Notes on Intuitionistic Fuzzy Sets
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On some new geometrically motivated operators over intuitionistic fuzzy sets

Peter Vassilev [1], **Kristina Zhivkova** [2], **Vassia Atanassova** [3], **Lyudmila Todorova** [4] **and Diana Petkova** [5]

Department of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
e-mails: [1] `peter.vassilev@gmail.com`,
[2] `kristinadimitrova.zh@gmail.com`,
[3] `vassia.atanassova@gmail.com`,
[4] `lpt@biomed.bas.bg`,
[5] `diana@biomed.bas.bg`

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Abstract: We consider four new geometrically motivated operators acting in the interior of the interpretation triangle used to depict intuitionistic fuzzy sets. The first operators uses the points resulting from the classical "necessity" and "possibility" operators, in combination with the point $\langle 0, 0\rangle$. The definition of the second operator employs the projections of the considered point onto the sides of the triangle, as viewed from the vertices $\langle 0, 0\rangle$, $\langle 0, 1\rangle$, and $\langle 1, 0\rangle$. The remaining two operators combine two of these projections with the result of the modal operators of the internal point. We also consider operators obtained as linear combinations of some of these four operators.
Keywords: Operator, Geometric interpretation, Interior point, Intuitionistic fuzzy set.
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1 Introduction

Intuitionistic fuzzy sets (IFSs) were introduced by K. Atanassov as an extension of L. Zadeh's fuzzy sets, first in a Bulgarian preprint [1], and later in English in the seminal paper in the journal *"Fuzzy Sets and Systems"*, [2]. In the first preprint [1], along with the original definition of IFSs, for the first time were defined the modal operators over IFSs "necessity" ($\Box$) and "possibility" ($\Diamond$). Many other modal, level and topological operators over IFSs have been introduced since then. We refer the interested reader to [3, 5, 6, 10, 11, 13–18, 20–23, 27, 30, 32, 35, 40–44, 48, 49, 52, 53] for a more comprehensive overview.

In the present paper, we concentrate on the most used geometrical interpretation of intuitionistic fuzzy set as shown on Figure 1. It represents an IFS-element x defined over the universe set E in the unitary intuitionistic fuzzy interpretation triangle.

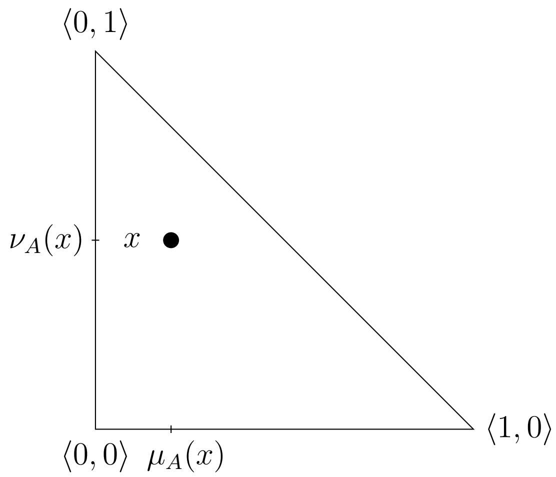

Figure 1. A geometrical interpretation of an element of an IFS

The set-up for this graphical interpretation is the first quadrant of the Cartesian coordinate system, with the two coordinates representing the membership (abscissa) and the non-membership (ordinate), and the origin of the coordinate system $\langle 0, 0\rangle$ representing the complete intuitionistic fuzzy "Uncertainty". The vertex $\langle 1, 0\rangle$ of the unitary triangle represents the complete intuitionistic fuzzy "Truth", and the vertex $\langle 0, 1\rangle$ corresponds to the complete intuitionistic fuzzy "Falsity". To demonstrate that IFSs are a valid extension of the ordinary fuzzy sets, the representation of the fuzzy set into the triangular interpretation is the projection onto the triangle's hypotenuse.

The triangular interpretation of IFSs is not only one of the earliest geometrical interpretations of the IFSs, dated back to as early as 1989 in the preprint [4], but also one that has no analogue in the case of Zadeh's ordinary fuzzy sets (cf. the graphical representation of IF membership functions, discussed in [31]). Employing the specifics and augmented nature of the concept of IFSs compared to fuzzy sets, in numerous recent researches, ideas sparking specifically from the geometry of triangle have helped for the interpretation of some theoretical and application aspects of the intuitionistic fuzziness, see [24,28,38,51]. The geometrical interpretability research further branches into:

- interpretation of extensions of intuitionistic fuzzy sets within the classical IF interpretational triangle [24, 36].
- interpretations of IFS in other planar triangles like the unitary equilateral triangle [34, 37, 47, 50].
- interpretations of IFS in the three dimensions [24].

One of the theoretical aspects of IFSs that has been most extensively researched from a geometrical point of view are the operators defined over IFSs. The very shape of an IFS has been discussed and modelled in [39]. The mentioned basic modal operators of "necessity" and "possibility", projected in points located onto the hypotenuse, over time have been elaborated and generalized to operators plottable in various regions within the interior of the interpretational triangle, e.g., [3, 7, 8, 13, 19, 51], as well as outside, e.g., [9, 25, 26].

In what follows, we shall primarily consider points that lie in the interior of the IF interpretational triangle. In Section 2 we shall provide a precise definition of that notion and will recall some basic facts regarding IFSs. Section 3 showcases several new operators and some proven assertions concerning them.

2 Preliminaries

We will start this section with the following brief recall of basic facts.

Definition 1 (cf. [1]). *Let E be a universe set, and let $A \subset E$. An intuitionistic fuzzy set is an object of the form*

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the mappings $\mu_A : E \to [0,1], \nu_A : E \to [0,1]$ are such that for all $x \in E$, we have:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

The degree of membership of the element x to the set A, is denoted by $\mu_A(x)$, while the degree of non-membership of the element x to the set A is denoted by $\nu_A(x)$. The degree of indeterminacy, sometimes also called hesitancy margin or intuitionistic fuzzy index is defined as:

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x).$$

Further, for simplicity of notation we shall omit the * and simply name the IFSs by A, B, etc.

We will use also the "inclusion" between two IFSs A and B, defined over the same universe set E. Following [1], we say that the IFS A is included in IFS B, and denote this fact by $A \subseteq B$ if and only if, for all $x \in E$, it is true that:

$$\begin{cases} \mu_A(x) \leq \mu_B(x) \\ \nu_A(x) \geq \nu_B(x). \end{cases}$$

Before we proceed we introduce the following definition, which will be required for our further considerations.

Definition 2. *A point* $\langle x, \mu_A(x), \nu_A(x)\rangle$ *is said to be interior for the interpretation triangle if and only if:*

$$\begin{cases} \mu_A(x).\nu_A(x) & \neq 0 \\ \mu_A(x) + \nu_A(x) & < 1. \end{cases} \tag{1}$$

Further, for brevity, we will refer to the point $\langle x, \mu_A(x), \nu_A(x)\rangle$ by simply denoting it as x.

3 Main results

In what follows we will use the following secondary non-interior points as constructed from the considered interior point (IP) (see Figure 2).

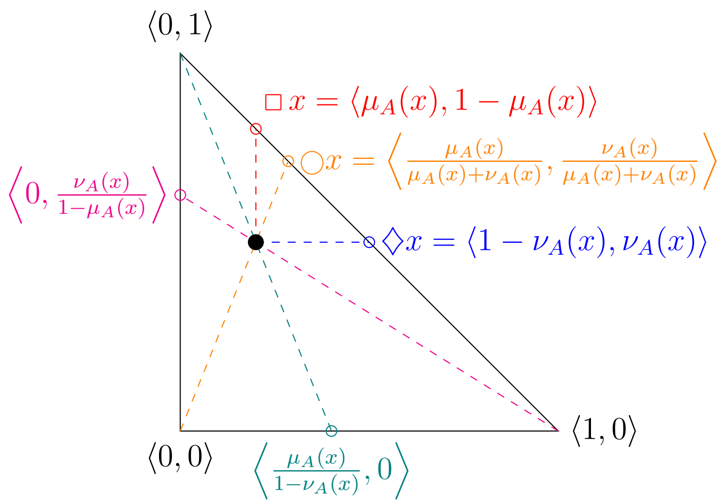

Figure 2. Points generated by an arbitrary interior point: "necessity" (red) and "possibility" (blue), and the projections of the vertices $\langle 0, 0\rangle$ (orange), $\langle 1, 0\rangle$ (magenta) and $\langle 0, 1\rangle$ (teal) onto the opposite triangle's sides.

Definition 3. *Let an IFS* A *be given. Then we can define the operator*

$$Z_1(A) = \begin{cases} \left\langle x, \frac{1}{3}\left(\mu_A(x) + \frac{\mu_A(x)}{1-\nu_A(x)}\right), \frac{1}{3}\left(1 - \mu_A(x) + \frac{\nu_A(x)}{1-\mu_A(x)}\right)\right\rangle & \text{if } x \text{ is an IP} \\ \langle x, \mu_A(x), \nu_A(x)\rangle & \text{otherwise.} \end{cases} \tag{2}$$

Definition 4. *Let an IFS* A *be given. Then we can define the operator*

$$Z_2(A) = \begin{cases} \left\langle x, \frac{1}{3}\left(\frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)} + \frac{\mu_A(x)}{1-\nu_A(x)}\right), \frac{1}{3}\left(\frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)} + \frac{\nu_A(x)}{1-\mu_A(x)}\right)\right\rangle & \text{if } x \text{ is an IP} \\ \langle x, \mu_A(x), \nu_A(x)\rangle & \text{otherwise.} \end{cases} \tag{3}$$

Definition 5. *Let an IFS* A *be given. Then we can define the operator*

$$Z_3(A) = \begin{cases} \left\langle x, \frac{1}{3}\left(1 - \nu_A(x) + \frac{\mu_A(x)}{1-\nu_A(x)}\right), \frac{1}{3}\left(\nu_A(x) + \frac{\nu_A(x)}{1-\mu_A(x)}\right)\right\rangle & \text{if } x \text{ is an IP} \\ \langle x, \mu_A(x), \nu_A(x)\rangle & \text{otherwise.} \end{cases} \tag{4}$$

Definition 6. *Let an IFS A be given. Then we can define the operator*

$$Z_4(A) = \begin{cases} \left\langle x, \frac{1}{3}\left(\mu_A(x) + 1 - \nu_A(x)\right), \frac{1}{3}\left(\nu_A(x) + 1 - \mu_A(x)\right)\right\rangle & \text{if } x \text{ is an IP} \\ \langle x, \mu_A(x), \nu_A(x)\rangle & \text{otherwise.} \end{cases} \quad (5)$$

Proposition 1. *The operators $Z_1, \ldots, Z_4$ are well-defined.*

Proof. The fact that $\mu_{Z_i}(x) \geq 0$ and $\nu_{Z_i}(x) \geq 0$ ($i = 1, 2, 3, 4$) is obvious.

If x is not an IP for A, then it follows that

$$\max(\mu_{Z_i}(x), \nu_{Z_i}(x)) \leq \mu_{Z_i}(x) + \nu_{Z_i}(x) = \mu_A(x) + \nu_A(x) \leq 1$$

since A is an IFS.

Let x be an IP for A. Then, using (1), we obtain:

$$\mu_A(x) < \frac{\mu_A(x)}{1 - \nu_A(x)} < 1 \quad (6)$$

and

$$\nu_A(x) < \frac{\nu_A(x)}{1 - \mu_A(x)} < 1 \quad (7)$$

and hence, for $i = 1, 2, 3$ from (2), (3), (4), (6) and (7), we have:

$$\max(\mu_{Z_i}(x), \nu_{Z_i}(x)) < \mu_{Z_i}(x) + \nu_{Z_i}(x) = \frac{1}{3}\left(1 + \frac{\mu_A(x)}{1 - \nu_A(x)} + \frac{\nu_A(x)}{1 - \mu_A(x)}\right) < 1.$$

For $i = 4$,

$$\max(\mu_{Z_4}(x), \nu_{Z_4}(x)) < \mu_{Z_4}(x) + \nu_{Z_4}(x) = \frac{2}{3} < 1.$$

This completes the proof. □

Theorem 1. *For any IFS A we have:*

$$Z_1(A) \subseteq Z_2(A) \subseteq Z_3(A).$$

Proof. If x is not an IP for A, we have

$$\mu_{Z_1}(x) = \mu_{Z_2}(x) = \mu_{Z_3}(x) = \mu_A(x),$$
$$\nu_{Z_1}(x) = \nu_{Z_2}(x) = \nu_{Z_3}(x) = \nu_A(x).$$

and the the result is clear. Hence, we need to consider only the IPs.

From (1), we obtain:

$$\mu_{Z_1}(x) - \mu_{Z_2}(x) = \frac{1}{3}\mu_A(x)\left(\frac{\mu_A(x) + \nu_A(x) - 1}{\mu_A(x) + \nu_A(x)}\right) < 0,$$
$$\mu_{Z_2}(x) - \mu_{Z_3}(x) = \frac{1}{3}\nu_A(x)\left(\frac{\mu_A(x) + \nu_A(x) - 1}{\mu_A(x) + \nu_A(x)}\right) < 0.$$

With the same reasoning we obtain:

$$\nu_{Z_2}(x) - \nu_{Z_1}(x) = \frac{1}{3}\mu_A(x)\left(\frac{\mu_A(x) + \nu_A(x) - 1}{\mu_A(x) + \nu_A(x)}\right) < 0,$$
$$\mu_{Z_3}(x) - \mu_{Z_2}(x) = \frac{1}{3}\nu_A(x)\left(\frac{\mu_A(x) + \nu_A(x) - 1}{\mu_A(x) + \nu_A(x)}\right) < 0.$$

Hence, for any point x we have $\mu_{Z_1}(x) \leq \mu_{Z_2}(x) \leq \mu_{Z_3}(x)$ and $\nu_{Z_1}(x) \geq \nu_{Z_2}(x) \geq \nu_{Z_3}(x)$. □

We can construct another two operators in the following manner.

Definition 7. *Let an IFS A be given. Let $\alpha \in (0,1)$. Then we can define the operator*

$$Z_{12}(A) = \langle x, \alpha\mu_{Z_1}(x) + (1-\alpha)\mu_{Z_2}(x), \alpha\nu_{Z_1}(x) + (1-\alpha)\nu_{Z_2}(x)\rangle. \tag{8}$$

Definition 8. *Let an IFS A be given. Let $\alpha \in (0,1)$. Then we can define the operator*

$$Z_{23}(A) = \langle x, \alpha\mu_{Z_2}(x) + (1-\alpha)\mu_{Z_3}(x), \alpha\nu_{Z_2}(x) + (1-\alpha)\nu_{Z_3}(x)\rangle. \tag{9}$$

Theorem 2. *For any IFS A we have:*

$$Z_1(A) \subseteq Z_{12}(A) \subseteq Z_2(A) \subseteq Z_{23}(A) \subseteq Z_3(A).$$

Proof. We will only consider the first inclusion as the remaining inequalities are verified in the same manner. From (8) we obtain:

$$\begin{aligned}\mu_{Z_{12}}(x) - \mu_{Z_1}(x) &= (1-\alpha)(\mu_{Z_2}(x) - \mu_{Z_1}(x)),\\ \nu_{Z_{12}}(x) - \nu_{Z_1}(x) &= (1-\alpha)(\nu_{Z_2}(x) - \nu_{Z_1}(x)).\end{aligned}$$

Due to Theorem 1 and the fact that $1-\alpha > 0$, we conclude that:

$$\begin{aligned}\mu_{Z_{12}}(x) - \mu_{Z_1}(x) &= \underbrace{(1-\alpha)}_{>0}\underbrace{(\mu_{Z_2}(x) - \mu_{Z_1}(x))}_{\geq 0} \geq 0,\\ \nu_{Z_{12}}(x) - \nu_{Z_1}(x) &= \underbrace{(1-\alpha)}_{>0}\underbrace{(\nu_{Z_2}(x) - \nu_{Z_1}(x))}_{\leq 0} \leq 0.\end{aligned}$$ □

We will end this section with some final remarks. As we have demonstrated, five of the proposed operators exhibit a nice inclusion property. This means, for instance, that if we consider the result of each of these operators as a way to rank an intuitionistic fuzzy alternative $\langle x, \mu_A(x), \nu_A(x)\rangle$, Z_1 will present the least favorable evaluation, while Z_3 will give it the highest score. If we have a second alternative $\langle x, \mu_B(x), \nu_B(x)\rangle$, and $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$, these evaluations will preserve that order. However, if either $\mu_A(x) < \mu_B(x)$, $\nu_A(x) < \nu_B(x)$, or $\mu_A(x) > \mu_B(x)$, $\nu_A(x) > \nu_B(x)$, it is unclear whether these operators may be considered effective for ranking the two alternatives. In the general case, there are several approaches to ranking intuitionistic fuzzy alternatives and we refer the interested reader to the following papers (and the references therein) for some insights on the theoretical challenges and the practical advantages and disadvantages of the methods that are generally used [29, 45, 54–56].

4 Conclusion

We introduced some new geometrically motivated operators over IFS and established some properties of theirs. In future works we will consider operators similar to the proposed above but depending on more parameters and study their properties.

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