \nu_{?\mathrm{A}}^{+}(x^p)\\
&= 1 - \nu_{\lozenge(?(\mathrm{A}))}^{+}(x^p)
\end{aligned}
$$

$$= \mu^{+}_{\lozenge(?\mathrm{A})}(x^p)$$

Therefore, $\mu^{+}_{?(\lozenge\mathrm{A})}(mxy) \le \mu^{+}_{\lozenge(?\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider $$\nu^{+}_{?(\lozenge\mathrm{A})}(mxy) = \min\left(\frac{1}{2}, \nu^{+}_{\lozenge\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \nu^{+}_{\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, supN_A(mxy)\right)$$
$$\ge \min\left(\frac{1}{2}, supN_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu^{+}_{\mathrm{A}}(x^p)\right)$$
$$= \nu^{+}_{?\mathrm{A}}(x^p)$$
$$= \nu^{+}_{\lozenge(?\mathrm{A})}(x^p)$$

Therefore, $\nu^{+}_{?(\lozenge\mathrm{A})}(mxy) \ge \nu^{+}_{\lozenge(?\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider $$\mu^{-}_{?(\lozenge\mathrm{A})}(mxy) = \min\left(\frac{1}{2}, \mu^{-}_{\lozenge\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \nu^{-}_{\lozenge\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \nu^{-}_{\mathrm{A}}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - infN_A(mxy)\right)$$
$$\ge \min\left(\frac{1}{2}, 1 - infN_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \nu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \mu^{-}_{?\mathrm{A}}(x^p)$$
$$= 1 - \nu^{-}_{?\mathrm{A}}(x^p)$$
$$= 1 - \nu^{-}_{\lozenge(?\mathrm{A})}(x^p)$$
$$= \mu^{-}_{\lozenge(?\mathrm{A})}(x^p)$$

Therefore, $\mu^{-}_{?(\lozenge\mathrm{A})}(mxy) \ge \mu^{-}_{\lozenge(?\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider $$\nu^{-}_{?(\lozenge\mathrm{A})}(mxy) = \max\left(\frac{1}{2}, \nu^{-}_{\lozenge\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \nu^{-}_{\mathrm{A}}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, infN_A(mxy)\right)$$
$$\le \max\left(\frac{1}{2}, infN_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu^{-}_{\mathrm{A}}(x^p)\right)$$
$$= \nu^{-}_{?\mathrm{A}}(x^p)$$
$$= \nu^{-}_{\lozenge(?\mathrm{A})}(x^p)$$

Therefore, $\nu^{-}_{?(\lozenge\mathrm{A})}(mxy) \le \nu^{-}_{\lozenge(?\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Therefore, $?(\lozenge \mathrm{A}) = \lozenge(?\mathrm{A})$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*. □

Theorem 11. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then !A is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $$\begin{aligned} \mu^+_{!\mathrm{A}}(mxy) &= \min\left(\tfrac{1}{2}, \mu^+_A(mxy)\right) \\ &= \min\left(\tfrac{1}{2}, supM_A(mxy)\right) \\ &\geq \min\left(\tfrac{1}{2}, supM_A(x^p)\right) \\ &= \min\left(\tfrac{1}{2}, \mu^+_A(x^p)\right) \\ &= \mu^+_{!\mathrm{A}}(x^p) \end{aligned}$$

Therefore, $\mu^+_{!\mathrm{A}}(mxy) \geq \mu^+_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $$\begin{aligned} \nu^+_{!\mathrm{A}}(mxy)) &= \max\left(\tfrac{1}{2}, \nu^+_A(mxy)\right) \\ &= \max\left(\tfrac{1}{2}, supN_A(mxy)\right) \\ &\leq \max\left(\tfrac{1}{2}, supN_A(x^p)\right) \\ &= \max\left(\tfrac{1}{2}, \nu^+_A(x^p)\right) \\ &= \nu^+_{!\mathrm{A}}(x^p) \end{aligned}$$

Therefore, $\nu^+_{!\mathrm{A}}(mxy) \leq \nu^+_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $$\begin{aligned} \mu^-_{!\mathrm{A}}(mxy) &= \max\left(\tfrac{1}{2}, \mu^-_A(mxy)\right) \\ &= \max\left(\tfrac{1}{2}, inf M_A(mxy)\right) \\ &\leq \max\left(\tfrac{1}{2}, inf M_A(x^p)\right) \\ &= \max\left(\tfrac{1}{2}, \mu^-_A(x^p)\right) \\ &= \mu^-_{!\mathrm{A}}(x^p) \end{aligned}$$

Therefore, $\mu^-_{!\mathrm{A}}(mxy) \leq \mu^-_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Consider $$\begin{aligned} \nu^-_{!\mathrm{A}}(mxy)) &= \min\left(\tfrac{1}{2}, \nu^-_A(mxy)\right) \\ &= \min\left(\tfrac{1}{2}, inf N_A(mxy)\right) \\ &\geq \min\left(\tfrac{1}{2}, inf N_A(x^p)\right) \\ &= \min\left(\tfrac{1}{2}, \nu^-_A(x^p)\right) \\ &= \nu^-_{!\mathrm{A}}(x^p) \end{aligned}$$

Therefore, $\nu^-_{!\mathrm{A}}(mxy) \geq \nu^-_{!\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Therefore, $!A$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 12. *If A and B are primary interval-valued intuitionistic fuzzy anti M group of G, then* $!(A \cap B) = !A \cap !B$ *is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $x, y \in B$ and $m \in M$.

Consider
$$\begin{aligned}
\mu^{+}_{!(A\cap B)}(mxy) &= \min\left(\frac{1}{2}, \mu^{+}_{A\cap B}(mxy)\right)\\
&= \min\left(\frac{1}{2}, \min(\mu^{+}_{A}(mxy), \mu^{+}_{B}(mxy))\right)\\
&= \min\left(\frac{1}{2}, \min(supM_{A}(mxy), supM_{B}(mxy))\right)\\
&\geq \min\left(\frac{1}{2}, \min(supM_{A}(x^{p}), supM_{B}(x^{p}))\right)\\
&= \min\left(\frac{1}{2}, \min(\mu^{+}_{A}(x^{p}), \mu^{+}_{B}(x^{p}))\right)\\
&= \min\left(\min\left(\frac{1}{2}, \mu^{+}_{A}(x^{p})\right), \min\left(\frac{1}{2}, \mu^{+}_{B}(x^{p})\right)\right)\\
&= \min\left(\mu^{+}_{!\mathrm{A}}(x^{p}), \mu^{+}_{!\mathrm{B}}(x^{p})\right)\\
&= \mu^{+}_{!\mathrm{A}\cap !\mathrm{B}}(x^{p})
\end{aligned}$$

Therefore, $\mu^{+}_{!(A\cap B)}(mxy) \geq \mu^{+}_{!\mathrm{A}\cap !\mathrm{B}}(x^{p})$, for some $p \in Z_{+}$.

Consider
$$\begin{aligned}
\nu^{+}_{!(A\cap B)}(mxy) &= \max\left(\frac{1}{2}, \nu^{+}_{A\cap B}(mxy)\right)\\
&= \max\left(\frac{1}{2}, \max(\nu^{+}_{A}(mxy), \nu^{+}_{B}(mxy))\right)\\
&= \max\left(\frac{1}{2}, \max(supN_{A}(mxy), supN_{B}(mxy))\right)\\
&\leq \max\left(\frac{1}{2}, \max(supN_{A}(x^{p}), supN_{B}(x^{p}))\right)\\
&= \max\left(\frac{1}{2}, \max(\nu^{+}_{A}(x^{p}), \upsilon^{+}_{B}(x^{p}))\right)\\
&= \max\left(\max\left(\frac{1}{2}, \nu^{+}_{A}(x^{p})\right), \max\left(\frac{1}{2}, \nu^{+}_{B}(x^{p})\right)\right)\\
&= \max\left(\nu^{+}_{!\mathrm{A}}(x^{p}), \nu^{+}_{!\mathrm{B}}(x^{p})\right)\\
&= \nu^{+}_{!\mathrm{A}\cap !\mathrm{B}}(x^{p})
\end{aligned}$$

Therefore, $\nu^{+}_{!(A\cap B)}(mxy) \leq \nu^{+}_{!\mathrm{A}\cap !\mathrm{B}}(x^{p})$, for some $p \in Z_{+}$.

Consider
$$\begin{aligned}
\mu^{-}_{!(A\cap B)}(mxy) &= \max\left(\frac{1}{2}, \mu^{-}_{A\cap B}(mxy)\right)\\
&= \max\left(\frac{1}{2}, \max(\mu^{-}_{A}(mxy), \mu^{-}_{B}(mxy))\right)\\
&= \max\left(\frac{1}{2}, \max(infM_{A}(mxy), infM_{B}(mxy))\right)\\
&\leq \max\left(\frac{1}{2}, \max(infM_{A}(x^{p}), infM_{B}(x^{p}))\right)\\
&= \max\left(\frac{1}{2}, \max(\mu^{-}_{A}(x^{p}), \mu^{-}_{B}(x^{p}))\right)\\
&= \max\left(\max\left(\frac{1}{2}, \mu^{-}_{A}(x^{p})\right), \max\left(\frac{1}{2}, \mu^{-}_{B}(x^{p})\right)\right)\\
&= \max\left(\mu^{-}_{!\mathrm{A}}(x^{p}), \mu^{-}_{!\mathrm{B}}(x^{p})\right)\\
&= \mu^{-}_{!\mathrm{A}\cap !\mathrm{B}}(x^{p})
\end{aligned}$$

Therefore, $\mu^{-}_{!(A\cap B)}(mxy) \leq \mu^{-}_{!\mathrm{A}\cap !\mathrm{B}}(x^{p})$, for some $p \in Z_{+}$.

Consider
$$\begin{aligned}
\nu^{-}_{!(A\cap B)}(mxy) &= \min\left(\frac{1}{2}, \nu^{-}_{A\cap B}(mxy)\right)\\
&= \min\left(\frac{1}{2}, \min(\nu^{-}_{A}(mxy), \nu^{-}_{B}(mxy))\right)
\end{aligned}$$

$$= \min\left(\frac{1}{2}, \min(infN_A(mxy), \inf N_B(mxy))\right)$$
$$\geq \min\left(\frac{1}{2}, \min(infN_A(x^p), \inf N_B(x^p))\right)$$
$$= \min\left(\frac{1}{2}, \min(\nu_A^-(x^p), \nu_B^-(x^p))\right)$$
$$= \min\left(\min\left(\frac{1}{2}, \nu_A^-(x^p)\right), \min\left(\frac{1}{2}, \nu_B^-(x^p)\right)\right)$$
$$= \min\left(\nu_{!\mathrm{A}}^-(x^p), \nu_{!\mathrm{B}}^-(x^p)\right)$$
$$= \nu_{!\mathrm{A}\cap !\mathrm{B}}^-(x^p)$$

Therefore, $\nu_{!(A\cap B)}^-(mxy) \geq \nu_{!\mathrm{A}\cap !\mathrm{B}}^-(x^p)$, for some $p \in Z_+$.

Therefore, $!(A \cap B) = !A \cap !B$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 13. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then ?A is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\mu_{?\mathrm{A}}^+(mxy) = \max\left(\frac{1}{2}, \mu_A^+(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \sup M_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, \sup M_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \mu_A^+(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^+(x^p)$$

Therefore, $\mu_{?\mathrm{A}}^+(mxy) \geq \mu_{?\mathrm{A}}^+(x^p)$, for some $p \in Z_+$.

Consider
$$\nu_{?\mathrm{A}}^+(mxy)) = \min\left(\frac{1}{2}, \nu_A^+(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \sup N_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, \sup N_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu_A^+(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^+(x^p)$$

Therefore, $\nu_{?\mathrm{A}}^+(mxy) \leq \nu_{?\mathrm{A}}^+(x^p)$, for some $p \in Z_+$.

Consider
$$\mu_{?\mathrm{A}}^-(mxy) = \min\left(\frac{1}{2}, \mu_A^-(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \inf M_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, \inf M_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu_A^-(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^-(x^p)$$

Therefore, $\mu_{?\mathrm{A}}^-(mxy) \leq \mu_{?\mathrm{A}}^-(x^p)$, for some $p \in Z_+$.

Consider $\nu^{-}_{?\mathrm{A}}(mxy)) = \max\left(\frac{1}{2}, \nu^{-}_{A}(mxy)\right)$

$= \max\left(\frac{1}{2}, \inf N_A(mxy)\right)$

$\geq \max\left(\frac{1}{2}, \inf N_A(x^p)\right)$

$= \max\left(\frac{1}{2}, \nu^{-}_{A}(x^p)\right)$

$= \nu^{-}_{?\mathrm{A}}(x^p)$

Therefore, $\nu^{-}_{?\mathrm{A}}(mxy) \geq \nu^{-}_{?\mathrm{A}}(x^p)$, for some $p \in Z_+$.

Therefore, $?A$ is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*. □

Theorem 14. *If A and B are primary interval-valued intuitionistic fuzzy anti M group of G, then* $?(A \cap B) = ?A \cap ?B$ *is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $x, y \in B$ and $m \in M$.

Consider $\mu^{+}_{?(A\cap B)}(mxy) = \max\left(\frac{1}{2}, \mu^{+}_{A\cap B}(mxy)\right)$

$= \max\left(\frac{1}{2}, \min(\mu^{+}_{A}(mxy), \mu^{+}_{B}(mxy))\right)$

$= \max\left(\frac{1}{2}, \min(\sup M_A(mxy), \sup M_B(mxy))\right)$

$\geq \max\left(\frac{1}{2}, \min(\sup M_A(x^p), \sup M_B(x^p))\right)$

$= \max\left(\frac{1}{2}, \min(\mu^{+}_{A}(x^p), \mu^{+}_{B}(x^p))\right)$

$= \min\left(\max\left(\frac{1}{2}, \mu^{+}_{A}(x^p)\right), \max\left(\frac{1}{2}, \mu^{+}_{B}(x^p)\right)\right)$

$= \min\left(\mu^{+}_{?\mathrm{A}}(x^p), \mu^{+}_{?\mathrm{B}}(x^p)\right)$

$= \mu^{+}_{?A\cap ?B}(x^p)$

Therefore, $\mu^{+}_{?(A\cap B)}(mxy) \geq \mu^{+}_{?\mathrm{A}\cap ?\mathrm{B}}(x^p)$, for some $p \in Z_+$.

Consider $\nu^{+}_{?(A\cap B)}(mxy) = \min\left(\frac{1}{2}, \nu^{+}_{A\cap B}(mxy)\right)$

$= \min\left(\frac{1}{2}, \max(\nu^{+}_{A}(mxy), \nu^{+}_{B}(mxy))\right)$

$= \min\left(\frac{1}{2}, \max(\sup N_A(mxy), \sup N_B(mxy))\right)$

$\leq \min\left(\frac{1}{2}, \max(\sup N_A(x^p), \sup N_B(x^p))\right)$

$= \min\left(\frac{1}{2}, \max(\nu^{+}_{A}(x^p), \nu^{+}_{B}(x^p))\right)$

$= \max\left(\min\left(\frac{1}{2}, \nu^{+}_{A}(x^p)\right), \min\left(\frac{1}{2}, \nu^{+}_{B}(x^p)\right)\right)$

$= \max\left(\nu^{+}_{?\mathrm{A}}(x^p), \nu^{+}_{?\mathrm{B}}(x^p)\right)$

$= \nu^{+}_{?\mathrm{A}\cap ?\mathrm{B}}(x^p)$

Therefore, $\nu^{+}_{?(A\cap B)}(mxy) \leq \nu^{+}_{?\mathrm{A}\cap ?\mathrm{B}}(x^p)$, for some $p \in Z_+$.

Consider $\mu^{-}_{?(A\cap B)}(mxy) = \min\left(\frac{1}{2}, \mu^{-}_{A\cap B}(mxy)\right)$

$= \min\left(\frac{1}{2}, \max(\mu^{-}_{A}(mxy), \mu^{-}_{B}(mxy))\right)$

$= \min\left(\frac{1}{2}, \max(inf\, M_A(mxy), \inf M_B(mxy))\right)$

$$\begin{aligned}
&\le \min\left(\frac{1}{2}, \max(inf M_A(x^p), \inf M_B(x^p))\right)\\
&= \min\left(\frac{1}{2}, \max(\mu_A^-(x^p), \mu_B^-(x^p))\right)\\
&= \max\left(\min\left(\frac{1}{2}, \mu_A^-(x^p)\right), \min\left(\frac{1}{2}, \mu_B^-(x^p)\right)\right)\\
&= \max\left(\mu_{?\mathrm{A}}^-(x^p), \mu_{?\mathrm{B}}^-(x^p)\right)\\
&= \mu_{?\mathrm{A}\cap ?\mathrm{B}}^-(x^p)
\end{aligned}$$

Therefore, $\mu_{?(A\cap B)}^-(mxy) \le \mu_{?\mathrm{A}\cap ?\mathrm{B}}^-(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}
\nu_{?(A\cap B)}^-(mxy) &= \max\left(\frac{1}{2}, \nu_{A\cap B}^-(mxy)\right)\\
&= \max\left(\frac{1}{2}, \min(\nu_A^-(mxy), \nu_B^-(mxy))\right)\\
&= \max\left(\frac{1}{2}, \min(\inf N_A(mxy), \inf N_B(mxy))\right)\\
&\ge \max\left(\frac{1}{2}, \min(\inf N_A(x^p), \inf N_B(x^p))\right)\\
&= \max\left(\frac{1}{2}, \min(\nu_A^-(x^p), \nu_B^-(x^p))\right)\\
&= \min\left(\max\left(\frac{1}{2}, \nu_A^-(x^p)\right), \max\left(\frac{1}{2}, \nu_B^-(x^p)\right)\right)\\
&= \min\left(\nu_{?\mathrm{A}}^-(x^p), \nu_{?\mathrm{B}}^-(x^p)\right)\\
&= \nu_{?\mathrm{A}\cap ?\mathrm{B}}^-(x^p)
\end{aligned}$$

Therefore, $\nu_{?(A\cap B)}^-(mxy) \ge \nu_{?\mathrm{A}\cap ?\mathrm{B}}^-(x^p)$, for some $p \in Z_+$.

Therefore, $?(A\cap B) = ?A \cap ?B$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 15. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then $\overline{?\,\overline{A}} = !A$ is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\begin{aligned}
\mu_{\overline{?\overline{\mathrm{A}}}}^+(mxy) &= \nu_{?\overline{\mathrm{A}}}^+(mxy)\\
&= \min\left(\frac{1}{2}, \nu_{\overline{\mathrm{A}}}^+(mxy)\right)\\
&= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^+(mxy)\right)\\
&= \min\left(\frac{1}{2}, sup M_A(mxy)\right)\\
&\ge \min\left(\frac{1}{2}, sup M_A(x^p)\right)\\
&= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^+(x^p)\right)\\
&= \mu_{!\mathrm{A}}^+(x^p)
\end{aligned}$$

Therefore, $\mu_{\overline{?\overline{\mathrm{A}}}}^+(mxy) \ge \mu_{!\mathrm{A}}^+(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}
\nu_{\overline{?\overline{\mathrm{A}}}}^+(mxy) &= \mu_{?\overline{\mathrm{A}}}^+(mxy)\\
&= \max\left(\frac{1}{2}, \mu_{\overline{\mathrm{A}}}^+(mxy)\right)\\
&= \max\left(\frac{1}{2}, \nu_{\mathrm{A}}^+(mxy)\right)\\
&= \max\left(\frac{1}{2}, sup N_A(mxy)\right)
\end{aligned}$$

$$\leq \max\left(\frac{1}{2}, supN_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \nu_{!\mathrm{A}}^{+}(x^p)$$

Therefore, $\nu_{\overline{?\overline{\mathrm{A}}}}^{+}(mxy) \leq \nu_{!\mathrm{A}}^{+}(x^p)$, for some $p \in Z_+$.

Consider $\mu_{\overline{?\overline{\mathrm{A}}}}^{-}(mxy) = \nu_{?\overline{\mathrm{A}}}^{-}(mxy)$

$$= \max\left(\frac{1}{2}, \nu_{\overline{\mathrm{A}}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, infM_A(mxy)\right)$$
$$\leq \max\left(\frac{1}{2}, infM_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \mu_{!\mathrm{A}}^{-}(x^p)$$

Therefore, $\mu_{\overline{?\overline{\mathrm{A}}}}^{-}(mxy) \leq \mu_{!\mathrm{A}}^{-}(x^p)$, for some $p \in Z_+$.

Consider $\nu_{\overline{?\overline{\mathrm{A}}}}^{-}(mxy) = \mu_{?\overline{\mathrm{A}}}^{-}(mxy)$

$$= \min\left(\frac{1}{2}, \mu_{\overline{\mathrm{A}}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, infN_A(mxy)\right)$$
$$\geq \min\left(\frac{1}{2}, infN_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \nu_{!\mathrm{A}}^{-}(x^p)$$

Therefore, $\nu_{\overline{?\overline{\mathrm{A}}}}^{-}(mxy) \geq \nu_{!\mathrm{A}}^{-}(x^p)$, for some $p \in Z_+$.

Therefore, $\overline{?\overline{\mathrm{A}}} = !A$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 16. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then $!(?A) = ?(!A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $\mu_{!(?A)}^{+}(mxy) = \min\left(\frac{1}{2}, \mu_{?A}^{+}(mxy)\right)$

$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \mu_A^{+}(mxy)\right)\right)$$
$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, supM_A(mxy)\right)\right)$$
$$\geq \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, supM_A(x^p)\right)\right)$$
$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \mu_A^{+}(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \mu_A^{+}(x^p)\right)\right)$$

$$= \max\left(\frac{1}{2}, \mu^+_{!A}(x^p)\right)$$
$$= \mu^+_{?(!A)}(x^p)$$

Therefore, $\mu^+_{!(?A)}(mxy) \geq \mu^+_{?(!A)}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned} \nu^+_{!(?A)}(mxy) &= \max\left(\frac{1}{2}, \nu^+_{?A}(mxy)\right) \\ &= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \nu^+_A(mxy)\right)\right) \\ &= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, supN_A(mxy)\right)\right) \\ &\leq \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, supN_A(x^p)\right)\right) \\ &= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \nu^+_A(x^p)\right)\right) \\ &= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \nu^+_A(x^p)\right)\right) \\ &= \min\left(\frac{1}{2}, \nu^+_{!A}(x^p)\right) \\ &= \nu^+_{?(!A)}(x^p) \end{aligned}$$

Therefore, $\nu^+_{!(?A)}(mxy) \leq \nu^+_{?(!A)}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned} \mu^-_{!(?A)}(mxy) &= \max\left(\frac{1}{2}, \mu^-_{?A}(mxy)\right) \\ &= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \mu^-_A(mxy)\right)\right) \\ &= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, infM_A(mxy)\right)\right) \\ &\leq \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, infM_A(x^p)\right)\right) \\ &= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \mu^-_A(x^p)\right)\right) \\ &= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \mu^-_A(x^p)\right)\right) \\ &= \min\left(\frac{1}{2}, \mu^-_{!A}(x^p)\right) \\ &= \mu^-_{?(!A)}(x^p) \end{aligned}$$

Therefore, $\mu^-_{!(?A)}(mxy) \leq \mu^-_{?(!A)}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned} \nu^-_{!(?A)}(mxy) &= \min\left(\frac{1}{2}, \nu^-_{?A}(mxy)\right) \\ &= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \nu^-_A(mxy)\right)\right) \\ &= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, infN_A(mxy)\right)\right) \\ &\geq \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, infN_A(x^p)\right)\right) \end{aligned}$$

$$= \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \nu_A^-(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \nu_A^-(x^p)\right)\right)$$
$$= \max\left(\frac{1}{2}, \nu_{!A}^-(x^p)\right)$$
$$= \nu_{?(!A)}^-(x^p)$$

Therefore, $\nu_{!(?A)}^-(mxy) \geq \nu_{?(!A)}^-(x^p)$, for some $p \in Z_+$. Therefore, $!(?A) = ?(!A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 17. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then* $!(\square A) = \square(!A)$ *is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\mu_{!(\square A)}^+(mxy) = \min\left(\frac{1}{2}, \mu_{\square A}^+(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \mu_A^+(mxy)\right)$$
$$= \min\left(\frac{1}{2}, supM_A(mxy)\right)$$
$$\geq \min\left(\frac{1}{2}, supM_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu_A^+(x^p)\right)$$
$$= \mu_{!A}^+(x^p)$$
$$= \mu_{\square(!A)}^+(x^p)$$

Therefore, $\mu_{!(\square A)}^+(mxy) \geq \mu_{\square(!A)}^+(x^p)$, for some $p \in Z_+$

Consider
$$\nu_{!(\square A)}^+(mxy) = \max\left(\frac{1}{2}, \nu_{\square A}^+(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu_A^+(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - supM_A(mxy)\right)$$
$$\leq \max\left(\frac{1}{2}, 1 - supM_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu_A^+(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu_A^+(x^p)\right)$$
$$= \nu_{!A}^+(x^p)$$
$$= 1 - \mu_{!A}^+(x^p)$$
$$= 1 - \mu_{\square(!A)}^+(x^p)$$
$$= \nu_{\square(!A)}^+(x^p)$$

Therefore, $\nu_{!(\square A)}^+(mxy) \leq \nu_{\square(!A)}^+(x^p)$, for some $p \in Z_+$.

Consider
$$\mu_{!(\square A)}^-(mxy) = \max\left(\frac{1}{2}, \mu_{\square A}^-(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \mu_A^-(mxy)\right)$$

$$= \max\left(\frac{1}{2}\ , inf M_A(mxy)\right)$$
$$\leq \max\left(\frac{1}{2}\ , inf M_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \mu_{!\mathrm{A}}^{-}(x^p)$$
$$= \mu_{\square(!\mathrm{A})}^{-}(x^p)$$

Therefore, $\mu_{!(\square\mathrm{A})}^{-}(mxy) \leq \mu_{\square(!\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$

Consider $\nu_{!(\square\mathrm{A})}^{-}(mxy) = \min\left(\frac{1}{2}, \nu_{\square\mathrm{A}}^{-}(mxy)\right)$

$$= \min\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - inf M_A(mxy)\right)$$
$$\geq \min\left(\frac{1}{2}, 1 - inf M_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \nu_{!\mathrm{A}}^{-}(x^p)$$
$$= 1 - \mu_{!\mathrm{A}}^{-}(x^p)$$
$$= 1 - \mu_{\square(!\mathrm{A})}^{-}(x^p)$$
$$= \nu_{\square(!\mathrm{A})}^{-}(x^p)$$

Therefore, $\nu_{!(\square\mathrm{A})}^{-}(mxy) \geq \nu_{\square(!\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$.

Therefore, $!(\square A) = \square(!A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

Theorem 18. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then* $?(\square A) = \square(?A)$ *is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider $\mu_{?(\square\mathrm{A})}^{+}(mxy) = \max\left(\frac{1}{2}, \mu_{\square\mathrm{A}}^{+}(mxy)\right)$

$$= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{+}(mxy)\right)$$
$$= \max\left(\frac{1}{2}\ , sup M_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}\ , sup M_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \mu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^{+}(x^p)$$
$$= \mu_{\square(?\mathrm{A})}^{+}(x^p)$$

Therefore, $\mu_{?(\square\mathrm{A})}^{+}(mxy) \geq \mu_{\square(?\mathrm{A})}^{+}(x^p)$, for some $p \in Z_+$.

Consider $\nu_{?(\square\mathrm{A})}^{+}(mxy) = \min\left(\frac{1}{2}, \nu_{\square\mathrm{A}}^{+}(mxy)\right)$

$$= \min\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{+}(mxy)\right)$$

$$= \min\left(\frac{1}{2}, 1 - supM_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, 1 - supM_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^{+}(x^p)$$
$$= 1 - \mu_{?\mathrm{A}}^{+}(x^p)$$
$$= 1 - \mu_{\square(?\mathrm{A})}^{+}(x^p)$$
$$= \nu_{\square(?\mathrm{A})}^{+}(x^p)$$

Therefore, $\nu_{?(\square\mathrm{A})}^{+}(mxy) \leq \nu_{\square(?\mathrm{A})}^{+}(x^p)$, for some $p \in Z_+$.

Consider

$$\mu_{?(\square\mathrm{A})}^{-}(mxy) = \min\left(\frac{1}{2}, \mu_{\square\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, infM_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, infM_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^{-}(x^p)$$
$$= \mu_{\square(?\mathrm{A})}^{-}(x^p)$$

Therefore, $\mu_{?(\square\mathrm{A})}^{-}(mxy) \leq \mu_{\square(?\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$.

Consider

$$\nu_{?(\square\mathrm{A})}^{-}(mxy) = \max\left(\frac{1}{2}, \nu_{\square\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, 1 - infM_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, 1 - infM_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, 1 - \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^{-}(x^p)$$
$$= 1 - \mu_{?\mathrm{A}}^{-}(x^p)$$
$$= 1 - \mu_{\square(?\mathrm{A})}^{-}(x^p)$$
$$= \nu_{\square(?\mathrm{A})}^{-}(x^p).$$

Therefore, $\nu_{?(\square\mathrm{A})}^{-}(mxy) \geq \nu_{\square(?\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$. Therefore, $?(\square A) = \square(?A)$ is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*. □

Theorem 19. *If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then* $!(\lozenge A) = \lozenge(!A)$ *is a primary interval-valued intuitionistic fuzzy anti M group of G.*

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\begin{aligned}
\mu^{+}_{!(\lozenge\mathrm{A})}(mxy) &= \min\left(\frac{1}{2}, \mu^{+}_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \min\left(\frac{1}{2}, 1-\nu^{+}_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \min\left(\frac{1}{2}, 1-\nu^{+}_{\mathrm{A}}(mxy)\right) \\
&= \min\left(\frac{1}{2}, 1-supN_A(mxy)\right) \\
&\geq \min\left(\frac{1}{2}, 1-supN_A(x^p)\right) \\
&= \min\left(\frac{1}{2}, 1-\nu^{+}_{\mathrm{A}}(x^p)\right) \\
&= \min\left(\frac{1}{2}, \mu^{+}_{\mathrm{A}}(x^p)\right) \\
&= \mu^{+}_{!\mathrm{A}}(x^p) \\
&= 1-\nu^{+}_{!\mathrm{A}}(x^p) \\
&= 1-\nu^{+}_{\lozenge(!\mathrm{A})}(x^p) \\
&= \mu^{+}_{\lozenge(!\mathrm{A})}(x^p)
\end{aligned}$$

Therefore, $\mu^{+}_{!(\lozenge\mathrm{A})}(mxy) \geq \mu^{+}_{\lozenge(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}
\nu^{+}_{!(\lozenge\mathrm{A})}(mxy) &= \max\left(\frac{1}{2}, \nu^{+}_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, \nu^{+}_{\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, supN_A(mxy)\right) \\
&\leq \max\left(\frac{1}{2}, supN_A(x^p)\right) \\
&= \max\left(\frac{1}{2}, \nu^{+}_{\mathrm{A}}(x^p)\right) \\
&= \nu^{+}_{!\mathrm{A}}(x^p) \\
&= \nu^{+}_{\lozenge(!\mathrm{A})}(x^p)
\end{aligned}$$

Therefore, $\nu^{+}_{!(\lozenge\mathrm{A})}(mxy) \leq \nu^{+}_{\lozenge(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}
\mu^{-}_{!(\lozenge\mathrm{A})}(mxy) &= \max\left(\frac{1}{2}, \mu^{-}_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, 1-\nu^{-}_{\lozenge\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, 1-\nu^{-}_{\mathrm{A}}(mxy)\right) \\
&= \max\left(\frac{1}{2}, 1-infN_A(mxy)\right) \\
&\leq \max\left(\frac{1}{2}, 1-infN_A(x^p)\right) \\
&= \max\left(\frac{1}{2}, 1-\nu^{-}_{\mathrm{A}}(x^p)\right) \\
&= \max\left(\frac{1}{2}, \mu^{-}_{\mathrm{A}}(x^p)\right) \\
&= \mu^{-}_{!\mathrm{A}}(x^p)
\end{aligned}$$

$$= 1 - \nu^-_{!\mathrm{A}}(x^p)$$
$$= 1 - \nu^-_{\lozenge(!\mathrm{A})}(x^p)$$
$$= \mu^-_{\lozenge(!\mathrm{A})}(x^p)$$

Therefore, $\mu^-_{!(\lozenge\mathrm{A})}(mxy) \leq \mu^-_{\lozenge(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}\nu^-_{!(\lozenge\mathrm{A})}(mxy) &= \min\left(\frac{1}{2}, \nu^-_{\lozenge\mathrm{A}}(mxy)\right)\\ &= \min\left(\frac{1}{2}, \nu^-_{\mathrm{A}}(mxy)\right)\\ &= \min\left(\frac{1}{2}, infN_A(mxy)\right)\\ &= \min\left(\frac{1}{2}, infN_A(x^p)\right)\\ &\geq \min\left(\frac{1}{2}, \nu^-_{\mathrm{A}}(x^p)\right)\\ &= \nu^-_{!\mathrm{A}}(x^p)\\ &= \nu^-_{\lozenge(!\mathrm{A})}(x^p)\end{aligned}$$

Therefore, $\nu^-_{!(\lozenge\mathrm{A})}(mxy) \geq \nu^-_{\lozenge(!\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Therefore, $!\,(\lozenge\, A) = \lozenge\,(!\,A)$ is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*. □

Theorem 20. If A is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*, then $?\,(\lozenge\, A) = \lozenge\,(?\,A)$ is a primary interval-valued intuitionistic fuzzy anti *M* group of *G*.

Proof. Consider $x, y \in A$ and $m \in M$.

Consider
$$\begin{aligned}\mu^+_{?(\lozenge\mathrm{A})}(mxy) &= \max\left(\frac{1}{2}, \mu^+_{\lozenge\mathrm{A}}(mxy)\right)\\ &= \max\left(\frac{1}{2}, 1 - \nu^+_{\lozenge\mathrm{A}}(mxy)\right)\\ &= \max\left(\frac{1}{2}, 1 - \nu^+_{\mathrm{A}}(mxy)\right)\\ &= \max\left(\frac{1}{2}, 1 - supN_A(mxy)\right)\\ &\geq \max\left(\frac{1}{2}, 1 - supN_A(x^p)\right)\\ &= \max\left(\frac{1}{2}, 1 - \nu^+_{\mathrm{A}}(x^p)\right)\\ &= \max\left(\frac{1}{2}, \mu^+_{\mathrm{A}}(x^p)\right)\\ &= \mu^+_{?\mathrm{A}}(x^p)\\ &= 1 - \nu^+_{?\mathrm{A}}(x^p)\\ &= 1 - \nu^+_{\lozenge(?(\mathrm{A}))}(x^p)\\ &= \mu^+_{\lozenge(?\mathrm{A})}(x^p)\end{aligned}$$

Therefore, $\mu^+_{?(\lozenge\mathrm{A})}(mxy) \geq \mu^+_{\lozenge(?\mathrm{A})}(x^p)$, for some $p \in Z_+$.

Consider
$$\begin{aligned}\nu^+_{?(\lozenge\mathrm{A})}(mxy) &= \min\left(\frac{1}{2}, \nu^+_{\lozenge\mathrm{A}}(mxy)\right)\\ &= \min\left(\frac{1}{2}, \nu^+_{\mathrm{A}}(mxy)\right)\\ &= \min\left(\frac{1}{2}, supN_A(mxy)\right)\end{aligned}$$

$$\leq \min\left(\frac{1}{2}, supN_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \nu_{\mathrm{A}}^{+}(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^{+}(x^p)$$
$$= \nu_{\lozenge(?\mathrm{A})}^{+}(x^p).$$

Therefore, $\nu_{?(\lozenge\mathrm{A})}^{+}(mxy) \leq \nu_{\lozenge(?\mathrm{A})}^{+}(x^p)$, for some $p \in Z_+$.

Consider $$\mu_{?(\lozenge\mathrm{A})}^{-}(mxy) = \min\left(\frac{1}{2}, \mu_{\lozenge\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \nu_{\lozenge\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \nu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \min\left(\frac{1}{2}, 1 - infN_A(mxy)\right)$$
$$\leq \min\left(\frac{1}{2}, 1 - infN_A(x^p)\right)$$
$$= \min\left(\frac{1}{2}, 1 - \nu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \min\left(\frac{1}{2}, \mu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \mu_{?\mathrm{A}}^{-}(x^p)$$
$$= 1 - \nu_{?\mathrm{A}}^{-}(x^p)$$
$$= 1 - \nu_{\lozenge(?\mathrm{A})}^{-}(x^p)$$
$$= \mu_{\lozenge(?\mathrm{A})}^{-}(x^p)$$

Therefore, $\mu_{?(\lozenge\mathrm{A})}^{-}(mxy) \leq \mu_{\lozenge(?\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$.

Consider $$\nu_{?(\lozenge\mathrm{A})}^{-}(mxy) = \max\left(\frac{1}{2}, \nu_{\lozenge\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(mxy)\right)$$
$$= \max\left(\frac{1}{2}, infN_A(mxy)\right)$$
$$\geq \max\left(\frac{1}{2}, infN_A(x^p)\right)$$
$$= \max\left(\frac{1}{2}, \nu_{\mathrm{A}}^{-}(x^p)\right)$$
$$= \nu_{?\mathrm{A}}^{-}(x^p)$$
$$= \nu_{\lozenge(?\mathrm{A})}^{-}(x^p)$$

Therefore, $\nu_{?(\lozenge\mathrm{A})}^{-}(mxy) \geq \nu_{\lozenge(?\mathrm{A})}^{-}(x^p)$, for some $p \in Z_+$.

Therefore, $?(\lozenge A) = \lozenge(?A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G. □

4 Conclusion

In this paper the main idea of primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group are a new algebraic structures of fuzzy algebra and it is used through the level operators. We believe that our ideas can also applied for other algebraic system.

References

[1] Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets: Theory and Application*s. Springer Physica-Verlag.

[2] Atanassov, K. T. (2020). *Interval-Valued Intuitionistic Fuzzy Sets*. Springer Cham.

[3] Balasubramanian, A., Muruganantha Prasad, K. L., & Arjunan, K. (2015). Bipolar Interval Valued Fuzzy Subgroups of a Group. *Bulletin of Mathematics and Statistics Research*, 3(3), 234–239.

[4] Chakrabarthy, K., Biswas, R., & Nanda, S. (1997). A note on union and intersection of intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 3(4), 34–39.

[5] Prasannavengeteswari, G., Gunasekaran, K., & Nandakumar, S. (2021). Primary Bipolar Intuitionistic M Fuzzy Group. *Journal of Shanghai Jiaotong University*, 17(4), 82–92.

[6] Prasannavengeteswari, G., Gunasekaran, K., &Nandakumar, S. (2022). Primary interval-valued intuitionistic fuzzy *M* group. *Notes on Intuitionistic Fuzzy Sets*, 28(2), 120–131.

[7] Rosenfeld, A. (1971). Fuzzy Groups. *Journal of Mathematical Analysis and Its Application*, 35, 512–517.

[8] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338–353.

[9] Zhang, W. R. (1998). Bipolar fuzzy sets. *Proceeding of FUZZ-IEEE*, 835–840.

[10] Zimmermann, H. J. (1985). *Fuzzy Set Theory and Its Applications*. Kluwer-Nijhoff Publishing Co.