Notes on Intuitionistic Fuzzy Sets
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The algorithm of correction of the unconscientious experts' evaluations in the interval-valued intuitionistic fuzzy sets case

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Abstract: In the interval-valued intuitionistic fuzzy environment unconscientious opinions may cause problems in the data processing. The problem is similar to those arising in the case of intuitionistic fuzzy sets. In this paper the previous ways of correction of the unconscientious experts' evaluations in the interval-valued intuitionistic fuzzy sets environment are presented and some new algorithms are proposed.
Keywords: Interval-valued intuitionistic fuzzy sets (IVIFSs), Unconscientious experts' evaluation.
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1 Introduction

Intuitionistic Fuzzy Sets (shortly: IFSs) are introduced by Krassimir T. Atanassov in 1983 as some extension of the Fuzzy Sets (shortly: FSs) introduced by Lotfi Zadeh in 1965. The latest developments of the theory are collected in monographs [4] and [5]. Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs) are some extension of the IFSs. The idea is presented by Atanassov and Gargov in [7]. The achievements on this subject are presented in the book [6].

The IFSs can be a promising tool in the decision making. In the algorithm of decision making in the intuitionistic fuzzy environment can arise the situation when the membership and non-membership degrees, given by expert as the evaluation of a variant, sums up to the value greater than 1. In this case we are dealing with the so-called Unconscientious Experts' Evaluation. The situation arise if an expert is more than 100% sure that the variant belongs either to the set or to the complement of this set. The problem of the Unconscientious Evaluation (shortly: UE) in the

IF environment is presented and some solving of this problem are given in [7, 10, 11, 12]. The similar situation can arise in the IVIFSs environment. In this paper we consider some kind of correction of such situations.

2 Basic definitions and properties

In a sequence of papers and books are discussed the intuitionistic fuzzy logic (IFL). The most important in this area is the monograph [5]. In the intuitionistic fuzzy logic the truth-value of logical variable x is given by ordered pair $\langle \mu, \nu \rangle$, where $\mu, \nu, \mu + \nu \in [0, 1]$. Symbolically we can write $V(x) = \langle \mu, \nu \rangle$. The pair $\langle \mu, \nu \rangle$ is called the *intuitionistic fuzzy value* or, better, *intuitionistic fuzzy pair* (IFP) (see e.g. [8]). The set of all IF pairs, seen as points with coordinates μ and ν, we will call the *IF interpretation triangle* (IFTr) (see e.g. [4], p. 38). Let us note that the pair $\langle \mu, \nu \rangle$ is sometimes called the *intuitionistic fuzzy number*, but this name should not be considered appropriate. The IFP $\langle \mu, \nu \rangle$ can be viewed as an evaluation (assessment) of some decision (variant, option, object) by an expert or experts (see e.g. [14]). It can also be viewed as the degree of truthfulness of some sentence in the intuitionistic fuzzy logic. In one of the subsections of [4] are discussed the issues regarding the use of experts' opinions for determination of the membership degree and the non-membership degree, with which the evaluated variant belongs/not belongs to the IF set of variants satisfying certain criterion. In this method of determining of membership / non-membership degrees may arise the situation of some inconsistency in the assessment of belonging and non-belonging values. In such case we deal with the unconscientious evaluations. Atanassov [4, 5] notes that the fact of existence of this kind of problems by the evaluation of events distinguishes the decision aid in the intuitionistic fuzzy environment from the decision aid in the (classical) fuzzy environment, where such unconscientious evaluations do not exist (or it is easy to correct).

More precisely / formally, we can describe this situation of inconsistency of the assessment in terms of membership- , and non-membership functions as follows.

Let E_i, $i = 1, \ldots, n$, be an *i*-th expert from the group of *n* experts. Following Atanassov ([4], p.12) we call the expert E_i *unconscientious*, if among his estimations $\{\langle \mu_{i,j}, \nu_{i,j} \rangle \mid j \in J_i\}$, where $J = \bigcup_{i=1}^{n} J_i$ is an index set (related to the evaluated variants), there exists an estimation for which $\mu_{i,j} \leq 1$ and $\nu_{i,j} \leq 1$, but $\mu_{i,j} + \nu_{i,j} > 1$. We call the IFP $\langle \mu_{i,j}, \nu_{i,j} \rangle$ for which $\mu_{i,j} \leq 1$ and $\nu_{i,j} \leq 1$, but $\mu_{i,j} + \nu_{i,j} > 1$ the UE of *j*-th variant (feature, event) by the *i*-th expert. From now on, the UE $\langle \mu_{i,j}, \nu_{i,j} \rangle$ we denote, for shortly, as UE $\langle \mu, \nu \rangle$. Geometrically, the set of all possible UEs together with IF interpretation triangle of all IF pairs can be viewed as a *unit square*.

To apply the intuitionistic fuzzy sets theory to the processing of evaluations, the UE $\langle \mu, \nu \rangle$ must be adjusted (convert) to the correct IFP $\langle \mu', \nu' \rangle$ where $\mu', \nu' \in [0, 1]$ and $\mu' + \nu' \in [0, 1]$. The ways of correction of unconscientious experts' evaluations in the case of intuitionistic fuzzy sets is presented in [1, 2], [4] (pp. 12–16), [10–12]. Earlier it was marginally mentioned also in the paper of Lakov [14], pp. 36–37.

Analogous situation of the inconsistency of evaluations may be considered in the interval-valued intuitionistic fuzzy sets. Some remarks on this case are presented in the monograph [6], pp. 16–22. For convenience of the reasoning they will be in this paper reminded.

Some generalization of the IFP is the *Interval-Valued Intuitionistic Fuzzy Pair* (IVIFP). It is an ordered pair $\langle M, N\rangle$, where $M, N \subseteq [0, 1]$ are closed sub-intervals of the unit interval [0, 1], given as $M = [\inf M, \sup M]$, and $N = [\inf N, \sup N]$, and satisfying the property that $\sup M + \sup N \leq 1$.

The IVIFP can be interpreted geometrically as in Figure 1 (see e.g. [6, 9, 13]). It is the standard geometric interpretation. We note incidentally that there exist other geometrical interpretations of the IVIFP (see e.g. [6, 9]).

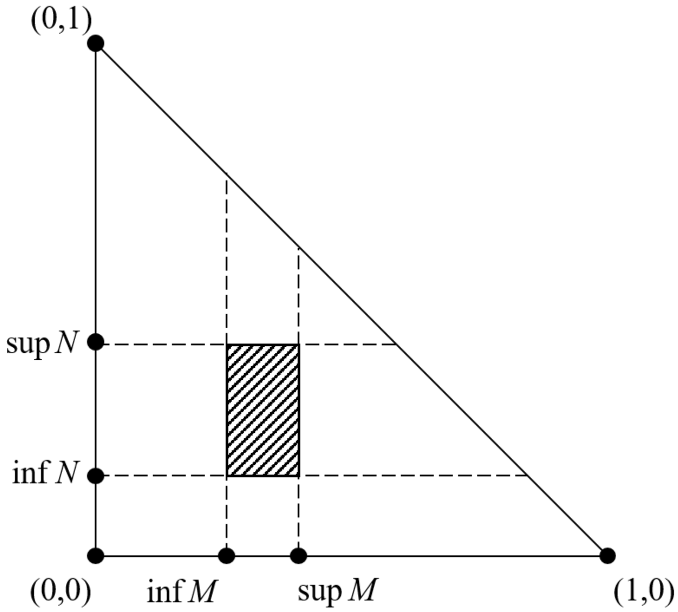

Figure 1. Geometrical standard interpretation of the IVIFP $\langle M, N\rangle$.

In the IVIFS case we can take into account also the hesitation margin. For the IVIFP $\langle M, N\rangle$ the hesitation margin is the interval $\Pi = [1 - \sup M - \sup N, 1 - \inf M - \inf N]$ (see e.g. [6], p.22).

In the IVIFSs environment, we can consider the definition analogous to the definition of the *IF Tautology* (IFT) and *IF co-Tautology* (IFcT) for IFSs. Let us recall, we call the IF pair $\langle \mu, \nu\rangle$ an IFT if and only if it holds that $\mu \geq \nu$, and, similarly, an IFcT if it holds that $\mu \leq \nu$. Formally, the name IFT deals to the Intuitionistic Fuzzy Logic and without background of the logic the name is not fully correct, therefore, we should use the name *Intuitionistic Fuzzy Tautological Pair* or *Intuitionistic Fuzzy co-Tautological Pair.*

The *Interval-Valued Intuitionistic Fuzzy Tautological Pair* (IVIFTP) is defined as the IVIFP $\langle M, N\rangle$ for which $M \geq N$, which we understand to be: $\inf M \geq \sup N$ (see: [6], pp. 14, 123). The *Interval-Valued Intuitionistic Fuzzy co-Tautological Pair* (IVIFcTP) we define as the IVIFP $\langle M, N\rangle$ for which $M \leq N$, which we understand to be: $\sup M \leq \inf N$.

3 Previous algorithms of the correction of the unconscientious experts' evaluations in the IVIFSs environment

When we use the IVIFP as an evaluation of some property (decision, variant, option, object), the intervals M and N are interpreted as intervals of degrees of membership and non-membership, or intervals of degrees of validity and non-validity (of property etc.).

The problem with the correction of the unconscientious experts' evaluations arises if an expert is *more than 100% sure* that the variant belongs either to the set or to the complement of this set. In the *interval-valued intuitionistic fuzzy* case, the *unconscientious evaluation* (shortly: IVIFUE) means fulfilling of the inequality $\sup M + \sup N > 1$. There can be fulfilled the inequalities $\sup M + \inf N > 1$, or $\inf M + \sup N > 1$, or $\inf M + \inf N > 1$ also, but it is not necessary.

Similar as for the IFSs and FSs the fact of existence of this kind of situation in the evaluation of events distinguishes the IVIFSs from the (classical) interval-valued fuzzy sets, where such unconscientious evaluations do not exist.

From now on we will use, for short, instead of $M = [\inf M, \sup M]$, and $N = [\inf N, \sup N]$ the notation $M = [a, b]$, and $N = [c, d]$.

In the monograph [6, pp. 16–22] and earlier in [1, 2, 3] are given transformations F and G and there are tools to make a correction of the UE and the IVIFUE also.

The F and G function are given as here:

$$F(\langle x, y\rangle) = \begin{cases} \langle 0,0\rangle & \text{if} \quad x = 0 \text{ and } y = 0 \\ \left\langle \dfrac{x^2}{x+y}, \dfrac{xy}{x+y} \right\rangle & \text{if} \quad x, y \in [0,1] \text{ and } x \geq y \text{ and } x+y > 0 \\ \left\langle \dfrac{xy}{x+y}, \dfrac{y^2}{x+y} \right\rangle & \text{if} \quad x, y \in [0,1] \text{ and } x \leq y \text{ and } x+y > 0 \end{cases},$$

and

$$G(\langle x, y\rangle) = \begin{cases} \left\langle x - \dfrac{y}{2}, \dfrac{y}{2} \right\rangle & \text{if} \quad x, y \in [0,1] \text{ and } x \geq y \\ \left\langle \dfrac{x}{2}, y - \dfrac{x}{2} \right\rangle & \text{if} \quad x, y \in [0,1] \text{ and } x \leq y \end{cases}.$$

For the UE in classical IF environment the function F and G are correct transformations of the UE to the IFP. Some doubts can arise looking on the hesitation degree of IFP after the F-, or the G-transformation. For the IFP $\langle x, y\rangle$ we have the hesitation degree $\pi = 1 - x - y$, while the corrected value π' depends only on a single value x or y. Namely, it is

$$\pi' = \begin{cases} 1 - x & \text{if} \quad x, y \in [0,1] \text{ and } x \geq y \\ 1 - y & \text{if} \quad x, y \in [0,1] \text{ and } x \leq y \end{cases}$$

However, regardless of the doubts, in the IVIF environment the transformation F and G, in pure form, are not correct, because the image of the rectangle (in terms of the standard geometrical interpretation) in the *F* or *G* mapping is not a rectangle. This fact can be formulated in the terms of the IVIF pairs as follows: the image of an IVIFP (or IVIFUE) is not an IVIFP.

Atanassov [6, p.18] noticed that the image $P'Q'R'S'$ of the rectangle *PQRS* with vertices $P(a, c)$, $Q(b, c)$, $R(b, d)$, $S(a, d)$ in the transformation *F* is only a quadrilateral but not necessarily a rectangle or a parallelogram. Because of this fact the transformation *F* is not directly suitable for correction of the IVIFUE.

The situation is similar in the case of transformation *G*. The transformation *G* is not directly suitable for correction of the IVIFUE also. Atanassov [6, pp. 18–19] gives the example showing that the rectangular is transformed to the parallelogram.

In general, the image of the rectangle *PQRS* with vertices $P(a, c)$, $Q(b, c)$, $R(b, d)$, $S(a, d)$ may not be a parallelogram. For example, let *PQRS* be the rectangle with vertices *P*(0.2, 0.2), *Q*(0.4, 0.2), *R*(0.4, 0.7), *S*(0.2, 0.7). The image of this rectangle in the *G*-transformation is a quadrilateral $P'Q'R'S'$ with vertices P'(0.1, 0.1), Q'(0.3, 0.1), R'(0.2, 0.5), S'(0.1, 0.6). We can see that on interpretation square in Figure 2.

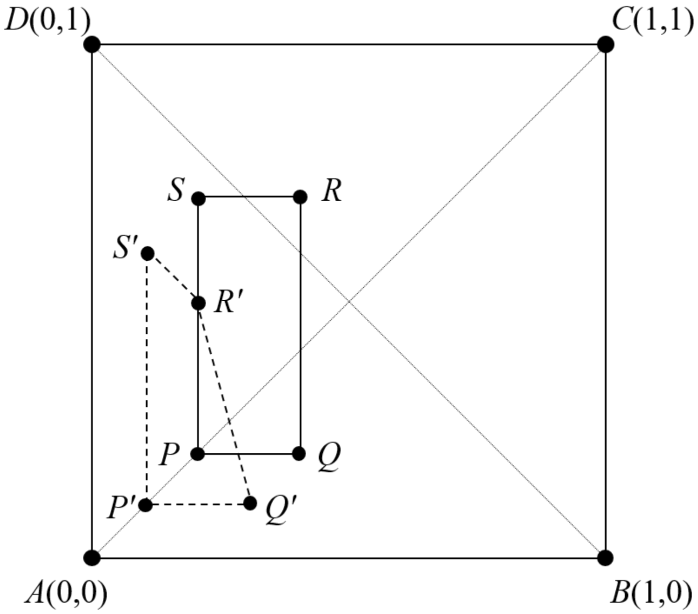

Figure 2. The image of the rectangle *PQRS* with *P*(0.2, 0.2), *Q*(0.4, 0.2), *R*(0.4, 0.7), *S*(0.2, 0.7) vertices in *G*-transformation.

Moreover, the image of the rectangle may not be a quadrilateral. For example, let *PQRS* be the rectangle with vertices *P*(0.2, 0.4), *Q*(0.8, 0.4), *R*(0.8, 0.8), *S*(0.2, 0.8). The image of this rectangle in the *G*-transformation is a triangle $P'Q'S'$ with vertices P'(0.1, 0.3), Q'(0.6, 0.2), S'(0.1, 0.7). The coordinates of the point R' are (0.4, 0.4). We can see that on interpretation square in Figure 3.

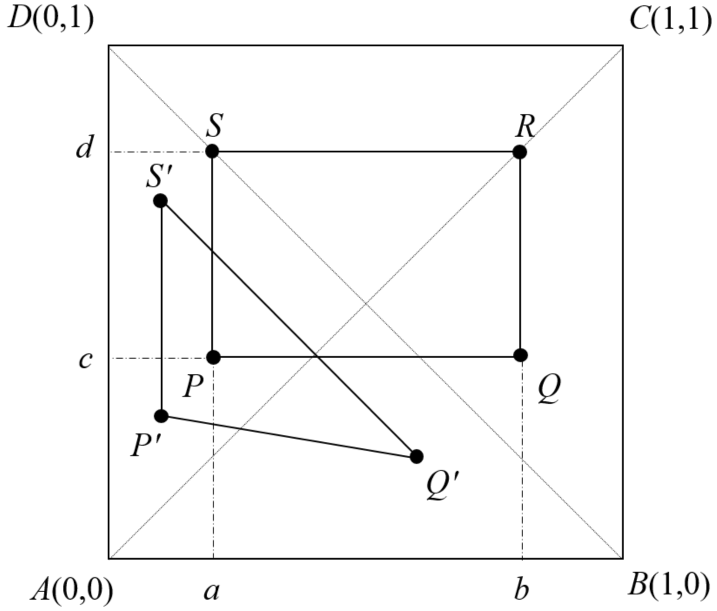

Figure 3. The image of the rectangle *PQRS* with *P*(0.2, 0.4), *Q*(0.8, 0.4), *R*(0.8, 0.8), *S*(0.2, 0.8) vertices in *G*-transformation.

It can be proved that if the rectangle *PQRS* with vertices *P*(*a*, *c*), *Q*(*b*, *c*), *R*(*b*, *d*), *S*(*a*, *d*) is contained in the triangle *ABC* or it is contained in the triangle *ACD*, then the image of the rectangular *PQRS* in the *G*-transformation is a parallelogram. In terms of coordinates of the rectangle *PQRS*, if $a \geq d$ or $c \geq b$ then the image of the rectangular *PQRS* in the *G*-transformation is a parallelogram.

As we see, transformations *F* and *G* cannot be directly used in the correction of IVIFUE. But they can be modified. The modification is proposed in [6]. The idea concerns on creating of a rectangle based on the image of the IVIFUE in the *F*- or *G*-transformation. The final result is given as the rectangle with opposite vertices *P′* and *R′*, where *P′* and *R′* are images of vertices *P*(*a*, *c*) and *R*(*b*, *d*). The sides of the rectangle are perpendicular / parallel to the axis of the Cartesian coordinate system. It is easy to show that the obtained rectangular is contained in the IF interpretational triangle. It is therefore an IVIFP. For the details see [6, pp. 18–22].

We will call the corrections, based on the functions *F* and *G*, **Way 1** and **Way 2** of the IVIFUE correction, respectively.

4 Properties postulated for the correction of the unconscientious experts' evaluations in the IVIFSs environment

Let us consider the pair $\langle M, N\rangle$, where $M, N \subseteq [0, 1]$ are closed sub-intervals of the unit interval [0, 1], given as $M = [a, b]$ and $N = [c, d]$. In the standard geometrical interpretation the IVIFP $\langle M, N\rangle$ is a rectangle *PQRS* with vertices *P*(*a*, *c*), *Q*(*b*, *c*), *R*(*b*, *d*), *S*(*a*, *d*). We call the pair $\langle M, N\rangle$ an IVIFUE if $b + d > 1$. Inequalities $b + c > 1$, or $a + d > 1$, or $a + c > 1$ can also be fulfilled.

To apply the interval-valued intuitionistic fuzzy sets theory, the sum $b + d$ must be smaller or equal to 1. In the case when $b + d > 1$ we need to make a correction of the sum $b + d$ (and can be another values).

Taking into account the correction of the IVIFUE, one may ask what properties of the correction should be fulfilled. In the previous literature no general condition has been given in order to the conversion of IVIFUE can being considered as proper. For the unconscientious intuitionistic fuzzy pair the properties of the conversion's mapping are studied in [10, 11, 12].

For the interval valued pair $\langle M, N\rangle$, we can consider some analogous properties. But, because the membership degree, non-membership degree and hesitancy margin are intervals, the comparison of the values is difficult.

The first, elementary property of correction of the IVIFUE is given as follows.

Property 1. The IVIFUE $\langle[a, b], [c, d]\rangle$ after correction must be the IVIFP $\langle[a', b'], [c', d']\rangle$ fulfilling the property $b' + d' \leq 1$.

The next postulated property deals with the IVIFTP and IVIFcTP. Namely, if the IVIFUE is 'rather true', then after the correction it must stay such. Similar, if the IVIFUE is 'rather false', then after the correction it must stay such. This property can be formally described as Property 2.

Property 2. If for the IVIFUE $\langle[a, b], [c, d]\rangle$ it is $a \geq d$ then, after correction, we must obtain the IVIFP $\langle[a', b'], [c', d']\rangle$ fulfilling the property $b' + d' \leq 1$ and $a' \geq d'$. If for the IVIFUE $\langle[a, b], [c, d]\rangle$ it is $c \geq b$, then, after correction, we must obtain the IVIFP $\langle[a', b'], [c', d']\rangle$ fulfilling the property $b' + d' \leq 1$ and $c' \geq b'$.

If we extend the definition of the IVIFTP and IVIFcTP to the IVIFUE then the Property 2 can be formulated in short form as Property 2′.

Property 2′. If the IVIFUE $\langle[a, b], [c, d]\rangle$ is an IVIFTP or IVIFcTP, then, after correction, we must obtain the IVIFP $\langle[a', b'], [c', d']\rangle$ that is an IVIFTP or IVIFcTP, respectively.

We propose to make the correction of the IVIFUE without going beyond the range of the current membership and non-membership values. This property can be written as follows.

Property 3. The IVIFUE $\langle[a, b], [c, d]\rangle$ after correction must be the IVIFP $\langle[a', b'], [c', d']\rangle$ fulfilling $[a', b'] \subseteq [a, b]$ and $[c', d'] \subseteq [c, d]$.

Let us recall for the IVIFP $\langle M, N\rangle$ the hesitation margin is the interval $\Pi = [1 - b - d, 1 - a - c]$. If we extend the concept for the IVIFUE we have $1 - b - d < 0$. I am not able to solve the problem how the hesitation margin with the left endpoint smaller than zero should be interpreted. Perhaps the "hesitation" that is "negative" can be understood as some "over-confidence"? Perhaps it is better, in such case, to call the interval Π an *uncertainty interval* and the "negative uncertainty" would be understood as "over-confidence", which is consistent with the statement that the expert is "more than 100% sure" that the variant belongs either to the set or to the complement of this set.

But, it can be seen, in all ways of corrections, presented below, that the uncertainty interval after correction is given as $\Pi' = [0, 1 - a - c]$.

5 New algorithms for the correction of the unconscientious evaluations in the interval-valued IF environment

Depending on how unconscientious the expert is (how unconscientious, negligent the evaluation is), we will consider several cases of correcting such evaluations. As the basic and most important case, we will take the Case 1. The remaining cases will be reduced to Case 1. All cases will be considered and explained based on the standard geometric interpretation of IVIFSs. The consideration will be done for a single IVIFUE only. For IVIFSs other than singletons (single-element sets), the procedure would be done separately for each elements of the universe.

5.1 Case 1

In this case we consider the situation when only one vertex of the rectangular *PQRS* is an UE. The illustration of the Case 1 is presented in the Figure 4. The coordinates of the vertex *R* have the property $b + d > 1$. Let us note that the transformations *F* and *G* generate the image of the rectangle *PQRS* through the generation of the image of all points of the rectangle. The below presented ways can be viewed in the same sense or as some 'cut' of the basic rectangle. We will not write the formula of the transformation explicitly because it is more important that we give the result in the IVIFP form. For this reason, we will describe corrections in the language of simple, elementary geometry.

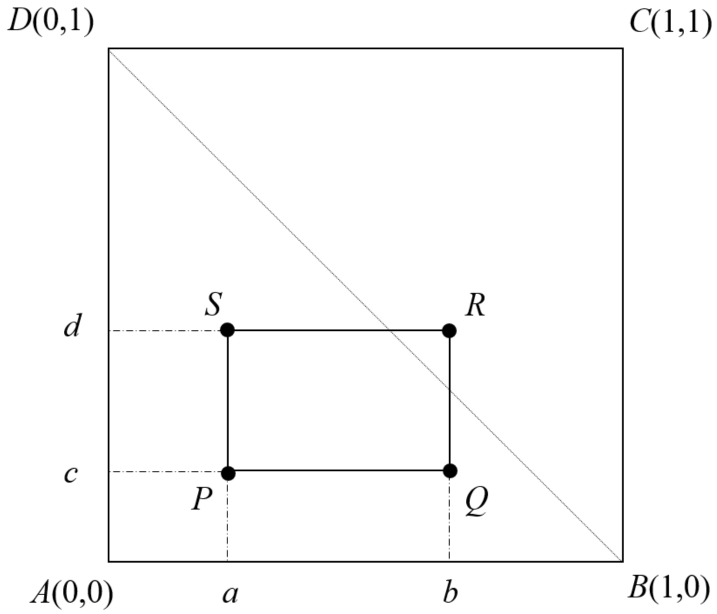

Figure 4. The Case 1 of the unconscientious evaluations in the IVIF environment.

The reduction of the sum $b + d$ can be done in three ways:

a) we reduce the value *d* leaving the value *b*,
b) we reduce the value *b* leaving the value *d*,
c) we reduce both of the values *b* and *d*,

so as to obtain $b + d \leq 1$.

The correction of the unconscientious evaluation, in the **Way 3** form, can be made as follows. We transform the vertex $R(b, d)$ to the vertex $R'(b', d')$, where

$$b' = \frac{b+1-d}{2},$$

$$d' = \frac{d+1-b}{2}.$$

The R' point is the one closest to R in the Euclidean metric sense on the plane. Therefore, we substitute the inappropriate R vertex with the nearest R' vertex. Of course, the quadrilateral $PQR'S$ is not a rectangle. The rectangle must be built on the basis of the P and R' vertices. The corrected rectangle will have vertices $P(a, c)$, $Q'(b', c')$, $R'(b', d')$ and $S(a, d')$.

Let us marginally note that in a particular case the figure $PQR'S$ can be deformed to a triangle (if $b + c = a + d = 1$), but the procedure of correction is analogous.

In a similar way, we can use the D_α operator (see e.g. [4]). It is defined for the IFS A in the universe U as

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha\,\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x)\rangle : x \in U\},$$

and for the IFP as

$$D_\alpha(\langle x, y\rangle) = \langle x + \alpha(1-x-y),\ y + (1-\alpha)(1-x-y)\rangle.$$

It is easy to show that the operator can be used not only for IFPs, but it can be extended for the pair $\langle x, y\rangle$, where $x, y \in [0, 1]$ and $x + y > 1$. Moreover, the IFP $D_\alpha(\langle x, y\rangle)$ is a classical fuzzy value because $x + \alpha(1-x-y) + y + (1-\alpha)(1-x-y) = 1$.

In the UE correction, the parameter α is some factor that informs on the direction of the projection of the R vertex on the diagonal BD.

The image of the R vertex is equal to $D_\alpha(\langle b, d\rangle) = \langle b + \alpha(1-b-d), d + (1-\alpha)(1-b-d)\rangle$.

If $\alpha = 0$, then $D_\alpha(R) = D_\alpha(\langle b, d\rangle) = \langle b, 1-b\rangle = \Box(\langle b, d\rangle)$.

If $\alpha = 1$, then $D_\alpha(R) = D_\alpha(\langle b, d\rangle) = \langle 1-d, d\rangle = \Diamond(\langle b, d\rangle)$.

The operators $\Box$ and $\Diamond$ are the well-known necessity and possibility operators defined for the IFS A as:

$$\Box A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle : x \in U\},$$

and

$$\Diamond A = \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle : x \in U\}.$$

The $D_\alpha(\langle x, y\rangle)$ can be therefore viewed as the convex combination of $\Box(\langle x, y\rangle)$ and $\Diamond(\langle x, y\rangle)$.

If $\alpha = 0.5$, then $D_\alpha(R) = D_\alpha(\langle b, d\rangle) = \left\langle \frac{b+1-d}{2}, \frac{d+1-b}{2} \right\rangle$, which is the result obtained as the point R', which is the closest to the point R in the Euclidean distance sense.

With the different values of the α, the type of the reduction of the sum $b + d$ is different:

a) for $\alpha = 0$ we reduce only the value d leaving the value b,

b) for $\alpha = 1$ we reduce only the value b leaving the value d.

c) for $\alpha \in (0, 1)$ we reduce both values b and d.

The correction of the UE $\langle b, d\rangle$ based on the D_α operator is illustrated in the Figure 5.

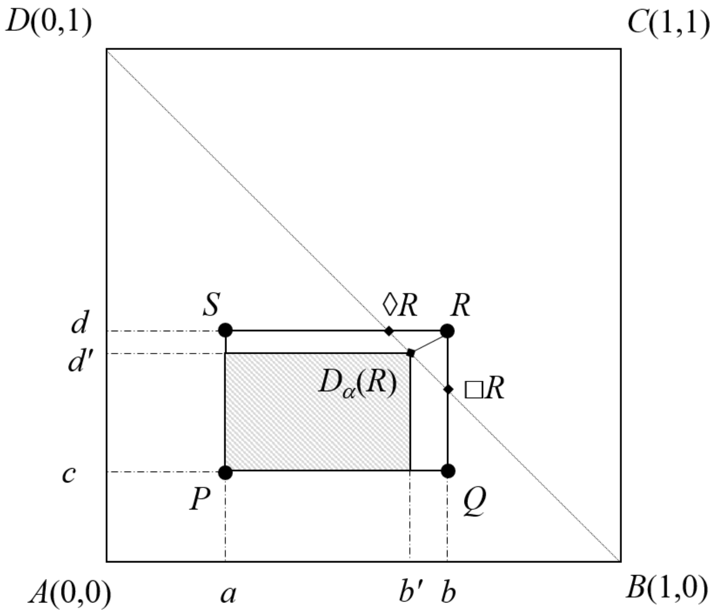

Figure 5. Case 1. Correction based on the D_α operator.

Similar as mentioned previously, if we take $R' = D_\alpha(R)$, the quadrilateral $PQR'S$ does not have to be a rectangle. It is a rectangle if $R' = \Box R$ or $R' = \Diamond R$. In the other cases the rectangle should be built on the basis of P and R' vertices, where $R' = D_\alpha(R) = \langle b', d' \rangle$. The corrected rectangle $PQ'R'S'$ will have vertices $P(a, c)$, $Q'(b', c)$, $R'(b', d')$ and $S'(a, d')$.

The rectangle $PQ'R'S'$ after correction should be 'similar' or 'close' to the rectangle $PQRS$ before the correction.

If we consider the classical, geometrical similarity of the rectangles, preserving the proportions of the length of the sides, the parameter α must be calculated as $\alpha = \dfrac{b-a}{b-a+d-c} \in [0, 1]$.

Some measure of the 'similarity', for rectangles with one common vertex P and sides of the $PQ'R'S'$ included in respectively slides of the $PQRS$, can be the area of the $PQ'R'S'$ rectangle, which should be as large as possible. The area of the $PQ'R'S'$ rectangle is the function of the parameter α given by:

$$\begin{aligned} P_{PQ'R'S'} &= (b+\alpha(1-d-b)-a)(d+(1-\alpha)(1-d-b)-c) \\ &= -(1-d-b)^2\alpha^2 + (1-d-b)(1+a-2b-c)\alpha + (b-a)(1-b-c) \\ &= P_{PQ'R'S'}(\alpha). \end{aligned}$$

The maximum of the quadratic function $P_{PQ'R'S'}(\alpha)$ is reached for $\alpha_{MAX} = \dfrac{1+a-2b-c}{2(1-d-b)}$. But it is necessary to check if $\alpha \in [0, 1]$. If not, the maximum of the $P_{PQ'R'S'}(\alpha)$ is reached for $\alpha = 0$ or $\alpha = 1$. The correction based on the D_α operator will be denoted as **Way 4**.

5.2 Case 2

In this case we consider the situation when two vertices of the rectangular $PQRS$ are UE. The Case 2 can be considered in two subcases Case 2.1 and Case 2.2. Both are presented in Figures 6 and 7.

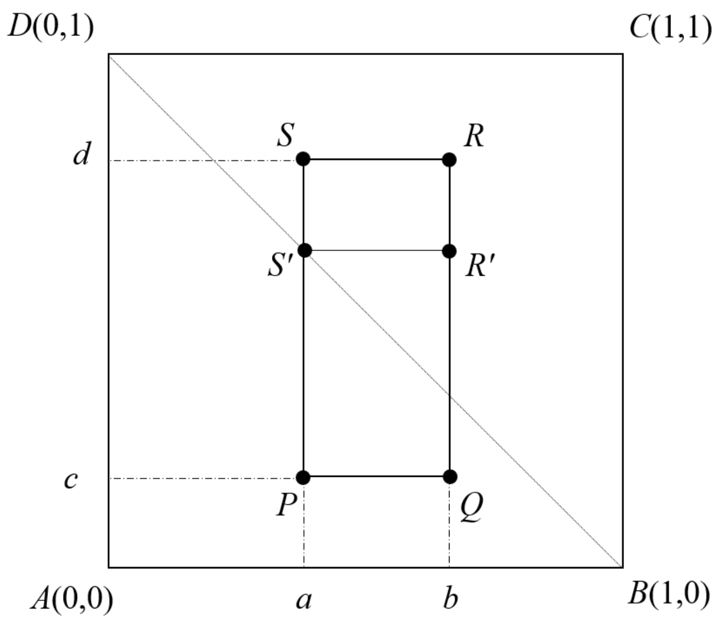

Figure 6. Case 2.1 of the unconscientious evaluations in the IVIF environment.

The correction can be made as follows:

Step 1. We correct the S vertex to the S' vertex with coordinates $a' = a$ and $d' = 1 - a$, and the R vertex to the R' vertex with coordinates $b' = b$ and $d' = 1 - a$.

Step 2. We apply the procedure from Case 1.

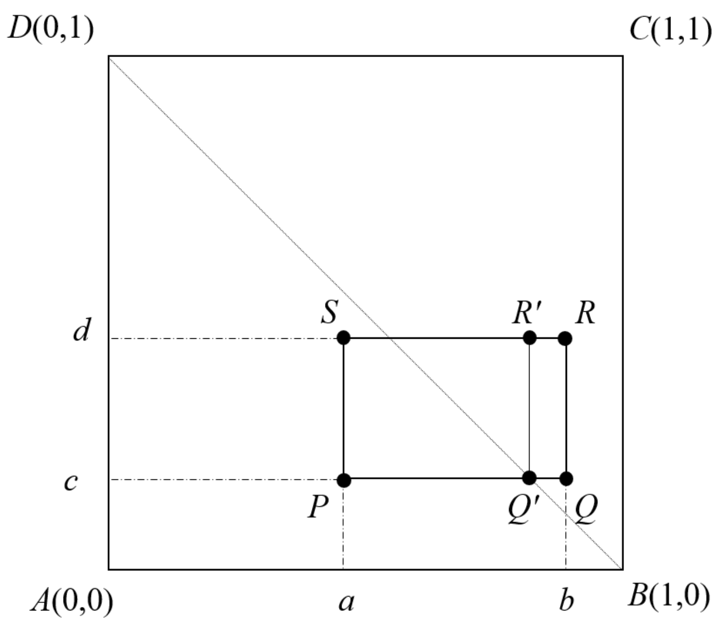

Figure 7. Case 2.2 of the unconscientious evaluations in the IVIF environment.

The correction can be made as follows:

Step 1. We correct the Q vertex to the Q' vertex with coordinates $b' = 1 - c$ and $c' = c$, and the R vertex to the R' vertex with coordinates $b' = 1 - c$ and $d' = d$.

Step 2. We apply the procedure from Case 1.

5.3 Case 3

In this case we consider the situation when three vertices of the rectangular *PQRS* are UE. Case 3 is presented in Figure 8.

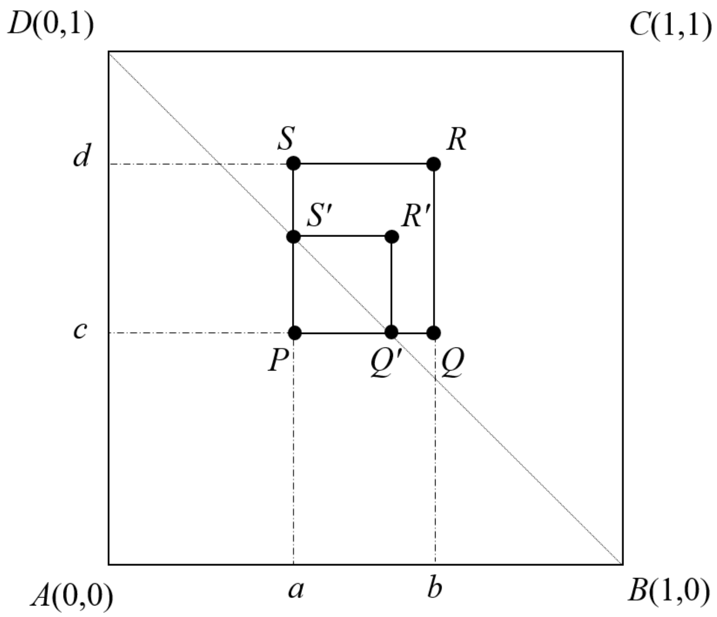

Figure 8. Case 3 of the unconscientious evaluations in the IVIF environment.

The correction can be made as follows:

Step 1. We correct the Q vertex to the Q' vertex with coordinates $b' = 1 - c$ and $c' = c$, and the S vertex to the S' vertex with coordinates $a' = a$ and $d' = 1 - a$, and the R vertex to the R' vertex with coordinates $b' = 1 - c$ and $d' = 1 - a$.

Step 2. We apply the procedure from Case 1.

6 Conclusion

In the interval valued intuitionistic fuzzy environment unconscientious opinions may cause problems in the data processing. In this paper the previous ways of the correction and the new ways of correction of the unconscientious evaluations are proposed. The basic properties, which should be fulfilled in order to the conversion can be considered as proper, are given.

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